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LOST IN TRANSLATION: THE 'BEAN SNARE' AS A CASE OF THE SITUATED–SYMBOLIC DIVIDE

Rozy Brar, Andrew J. Galpern, and Dor Abrahamson
University of California, Berkeley
{rozy, galpern, dor}@berkeley.edu
http://edrl.berkeley.edu

The NCTM Standards (2000) recommend that instructional programs enable all students to create, use, and translate between mathematical representations. Such advocacy for developing and supporting students' representational fluency builds on the work of scholars who argued that knowledge is grounded in interaction and that learning environments should therefore foster opportunities for such interaction (Montessori, 1917; Dewey, 1937; Bruner, Goodnow, & Austin, 1956; Piaget, 1960). However, while several studies have demonstrated that the use of various kinds of representations can facilitate children's understanding of mathematical concepts (e.g., Resnick & Omanson, 1987), other studies have found no consistent advantage to the use of concrete representations (Sowell, 1989) and have demonstrated that children sometimes fail to transfer what they have learned using concrete representations to mathematics notation (Martin & Schwartz, 2005).

We propose to illuminate one issue of this debate by demonstrating how critical shades of meaning are liable to be lost in re-representing situations in the form of mathematical symbols. Also, we explore the pedagogical value of having education practitioners engage in analysis of such semiotic breakdown generating apparent paradoxes of translation.

Methodology

The *Bean Snare* (see Figure 1, across) is a "kick-off" activity (Edelson, Pea, & Gomez, 1996), which *our* third author designed to spark discussion among graduate students or pre/in-service teachers around issues of mathematics education. Note how the presentation intentionally leads us down the garden path toward an incorrect mathematical statement—it is "mathematical sleight of hand." Discussing such an activity may sensitize participants to the complexity of constructivist design, teaching, and learning, i.e., to the subtle interactions of content, context, and the multi-media, multi-modal, and multi-representational aspects of collaborative reasoning about a situated mathematical problem. Thus, the activity may train participants to rigorously think about their own thinking about mathematics as a frame of mind that is potentially conducive to effective design, research, and teaching. Participant graduate students came from diverse backgrounds in education, including multi-

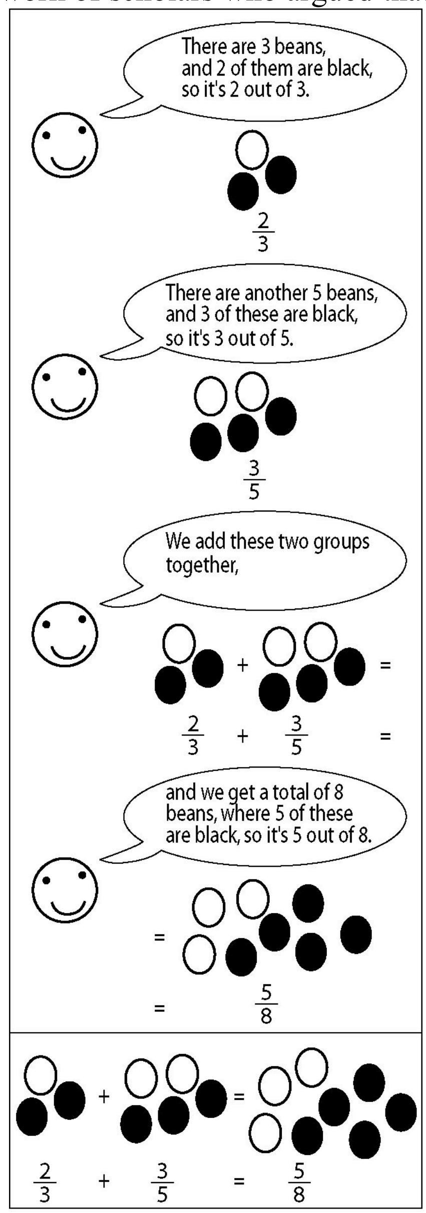

Figure 1. Phases in creating the 'Bean Snare' and a view of the completed picture.

media design, classroom teaching, and tutoring. The research group worked on the problem for a total of roughly 12 hours, including 8 hours of vigorous debate in 4 weekly meetings that spanned three weeks, interspersed by very detailed after-hours conversations and daily e-mail exchanges.

Results

Group members attempted to break down to microgenetic scale their personal interactions with the problem, e.g., “As I write ‘2/3,’ I am referring to the beans, but then my gaze shifts away from the icons, and so I lose crucial aspects of the context.” In hindsight, discussion was characterized by group members spontaneously assuming three different personas—‘teacher,’ ‘grade-school student,’ and ‘researcher’—and then explicitly attempting to coordinate perspectives emerging from these three positions toward a complete understanding of the complex educational situation. Wearing the ‘teacher’ hat, we hurried to *diagnose* the symptoms, e.g., “That's not how you add fractions!” or *remediate*, e.g., “Let me show you an area model that will preclude all these problems.” As ‘students,’ we said in earnest, “But I don't see what I did wrong.” As ‘researchers,’ though, we attempted to suspend our urge to diagnose or remediate and, instead, to dwell within the problem space.

What was lost in the translation between the situation (combining groups of beans) and the standard mathematics notation (adding fractions), we realized, is a crucial fragment of the contextual meaning implicit within the ostensive statement (“…and 2 *of them* are black”) and deictic gesture (indicating the *whole group*). Namely, in using fraction notation to inscribe the activity of combining beans, the multiplicand (the size of each group of beans) is omitted. This omission may pass unnoticed, because by default the multiplicand of a fraction-as-operator is assumed to be 1, which is why fractions may stand alone, e.g., “2/3” (and not “2/3 * 1”). In the Bean Snare, however, not only is the multiplicand unequal to 1, but it also changes from group to group. If we include the multiplicands in the notation we have $2/3 * \mathit{3} + 3/5 * \mathit{5} = 5/8 * \mathit{8}$, that is $2 + 3 = 5$, hence arriving at a mathematically true statement. The ambiguity of signs (icons, symbols, and numerals), the ambiguity of rational numbers (‘2/3’ as fraction or ratio), the ambiguity of language (2/3 “of these” as opposed to 2/3 “of this group of beans” which might create a clearer lexical placeholder for the missing multiplicand), and the crucial role of gestures and perception all combine to undermine the translation between a situation and its notation.

In summary, the The Bean Snare supported discussion of perception and ambiguity, the nature of mathematical reasoning and learning, challenges of pedagogy, didactics, and design, and issues of policy making around the “math wars.” Documenting this interaction constructed a progressively richer shared understanding of core aspects of mathematics education, honed a set of theoretical and empirical questions, and helped in shaping an agenda for our research group’s future studies. For instance, we will interview in-service teachers, using the bean snare, to investigate potential tension between their pedagogical beliefs (e.g.,. constructivist) and their formative-assessment practices.

Implications for Math Education in Line With the PME-NA Goals and Conference Theme

Situated mathematics can help ground mathematical meaning. Yet, if designers and/or teachers fail to recognize potential pitfalls inherent in mathematization, then the concrete contexts may constitute a disservice. The Bean Snare is a case of an activity that can generate insight into the intricacies of situated-mathematics learning. By *focusing on their own learning*, practitioners ultimately *focus on students*, by way of improving design and teaching.

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