Kinematics: Kinetically Induced Mathematical Learning—Overview of Rationale

Dor Abrahamson & Mark Howison
Embodied Design Research Laboratory
University of California, Berkeley

We are investigating conjectures central to research on embodied mathematical cognition. Specifically, we are carrying out an empirical study designed to evaluate the plausibility of theoretical models that argue for an embodied basis of mathematical learning and reasoning. Prior arguments for embodied mathematics either build on theoretical analyses of human reasoning (Barsalou, 1999; Goldin, 1987; Lakoff & Núñez, 2000), empirical studies of human’s activity in general (Barsalou, 2008; Hatano, Miyake, & Binks, 1977), interpretations specifically of mathematics student behaviors (Fuson & Abrahamson, 2005; Nemirovsky, Tierney, & Wright, 1998), or, in particular, evidence of gestures accompanying speech acts in the solution of mathematical problems (Alibali, Bassok, Olseth, Syc, & Goldin-Meadow, 1999; Edwards, Radford, & Arzarello, 2009). Whereas these studies furnish strong support for the potential viability of the embodied conjecture, and whereas they have demonstrated the plausibility of an embodied substrate for working memory (in-the-moment cognitive operations), they have not established conclusively a *sine qua non* role of multi-modal imagery in the ontological development of mathematical concepts. Namely, it has yet to be shown compelling that imagery plays more than a supportive or epiphenomenal role in the communication of essentially abstract concepts.²

To the extent that the embodied conjecture obtains, a central challenge in evaluating the roles of imagery in mathematical reasoning has been that these psychological constructs are currently inaccessible for measurement. That is, because we cannot see people’s “pictures in the head,” we instead rely on indirect means of investigating these images. For instance, to examine the nature of procedural fluency deeply rooted in object-based visual–kinesthetic experiences, Hatano et al. (1977) had expert abacus users solve arithmetic problems in the absence of an actual abacus yet while performing interfering finger-tapping tasks (all but master users’ performance at “mental abacus” degraded). Such experimental designs are well geared for studying mathematical imagery, because the researcher can confidently identify the experiential basis of the mental operations, and especially because the experience is based upon interaction with a known object. However, cross-sectional studies, such as the above, do not examine individual learning trajectories. Abrahamson (2004) introduced new mathematical objects into a middle-school environment...
classroom and examined students’ gestural acts as a window onto the imagery underlying their mathematical learning. Yet, it is difficult to appraise whether these gestures, performed in authentic classroom discussions, were pivotal to the students’ reasoning or only epiphenomenal manifestations inherent to discursive norms. Therefore, a study is needed that both monitors individual object-based learning and does so through an experimental design that confidently implicates this object as the source of elicited imagery. Thus, rather than seeking out an existing mediating physical artifact, such as the abacus, or waiting for spontaneous gestures to emerge, such as in mathematical argumentation, we take a more proactive approach in this study by directly inducing an unfamiliar image then asking participants to begin reasoning with it mathematically. Prior research has not examined the initial construction of imagery from multimodal embodied experiences nor how it begins to support mathematical reasoning.

Our research begins with the premise that mathematical learning is indeed image-based and mathematical reasoning is image-simulated (Case & Okamoto, 1996; Dahaene, 1997; Freudenthal, 1986; Lakoff & Núñez, 2000; Martin, 2008; Nemirovsky & Borba, 2004; Nemirovsky & Ferrara, in press; Nemirovsky et al., 1998; Pirie & Kieren, 1994; Presmeg, 2006). We further assume that people who demonstrate difficulty in understanding a particular mathematical concept nevertheless possess the cognitive wherewithal for leveraging imagery toward mathematical understanding, and conjecture that what these people lack is instead the initial, requisite image for that concept. Implicit in our approach is the view that teaching is a means of creating opportunities for students to develop a family of images that become resources for reasoning about a class of mathematical situations. Our conjecture is that some mathematical concepts may be difficult to learn precisely because our everyday experience fails to provide adequate imagery for those specific concepts. To evaluate this conjecture, we are conducting a study in which we first provide students with a “ready-made” visual–kinesthetic basis for developing imagery pertaining to a difficult mathematical concept and then measure the effects of such provision on their understanding of the targeted mathematical concept. Thus, our approach—which we describe as “image-before-concept”—is to kindle and investigate the incipient processes by which multi-modal imagery is developed, then to monitor how that imagery is subsequently leveraged as a cognitive resource for constructing and applying mathematical knowledge. So doing, we are also problematizing claims that students’ “everyday sensual experience” (Botzer & Yerushalmi, 2008) is a critical developmental resource for embodied knowing, as the experience we provide our participants is extra-ordinary, contrived, and perhaps necessarily so due to the scarcity or even absence of its everyday embodied manifestation.

The specific content we target is the notion of a proportional progression, e.g., a sequence of equivalent fractions whose numerators and denominators respectively increase by fixed amounts at each count (i.e. the sequence 2/3, 4/6, 6/9, etc.). Our work builds on a pilot study conducted by Fuson and Abrahamson (2005), which demonstrated that students’ conceptual difficulties in understanding such progressions coincided with their physical inability to act out the progressions with their hands (e.g., one hand continuously “grows” by 2 units every second while the other hand simultaneously grows by 3). Many students maintained a constant distance between their moving hands, acting out an additive rather than multiplicative relationship—a physical performance error that is reflected in mathematical errors reported throughout the literature on children’s development of rational number concepts (e.g., De Bock, 2002; Gelman
We conjecture that students’ difficulties with proportionality, and in particular their additive “same difference” errors, stem from their lack of a suitable dynamic image in which to ground their understanding of proportionality. Furthermore, it is unlikely that students construct the relevant imagery spontaneously because of the challenge of ambidextrously executing the physical actions themselves. Therefore, we decided to create an opportunity for study participants to undergo this ambidextrous multi-modal embodied experience—initially experiencing the physical concept passively and then gradually assuming physical command of the motion—thus allowing us to subsequently monitor any personal construction of meaning for this physical action.

The initial product of our experimental paradigm is a gesturoid—an embodied artifact for mathematical inquiry—that enables us to study how meaning for a mathematical notion emerges from, or vis-à-vis, an embodied sense of that notion. We refer to our artifact as a gesturoid because it has gesture-like properties yet is initially devoid of representational intentionality: it is instigated as a passive, physically-induced experience that has no function beyond the immediate discursive context, yet a participant may then assume agency in performing this physical action and it may acquire meaning within a familiar system of practice. One difference between a gesturoid and a physical manipulatable artifact such as the abacus is that the gesturoid does not admit straight-forward indexical semiotic acts. That is, pointing to parts of the gesturoid is difficult because it is moving and ephemeral, and anyway both hands are involved in performing this particular gesturoid so that neither hand it available for deixis. In contrast, an abacus functions almost solely on simple dislocational–indexical operations, configuring beads to indicate number, which effect spatial encodings that are cognitively ergonomic for performing arithmetic. A crucial research question is,

- What, if any, is the relation between a capacity to perform a physical action and the construction of the mathematical meaning it may be taken to represent?

Moreover, because the gesturoid is inherently and a priori embodied, it throws into question the learning affordances of various instructional media. In particular,

- Is the body, as a medium, more conducive for constructing a multi-modal mathematical image as compared to artifacts embedded in other media, such as a video or textbook?

We hypothesize that just because a physical activity is designed to embody a mathematical notion, we cannot assume that performing this activity necessarily implies an understanding of the notion, just as walking or breathing do not in-and-of-themselves bespeak rhyme, rate, or reason. This is an alternative a naïve strong claim that performance indicates understanding, or that doing is knowing. However, we believe that, at the very least, practicing the physical activity creates a context for constructing mathematical meaning through the guided re-inventing of the mathematical notion and systematic coordination with normative mathematical semiotic means, such as diagrams and symbolical notation. More pointedly, the gesturoid is first conducive for taking on meaning when it has become a gesture, gesticulation, or—semantics aside—an intentional choreographed coordination whose systematicity can be articulated.
How does our gesturoid activity relate to extant mathematical conceptualizations of proportionality, as seen in standard curricular units? There are quite a number of instructional designs for the topic of proportionality (Lamon, 2007). Whereas these range along many superficial parameters, they differ with respect to how proportional relations are modeled and the consequence of this modeling for the possibility of an embodied experience of proportion. In particular, these models differ with respect to their construction of the equivalence between the two ratios involved in the proportion—whether this equivalence is rooted in an acknowledgement of a constant process (e.g., same pair of quantities added over and over) or in identical sensory perceptions of the product (e.g., same sweetness).

Vergnaud (1983) differentiates between two types of situation X:Y ratios. He names isomorphism of measure cases in which X and Y are functionally related but phenomenologically separate, such that there is no immediate meaning to a ‘mix’ of the two measures. Such is the case of a 3:5 ratio between the number of boys and girls in a classroom. In this classical “recipe” model—e.g., 3 units of X per 5 units of Y—two separate pools of objects increase as the consequence of iterated adding of fixed quotas, yielding 3 & 5, then 6 & 10, then 9 & 15, etc. Proportional equivalence between two ratios is then defined as co-membership in a class of ratios, e.g., the 3 & 5 class. This co-membership or equivalence can be determined either directly, through iterated adding/subtracting of the base ratio or through a multiplication shortcut, or through common kinship with a base ratio, e.g., 6:10 and 9:15 are related transitively through their respective relations with 3 & 5. An isomorphism-of-measure introduction to proportional equivalence would thus lean on an essentially algorithmic criterion relating to situations of iterated adding or scaling up of a recipe. Note, in particular, that there is no immediate perceptual basis for evaluating the proportionality of two ratios of this type, e.g., in comparing 6:10 and 9:15.

Vergnaud (1983) also speaks of product of measures, e.g., sweetness, area, slope, or chance—cases in which the multiplicative relation of two measures is instantiated in terms of an ontologically distinct phenomenon. For situations embodying products of measures, humans may have a sensory basis for evaluating proportionality, even before performing measurement and calculation. For example, if two liquids of different volume have the same sweetness, we can directly experience equivalence per se—though not yet proportional equivalence—through our gustatory sense. That is, a liquid solution of 6 units of sugar and 10 units of water would have the identical flavor as a respective mix of 9 and 15. Later, one might attend to these volume measures and re-articulate these pairs of numerical measures as ratios between indicated quantities of sugar and water.

Our ambidextrous activity might be viewed as a hybrid of these previous meta-designs for proportionality, carrying elements both of isomorphism and product of measures. Indeed, the activity a priori recruits an embodied sense of continuity for each of two individual progressions (the respective motions of the right hand and the left hand) rather than focusing on the ratio between the quantities, let alone the equivalence of two ratios. Thus, within each such continuity, we might experience constant products of measure, velocity being expressed as the ratio of distance and time. However, the two moving hands are a priori spatially and ontologically separate entities—a case of isomorphism of measure. Moreover, whereas the hand motion is inherently continuous, these two motions can be discretized and rendered as a recipe, e.g., rising 3 inches here for every 5 inches there, over and over. In fact, we expect that the process of
discretizing the motion will be imperative for describing and reasoning with the gesturoid—this process would hence provide a context for re-describing the gesturoid symbolically.

We have presented here an artifact for mathematical investigation that is novel along at least two dimensions that, in turn, correspond to two study phases. First, the artifact induces and makes available for simulation a kinesthetic image that is potentially conducive for mathematical reasoning about proportional progression. Second, the artifact invites an instructional design for proportional progression that combines aspects of previous designs, providing both an isomorphism of measure and a perceptual basis for evaluating proportionality. With this artifact “in hand,” we aim to test the claim that embodied imagery is a prerequisite for certain conceptual mathematical understandings, while also demonstrating a new design for proportionality that draws on such imagery. Our future publications will present the supplementary technological devices we are creating for this project as well as our rationale, experimental design, and instruments for measuring and interpreting any learning gains that may be fostered by articulating the gesturoid.

References


3 See http://edrl.berkeley.edu/projects/kinemathics/MIT.mov (and thank you, Becky Blessing!)


