The Specifications Game: Developing Mathematical Practices of Defining

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Objective: Foster, Investigate, and Model the Mathematical Practices of Defining

Defining, a central practice in mathematics, is different from its daily practice. Whereas students may know many mathematical definitions, these are often presented as fact. Consequently, students often have trouble formulating definitions, interpreting new definitions, and using definitions in proof and problem solving (Vinner, 1991). The rationale of the design project is that students may spontaneously engage in generating mathematical definitions as their pragmatic solution to a problem involving specificity of information, such as in responding to a problem of ambiguity.

Background

Commognitive Interpretive Framework. Mathematics learning can be modeled as the process of internalizing mathematical discourse (Sfard, 2007). The Specifications Game (SG) is designed to create an engaging context that generates the type of mathematical discourse often involved with the formulation of definitions.

Developing Mathematical Definitions. Mathematical definitions, I submit, are not inherent truisms but rather meaning-relations built to fulfill particular purposes, often via an iterative process of formulating and evaluating a definition with respect to its utility, conformity to intuition, or other criteria. A definition, I further submit, could be viewed as an *epistemic form* (a target structure guiding inquiry), the end product of an *epistemic game* (Collins & Ferguson, 1993). SG was designed to prompt students to reinvent one such epistemic game. SG draws on familiar cultural practices, sets up mathematical language as a potential solution to ambiguity, distributes the process over two participants and a set of materials, and encourages reflection, mathematization, and the practice of defining.

Design: The Specifications Game

In SG, two participants take on the role of 'specifier' and 'checker', respectively. The specifier is provided with a game board out of which the 'goal shape' has been cut out (Figure 1, left). The checker is provided with the goal shape, as well as a set of shapes that share some properties with the goal shape (Figure 1, right) and some post-it notes on which to draw additional shapes. Both players have visual access to the game board, but only the checker has visual access to the set of game pieces. During each turn, the specifier provides one specification describing his shape. The specifications may not include shape names nor refer to shapes that were previously used. During the checker's turn, the checker provides the specifier with a shape that satisfies all specifications provided thus far. This shape may be selected from the set of game pieces or may be drawn on paper. If the shape does not fit the specifier's board precisely, it is returned to the checker to be used again.

Figure 1: A game board (left) and associated game pieces (right).

The specifier's goal is to obtain the goal shape in as few turns as possible. The checker's goal is to maximize the number of turns played. To this end, the checker tries to identify and provide shapes that, while satisfying all specifications given, are not the goal shape. When the specifier obtains the goal shape, the game ends.

Four dyads (ages 11 and 15, as well as pre-service teachers) participated in a pilot implementation of SG. Each dyad played the game at least twice, alternating roles. Next, I prompted them to: (a) examine and modify their list of requisite properties into a conjectured definition; (b) compare and evaluate alternative definitions for the same object; and (c) identify or generate definitions for unrelated mathematical concepts. All sessions were videotaped and then carefully micro-analyzed.

Conclusions

The practice of defining is developed via apprenticing into the field of mathematics. In this process, students move away from a view of mathematical definitions as arbitrary systems of constraints couched in non-normative language and disconnected from intuitive ways of thinking about the world, towards a view of definitions as epistemic forms satisfying specific properties that allow them to fulfill particular purposes. By distributing the defining process over two participants and a set of materials, SG supports students in enacting an epistemic game of formulating definitions. In this process and the subsequent guided reflection and mathematization, students become sensitized to properties of definitions such as specificity and minimality, resulting in a discursive shift away from mundane practices of defining.

References

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