Dor Abrahamson – Promotion Review Fall 2010
Essay on Research, Professional Activity, Teaching, and Service

Research and Creative Work

The following text offers overviews of the two dominant strands in my educational research on mathematical cognition and instruction: *learning theory* and *design theory*. An introductory section will first explain how these two research strands are related dialectically within the design-based research approach. Following, each of the strands will be treated separately. These treatments will highlight the seven publications that I submitted for external review. I shall also outline several additional publications, some of them still under review, so as to enable the reader to build a sense of the trajectory of my research beyond the publications already in print. The text is, for the main, descriptive, that is, I avoided excessive citations in an attempt to preserve coherence. The reader is referred to the publications themselves for the resources informing the assertions and conjectures laid forth in what follows.

1. Theory and Practice of Math Education as Mutually Informing Research Domains

1.1 Overview and general approach. I study phenomena at the interdisciplinary domain intersection of cognitive science, pedagogical theory, and interaction design. Focused on the mathematics discipline, I research and theorize learning processes that take place in social contexts, and in particular how students build personal meaning for mathematical concepts through participating in guided discovery-based problem-solving activities involving carefully designed pedagogical objects. So doing, I have attempted to contribute to the field’s understanding of:

- the roles of symbolical artifacts in mediating mathematical concepts;
- the nature of students’ situated quantitative intuitions;
- how these intuitions differ from formal mathematical analyses of situations;
- how these intuitive-vs.-formal differences underlie covert communication breakdown in instructional contexts; and
- how participants to the educational process—students, instructors, and designers—ultimately achieve conceptual change as negotiated syntheses of intuitive and formal views of situations.

I also develop frameworks for instructional design consistent with my theoretical models of mathematical learning. In particular, I engineer and research instructional activities that incorporate mixed-media materials (both traditional and computer-based) to support students’ meaningful appropriation of mathematical concepts. For the main, I work in the design-based research approach, which I shall now explain.

1.2 Design-based research. My dual scholarly foci on the development of both theoretical and practical knowledge serving mathematics education is characteristic of "design-based research," a young investigative paradigm inaugurated several decades ago with the incipience of the learning-sciences disciplinary field and now considered one of its major methodological approaches. In its inaugural years, learning-sciences leaders suggested the practice of instructional design as a promising context for articulating theoretical models of learning. Working in the design-based research approach, scholars of cognition and instruction in the disciplines operate as theoretically informed designers and design-informed theoreticians. Namely, the practice of design-based researchers of mathematics education alternates between building design and building theory, as follows. Design-based researchers:

- set off from conjectures respecting untapped educational potential, such as alleged cognitive mechanisms that students rarely engage in studying some particular targeted subject matter; They then
- engineer pedagogical artifacts and activities to create opportunities for evaluating these conjectures; Next, they
- implement these designs in empirical settings, documenting the sessions for subsequent analysis of student behavior; Finally, they
- develop and iteratively modify theoretical models in light of patterns discerned in these data.

Thus, as in Escher's *Drawing Hands*, the development of design and theory co-inscribe.

1.3 Methodology and theory. In accord with my focus on the process of learning, I use by-and-large qualitative analytic methods to produce nuanced descriptions of conceptual change students undergo through engaging in discourse-embedded manipulation, representation, and explication tasks facilitated by an instructor. My analyses draw on a range of perspectives primarily from two learning-sciences legacies traditionally viewed as incompatible, namely theories emphasizing either students' agency or sociocultural forces shaping conceptual change. Through combined-perspective analyses of my empirical data, I evaluate apparent differences between these perspectives by untangling their implicit epistemological assumptions, demonstrating both assumptions as incomplete, and offering alternatives that reposition the theories as potentially synergistic. This essentially theoretical work both draws on and reflexively informs my empirical studies, as follows.

My empirical design-based research studies are driven by theoretically framed conjectures with respect to dimensions of instructional processes whose modification could potentially enhance the quality of students' mathematical learning. To evaluate these conjectures, I require data of student behavior in instructional studies aligned with the conjectures. And yet, design-based research is, in a sense, the study of the possible, not of the existing. That is, our conjectures are with respect to hypothetical situations that do not in fact exist, so that I am required to engineer and build experimental instructional materials as a condition for creating the requisite empirical settings wherein I can gather appropriate data. Consequentially, a major facet of my practice is the invention and research-based development of physical and virtual pedagogical artifacts, some of which have ultimately been incorporated in curricula that have enjoyed nation-wide adoption (my dissertation ratio-and-proportion unit appears in Karen Fuson's "Math Expressions"; my probability mathematical object, the "combination tower," is integrated into recent versions of Cliff Konold's TinkerPlots computational modules). My empirical instructional studies thus serve to evaluate and modify both the conjectures that drive the studies and the objects and activities themselves. As a result, I use the empirical data also as a context to develop a principled heuristic framework giving rise to a design theory for mathematics education informed by and informing my emerging theoretical models of mathematics learning.

1.4 Design Rationale. In empirical instructional contexts shaped by my emerging design framework, students participate in inquiry activities designed to engage targeted cognitive mechanisms, whose innate capacity has been demonstrated by cognitive-developmental scholars conducting laboratory studies (e.g., evidence of infants' "statistical" intuitions). In my studies, students are asked to draw intuitive inferences regarding quantitative properties of a problem situation. These situations have been engineered explicitly so as to accommodate the targeted cognitive mechanisms, thus enabling the students to engage perceptual judgment and draw intuitive inferences that are mathematically-sound qualitative approximations of these quantitative properties. Students are then guided to model the same situations mathematically by performing analyses, even while they do not initially understand the logic or utility of these algorithmic operations and constructions. In particular, the students do not understand why the analyses attend to features or dimensions of the situations ostensibly irrelevant to the inquiry as stated. As a result of this process, two "objects" become juxtaposed for the student in the problem space: a realistic situation and its purported mathematical model. The *educational* challenge of meaningful learning is thus honed into the objective that students understand how the mathematical analysis, articulated in linguistic and inscriptional conventions, agrees with and empowers their earlier holistic inference. The educational *research* challenge is thus analogously honed, in that these paradigmatic "nature vs. nurture" empirical moments are auspicious for juxtaposing and reconciling theoretical models of human learning (see above) that typically interpret learning processes by emphasizing either "bottom up" ontogenetic microgenesis or "top down" social enculturation.

The informed reader will recognize, in the above rationale, seminal arguments from Michelle Artigue, M. Alessandra Mariotti, Pierre Rabardel, Luis Radford, Anna Sfard, and other leading Vygotskian interpreteurs. I hope to be able to foster dialogue between those scholars and, complementarily, other researchers interested in humans' innate, evolved cognitive capacity to perceive the world in particular ways, for example, Rochel Gelman, Gerd Gigerenzer, Elizabeth Spelke, and Fey Xu. Through appreciating that some situated phenomenal properties are perceptually privileged, I believe, educators and educational researchers can be better equipped to understand and cater to students' challenges as they are guided to adopt disciplinary views inherent to mathematical practice. I attempt to actuate this belief in the form of instructional activities in which students are guided to negotiate immediate and mediated perceptions of situations whose quantitative properties are under inquiry, and I attempt to inscribe this belief in the form of emerging theoretical models that encompass both parties to the dialogue (what diSessa calls the ‘dialectical’ approach). As I explain below, my quest for dialectical theory and design informs my selection of particular research problems.

2. Theory Development: Investigating Cognitive Processes

My research takes on a persisting theoretical problem in the learning sciences concerning how students make sense of mathematical solution procedures. I cast my research on this theoretical problem within the zeitgeist of reform-oriented mathematics education, wherein students’ sense-making is viewed as a desirable pedagogical goal. This section overviews findings appearing in publications that emerged from analyzing implementations of four different designs: ratio and proportion (Fuson & Abrahamson, 2005), statistics (Abrahamson & Wilensky, 2007), probability (Abrahamson, 2009a, 2009b, 2009c; Abrahamson, Gutiérrez, & Baddorf, in press), and again ratio and proportion (Reinholz, Trninic, Howison, & Abrahamson, in press).

2.1 Body-based conceptual integration in instruction and learning. In our 2005 chapter, “Understanding ratio and proportion as an example of the Apprehending Zone and Conceptual-Phase problem-solving models,” In J. Campbell (Ed.), *Handbook of mathematical cognition*, we (Fuson and Abrahamson) present theoretical models that emerged from the analysis of two 3-week implementations of my dissertation design for ratio and proportion in urban/suburban 5^{th}-grade classrooms. Specifically, my theoretical model of mathematical learning, The Apprehending Zone, highlights the tacit yet pivotal role of the teacher and students’ body-based discursive activity, with and about mathematical objects, in students’ integration of fragments of quantitative narratives into new conceptual composites. For example, two “multiplication stories” about independent agents each accumulating some substance at fixed, yet different, daily rates become a single “proportion story” about these agents’ coordinated actions (e.g., 3 acorns per day for Chip and, separately, 5 acorns per day for Dale).

At the center of this design were innovative mathematical objects as well as familiar objects that were put to new uses. In particular, I utilized students’ fluency with the multiplication table as a means of revisiting the arithmetic operation of multiplication as the recursive iteration of fixed units (“repeated addition”) down the multiplication-table columns; and as a means of thus grounding proportion modeled as recursive *twinned* iterations of different fixed units (e.g., 3 and 5, 3 and 5, etc.) stepping “hand in hand” down the columns, with the left-most 1-Column keeping track of the iterations. Using our designed solutions procedures, which were grounded in and emerged from the multiplication-table approach, our study students out-performed older students on comparable post-test items drawn from previous research and national assessments.

This publication built on the field’s rising interest in the implicit roles of gesture in the communication of multimodal action plans. The presentation-and-analysis format utilized in this study—the “transcliptions,” consisting of snapshot photograph sequences with “thought bubble” diagrammatic overlays of students’ imagined mathematical forms (see chapter)—constituted an innovation and drew some attention of leading discourse scholars (e.g., Frederick Erickson of UCLA). In particular, this presentation format enabled me to reify, monitor, and analyze students' individual and collective 3-week-long gradual schematization, rehearsal, and implicit gesture-based exchange of problem-solving templates that originated in inscriptional actions upon shared material pedagogical objects yet emerged as new epistemic forms that shifted the classroom's concept-specific discursive practices. These "transparent" incorporeal processes, I submit, are important for conceptual learning, and so it is important that researchers and educators alike sensitize to these embodied actions.

Through presenting this work, I became involved in the growing sub-field of embodied mathematical cognition. For example, I presented my studies at Susan Goldin-Meadow's University of Chicago laboratory and at a conference of the burgeoning International Gesture Society, I conferred with Rogers Hall and Ricardo Nemirovsky, and was invited to submit to a JLS special issue on embodied cognition. It has been a great joy and honor to be part of a very exciting intellectual endeavor that is trying hard to rigorously and graciously elbow itself into mainstream thought. Below, I edge further into the types of formalizations I am seeking as I attempt to build confluent cognitive–sociocultural views informed by the rising paradigm of embodied mathematical reasoning.

2.2 Mathematical learning at the interface of intuitive and formal views of situations. My fundamental epistemological stance, theoretical disposition, and concomitant methodological approach is to operationalize mathematical concepts as cultural analyses of concrete situations that are *a priori* meaningful to students, even if these meanings are personal, naïve, intuitive, unschooled, etc. Thus, on the one hand, situations can have *phenomenological immediacy* for the students, contingent on particular framing tasks, such as manipulation, representation, and judgment, that orient students' perceptions of the situations. Yet, on the other hand, mathematical analyses of situations are embodied in disciplinary inscriptions, such as diagrams, graphs, tables, models, and symbolical notation. These two "objects"—the situation, the analysis—are co-present during instruction. In particular, mathematical structures created through professional treatment of a situation are suggested to students as embodying the mathematical solution concerning the situation's quantitative properties under inquiry. And yet deciphering these structures is contingent on the individual's fluency with the sign system mediating the information—the inference is *semiotically mediated*. From the perspective of my empirical contexts, I essentialize my thematic research problem respecting mathematics learning by asking how students engaged in problem-solving activities involving the analysis of a realistic situation coordinate phenomenologically immediate and semiotically mediated inferences respecting the situation's quantitative properties.

In my *Cognition and Instruction* paper, "Orchestrating semiotic leaps from tacit to cultural quantitative reasoning—the case of anticipating experimental outcomes of a quasi-binomial random generator," I take on this research question by presenting a case analysis of a student struggling to align intuitive and formal views of a mathematical object designed for the study, a random generator involving a tub of green and blue marbles and a scooper that draws out four at a time. By surveying several learning-sciences theoretical perspectives on the empirical data, I argue that no single perspective can furnish a complete account. In particular, I contend that the case-study student employed a perceptual heuristic (a domain-general skill) enabling him to draw from the mathematical model (a cultural artifact) an inference aligned with the inference he had drawn from the situation (an innate capacity). Only then, I argue, could the student view as meaningful the *process* by which he himself had been guided to build the model. I conclude, on the one hand, that meaningful learning of mathematical analysis draws on intuitive situated skills. This view is well in line with theoretical models focusing on the evolution of mental schemes. Yet, on the other hand, and counter to epistemological and cognitive assumptions inherent in those theories, I argue that students make sense of mathematical analysis retrospectively, only after they have accepted as meaningful the products of these analyses. This position, in turn, is aligned with the view that humans first appropriate sociocultural artifacts as meaningful due to their function as instruments in an activity system of participatory practice, and only later they might decipher the inner workings of these mechanisms (see for example the respective and collaborative work of Michael Cole and James Wertsch).

These dual research conclusions about symbolical and epistemic cultural artifacts, above, might themselves be rendered intuitive, if we consider the case of substantive cultural artifacts, such as cars, which are *meaningful* to us as objects whose function is inextricably woven into our daily practice, even as we may not make *sense* of their inner workings. Yet in the case of particular epistemic cultural artifacts, such as mathematical solutions procedures, our cultural goal, which is embodied in national standards, is for students indeed to understand the inner workings. For example, we wish for students to understand the logic of the formula for modeling and conducting combinatorial-analysis solution procedures for predicting outcomes of binomial random generators, such as the marbles scooper. My work suggests that understanding this inner logic is in fact the result of heuristic alignment between intuitive and formal inferences. In the same paper, I characterize students' achievement of intuitive-to-formal alignment as a "semiotic leap" from tacit to cultural knowledge, a construct that draws on the work of the American semiotician C. S. Peirce on "abduction." I leverage Peircean models also to conceptualize mathematics students' intuitive-to-formal cognitive alignment as, in turn, epitomizing and resolving researchers' cognitive-vs.-sociocultural theoretical debate. (I am currently revising a submission on abduction to the *Journal of the Learning Sciences*.[1])

Finally, I suggest that the classicist expression of probability—the quotient of "favorable events" and "all possible unique events"—is but one of several mathematical constructs of type *a/b* presenting discontinuous learning progressions from non-analytic unmediated perception of identity to analytically mediated constructions of similarity. Other *a/b* constructs of this character I have published on with my colleagues and students include density, slope, and proportion. For all of these perceptually privileged intensive quantities, students can ground a sense of invariance in unmediated perception, for example recognizing the identical slope of two parallel lines of different length. Yet this sense of simple invariance becomes temporarily befuddled as students measure internal elements of the phenomena and restructure them as proportional invariance. I study

[1] Jeremy Roschelle, Editor in Charge

student–instructor discursive interactions as students are ushered to cross this tacit–cultural epistemological limbo, a process I view as semiotic breakdown and heuristic repair.

The next paper I summarize deals with but one type of cognitive–discursive mechanism students may employ as they struggle in the tacit–cultural epistemological limbo.

2.3 Students' idiosyncratic metaphors bridge the phenomenological and logical. A number of scholars of mathematics education posit that students engaged in problem-solving activities develop concepts by enlisting available objects in the immediate psychological and material environment, including symbolical notation, as semiotic means of concretizing pre-articulated properties, relations, or patterns they discern in the problem space. These semiotic means might present themselves in speech, gesture, and a variety of inscriptional forms. So doing, learners give form to their intuitions, rendering the intuitions into mental objects that support further conversation, guidance, reasoning, and discovery. However, previous research contributions to understanding the process of objectification did not sufficiently articulate nuances of students' cognitive work as they searched for a fit between, on the one hand, their emergent notions and, on the other hand, available artifacts introduced into the learning environment. In particular, previous literature left open the question of how students make sense of complicated mathematical artifacts, such as new procedures or complex representations, that students are expected to engage and learn. From the perspective of my research framework, the question is how students coordinate between a phenomenologically immediate impression from a situation and a challenging, unfamiliar mathematical structure offered either as a model of the same situation or as a technique for analyzing the situation.

In "Try to see it my way: the discursive function of idiosyncratic mathematical metaphor" (accepted for *Mathematical Thinking and Learning*) I, together with my graduate and undergraduate students José Gutiérrez and Anna Baddorf, propose that metaphor should be supplemented to the roster of discursive technologies studied as semiotic means of objectification and, moreover, metaphor should be recognized as a unique means. Although metaphors are expressed linguistically, in speech and gesture—both being semiotic systems and modalities that have already been previously implicated in the literature as offering means of objectification—metaphors serve a critical function in students' learning. Namely, students' idiosyncratic analogical reasoning both affords powerful bridging between situations and their purported models and facilitates the application of new strategies.[2]

We furnish several examples of a student and an interviewer–researcher who conjured elaborate, extemporized metaphors apparently as a means of helping the student make sense of an unfamiliar mathematical artifact or manage the performance of a challenging mathematical solution procedure. We suggest that these metaphors enlist embodied experiences from everyday phenomenology and, so doing, co-opt existing and well-rehearsed mental schemes that facilitate drawing and elaborating on relevant inferences

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[2] Katie Makar and Dani Ben-Zvi, Guest Editors; Lyn English, Editor in Chief.

as well as better managing cognitive resources. The illustrative examples in the paper are put forth in an attempt to demonstrate both the promise and fragility of spontaneous idiosyncratic metaphors as discursive supports. (I have just submitted a revised, different submission to the *Journal of the Learning Sciences* on perceptual mathematics education, "Mathematical vision: perceptual reasoning as conceptual learning.[3])

Very recently, my laboratory's dialectical research on mediated discovery has led us to develop new constructs that we offer as potentially important for reconciling opposing views on the nature of conceptual change. We have named these constructs "hooks and shifts," as follows below.

2.4 "Hooks and shifts" in embodied-interaction design (Reinholz et al., in press)
The Embodied Design Research Laboratory is currently analyzing empirical data consisting of video footage collected over 2009-2010 in a design-based research study investigating the emergence of mathematical reasoning from embodied interaction. The study is motivated by the bold conjecture that some mathematical concepts are challenging to students, because everyday life does not afford opportunities to develop appropriate fundamental schemas requisite as cognitive substrate for building personal meaning for the concepts. For example, whereas the mathematical operation of addition draws on simple ideas of grouping, accumulation, extension, stacking, etc., the analytical notion of proportion is not equally nurtured through mundane interactions. Consequently, the design rationale of our "Kinemathics" study is to create opportunities for students to learn the physical dynamics of proportional transformation prior to its mathematization.

At the center of Kinemathics is the "Mathematics Image Trainer," a type of *embodied-interaction* design. Embodied-interaction is a pioneering design genre practiced by an increasing number of mathematics education researchers, often in collaboration with HCI (Human–Computer Interface) engineers. In my interpretation of embodied-interaction design, *the physical solution procedure to artifact-mediated manipulation problems dynamically inscribes the normative conceptual metaphor of the target content*. The pedagogical objective of this body-based instructional activity is for students to discover, enact, and rehearse "firsthands" the mathematical principle that the designer has embedded into the mediating artifact in the form of its interactive, emergent solution strategy.

The Mathematics Image Trainer (MIT) activity consists of remote-controlling virtual objects on a screen, using Wii Nintendo technology that my laboratory has modified ("hacked"). Working with the MIT, students initially stumble on a correct production, then on another, and yet another, and this set of correct productions gives rise to an equivalence class, a new and yet unnamed ontology. Students articulate a solution procedure—a naïve, qualitative principle such as the covariation, "The higher you go, the bigger the distance between the hands should be" (what Batolini Bussi and Mariotti might call an "artifact-sign"). The instructor steers the student to re-articulate this situated

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[3] Rogers Hall and Ricardo Nemirovsky, Guest Editors; Reed Stevens, commenter; Jeremy Roschelle, Editor in Charge.

rule using mathematical instruments and forms. For example, the instructor introduces into the working space a grid, thus rendering a blank space into a Cartesian plane. Once students re-articulate their principle with the mathematical reference frame, their continuous actions surreptitiously become discrete and enumerated, and consequently students notice new patterns, such as the mathematically commensurate covariation, “For every one on the left, it’s two on the right.” Thus, students are guided to reinvent mathematical forms by virtue of engaging in discursive activity with symbolical artifacts introduced into the learning environment (see elaboration below).

In our most recent writing (e.g., a manuscript under review for AERA 2011), we characterize students’ discoveries as emerging from dialectical relations between the embodied message and the mathematical medium. We name this phenomenon “hooks and shifts.” We show empirical examples of students who set off from enacting an embodied-interaction solution procedure and then “inadvertently” bootstrap to qualitatively disparate, higher mathematical grounds through the process of engaging objects introduced into the problem space as new discursive forms. For example, the student engages a proposed mathematical instrument as a discursive means of explicating a gestured or mimed embodied strategy; so doing, the sheer communicative constraints of describing simultaneous action verbally casts the bimanual ambidextrous strategy into linear narrative, wherein linguistic structures usurp and morph the coordinated embodied enactment into a decomposed hand-by-hand description with accompanying sequential manipulation. This semiotic restructuring of the initial strategy, in turn, subsequently guides linear bimanual enactment of the solution, so that the interaction strategy is transformed to accommodate progressively mathematical form. Thus, for example, the covariation, “The higher, the bigger,” which had been enacted by raising both hands at different rates, is implicitly transformed into the new action plan, “For every one on the left, it’s two on the right,” which the student then carries out in left-then-right disjointed ratcheting gestures up the gridded plane, until hooked by the next mathematical form, and so on reflexively. In our microgenetic analyses, we show how shifts are onset by the students’ minute communicative gestures toward virtual elements in the problem space, gestures carried out as regulatory means of inviting the interviewer to co-attend to the perceptual cues that guide the student’s physical enactment of the body-based solution procedure.

The theoretical significance of the hooks-and-shifts construct is in the synergy it demonstrates between students’ cognitive capacity and the cultural artifacts that mediate and distribute the communication of the students’ inventions. Often, inventiveness and discovery are associated with the radical-constructivist bent of design, whereas notions of cultural artifacts mark sociocultural parlance. I argue that what appears as *ex nihilo* personal discovery (abductive inference) might, at least in some cases, be explicated as emerging from nuanced dialectics of embodied cognitive content and discursive semiotic form. Reciprocally, what appears as discontinuous reconfiguration of conceptual schemes through the guided appropriation of cultural artifacts might, at least in some cases, be explicated as students’ agentive modification of their own solution strategies. These theoretically complementary interpretations of empirical data are offered as contributing to the dialectical explication of mathematical learning.

As was probably evident in the above treatment of learning theory, it is difficult to extract the theoretical from the pragmatic in discussing findings and conjectures from design-based research studies. I shall now turn to focus on aspects of design theory arising from our work, yet these shall be “tainted” by some contextual discussion of learning theory.

3. Scientificating the Craft of Instructional Design

Whereas the design-based research approach is broadly viewed by members of the learning sciences community as a promising context for articulating theoretical models of learning, the practice of design itself, and in particular the collaborative practice of designing mathematical objects, has remained undertheorized. A design theory, it has been argued, could furnish a gap between, on the one hand, broad pedagogical philosophies nurturing from ‘big’ theories of learning and, on the other hand, specificities, contingencies, and technicalities of local contexts, including targeted concepts, available media, participant populations, and site logistics. One prevailing sentiment is that the practice of instructional design is as much an art as it is a deductive scientific craft. This sentiment would render futile the prospects of scientificating design, because doing so would be tantamount to pinning down psychological chimera such as inspiration. Yet, whereas talented designers for mathematics education have put forth heuristic frameworks in an attempt to guide novices to the practice, operating by-the-book within these construction recipes is liable to result in stodgy materials and activities. Some of the design magic, if you will, is not yet under a spell.

This section addresses my laboratory’s recent publications related to the methodology of design-based research practice, including papers on the design of mathematical objects and, stepping back, on the practice of design-based research teams engaged in collaborative development of mathematical artifacts and learning theory over sequences of iterative empirical intervention studies.

3.1 Scientificating the craft of designing mathematical objects. A lacuna in the field of mathematics education is that there do not exist detailed, research-informed heuristic design frameworks for implementing constructivist pedagogy in the form of viable learning materials. Granted, several math.-ed. design giants, such as Zoltán Pál Diénès, have authored frameworks that yielded effective, elegant materials, however these frameworks are tautological, in that they are not grounded in research but only describe principles of the intuitive design itself. A closer candidate is the Realistic Mathematics Education work, however their inspiring design treatises are only recently becoming specific enough so as to enable “outsiders” to attempt to emulate their practice (see also the European TELMA project).

I have argued for a design-based research approach to design theory. In our *International Journal of Computers for Mathematics Learning* paper, “Learning axes and bridging tools in a technology-based design for statistics,” we (Abrahamson and Wilensky) develop an instructional design framework for creating objects, activities, and facilitation guidelines and, so doing, developing theory of learning and theory of design. Building on cognitive-science research on conceptual development and creativity, the framework assumes that learning consists of coordinating cognitive elements into new conceptual structures.

The framework explains how to prompt conceptual blending through problem-solving activities involving carefully designed ambiguous objects. Under the condition of appropriate framing, we contend, these ambiguous objects elicit each of two embedded schemes as vying disambiguations. Students are guided first to attend to, and recognize the competing disambiguations of the object and then reconcile the resulting cognitive conflict by generating new cognitive structure. A simple rhetorical example would be an array of 2-by-3 dots that can be viewed either as 2 rows of 3 dots each or 3 columns of 2 dots each. These two perceptual constructions are in conflict, and yet they can be reconciled as transitively co-referring to one and the same set of cardinality 6. So doing, a new principle emerges as a residual effect of the reconciliation effort—the commutative property of multiplication, 2 * 3 = 3 * 2.

The paper draws on empirical data from a design-based research study of the growth of statistical reasoning. The study consisted of a networked-classroom implementation of ProbLab, a suite of computer-based probability activities that I authored in NetLogo, during my postdoctoral fellowship at the Center for Connected Learning and Computer-Based Modeling (Uri Wilensky, Director). At the heart of the implementation was S.A.M.P.L.E.R., *Statistics As Multi-Participant Learning-Environment Resource,* a participatory simulation activity that I built in NetLogo HubNet and implemented in two middle-school urban classrooms. The paper presents several vignettes and analyzes them as students demonstrating mathematical insight into the design's targeted concepts through struggling to reconcile competing disambiguations of interactive objects.

In my *Educational Studies in Mathematics* paper, "Embodied design: constructing means for constructing meaning," I propose that design-based researchers should articulate heuristics not for creating objects per se but for creating objects in light of closely observing students' interaction with objects, as I elaborate below.

At the center of my design framework is a principle from cognitive linguistics and educational semiotics, by which discourse is an individual's situated psychological means of reifying preconscious emergent notions (treated in an early section, above). This microprocess of multimodal "objectification"—signifying emergent notions in the form of words, gestures, or artifacts—is reflexive, in that by virtue of rendering tacit cognitive content accessible as an object of thought, this thought can then be nominalized and elaborated into further-evolving structures. Other scholars have implicated the necessarily discursive constitution of mathematical thought to underscore the intrinsically social nature of mathematical reasoning. I adopt these views and propose the practice of evaluating instructional efficacy of particular designed mathematical objects in light of the objects' apparent capacity to enable students' discursive objectification of preconscious inferences drawn from intuitive perceptual judgment. That is, designed objects that embody mathematical analysis of situated phenomena should appear to students as viable means of capturing and expressing their holistic phenomenology of these same 'raw' situations. In other words, pedagogical mathematical models should structure the semiotically mediated as meaning the phenomenologically immediate. In sum, design-based researchers' task is to develop cognitively ergonomic pedagogical artifacts—spatially embodied objects embedding potential disciplinary structures in forms that learners can heuristically identify as semiotic means of expressing tacit inference concerning properties of situated phenomena.

Still, whereas designers may succeed in engineering and framing pedagogical mathematical models that engage students' preconscious judgments just as raw situations do, the models are nevertheless constituted according to the mathematical analysis, not the intuitive inference. Thus, once students perceive and appropriate the models holistically as implying an inference tantamount to their intuitive inference from the situation itself—that is, once the students perform a "semiotic leap"—still the students must resolve the entailing incoherence or cognitive dissonance between the mathematical objects' macro discursive structure and micro analytic elements. In resolving this conflict, students "reluctantly" appropriate the mathematical *process*, not just its meaningful product.

The practice of design, from this perspective, is to listen closely as students attempt to capture their intuitive inferences using the pedagogical objects available in the learning environment. Students' failure to do so is interpreted as the designer's incomplete cognitive analysis of the emergent intuitions and the consequent inappropriateness of the designed objects under-development to lend themselves as semiotic means of objectification. The paper includes several empirical cases, the last of which is elaborated into an explanation of how listening closely to one undergraduate Statistics major resulted in the creation of a new type of interactive computer-based mathematical object for learning probability. (The refereed electronic supplementary material, including the new NetLogo interactive dynamical symbolic display for understanding the binomial, can be found here: http://edrl.berkeley.edu/publications/journals/ESM/Abrahamson-ESM/ .)

3.2 Scientificating the process of collaborative instructional design. Any useful commercial artifact has evolved through successive phases of design and usability testing. Along the process, each iterative product improvement embodies a shift in the designers' conceptualization of the product's structure and, possibly, emergent horizon of functionality. For design-based researchers of mathematics education, students' suboptimal engagement of instructional materials is an opportunity to reconceptualize pertinent elements of both the pedagogical philosophy and theoretical models of learning that informed the product's development. In short, what designers build is what they know, and what they change is what they have learned.

Thus, a hallmark of design-based researchers' practice is the iterative modification of emergent theoretical models in light of rich empirical data from usability testing. And yet, authors of academic publications on design-based research projects often abridge or elide reports on the succession of iterations leading up to the final product, or they describe the iterations without explicating the dialectics of theory and product. I agree with other theoreticians of design-based research in believing that poor documentation of design process is unfortunate, arguing that: (a) both the public accountability of product improvement and the intellectual legitimacy of emergent constructs are rooted in micro-decisions designers make en route to the final, publicly familiar product; moreover, (b) rich accounts of design process are invaluable sources for the development and dissemination of design methodology; and, finally, (c) it may behoove educational designers to publicize how they learn from failure, seeing as many of them preach this as a desirable pedagogical practice for their end-client students.

In my recent *International Electronic Journal of Mathematics Education* paper, "A student's synthesis of tacit and mathematical knowledge as a researcher's lens on bridging learning theory," I respond to calls to publicize process by narrating the evolution of the "semiotic leap" construct over four years of data analysis. Specifically, I describe how my research team members, who were analyzing a set of interviews, noted a pattern of apparent miscommunication between a researcher–tutor and high-achieving 6^{th}-grade mathematics students. These miscommunications were initially perplexing, because there was strong reason to believe that the tutor and student were referring to the same object, and yet their inferences regarding quantitative properties of this object were diametrically opposed with respect to the didactical content of the intervention. I describe the literature enlisted by the research team in an attempt to make sense of these data and how the application of literature modified the analyses and gave rise to a hypothetical construct that initially enjoyed some stability but later proved but intermediary toward subsequent insights. The researchers concluded that although the tutor and student perceived the same objective stimulus, they were each mentally constructing the stimulus in accord with their subjective interpretation of the task contingent on their mathematical knowledge. This stable dichotomy of views in the researchers' empirical data marked for them the utility of theoretical models emphasizing disparity between naive and expert analyses of situations; enlisting these new readings, in turn, promoted further insight, etc.

In addition to the three 2009 journal publications cited above, I published a chapter (in press) in an edited volume, *Developmental cognitive science goes to school* (Nancy Stein & Stephen Raudenbush, Eds.), "Towards instructional design for grounded mathematics learning: the case of the binomial." The book is essentially the extended proceedings of a mini-conference by invitation only, sponsored by the Spencer Foundation. The objective of the conference, attended by less than two-dozen scholars from across the USA, was "to establish intellectual exchanges and interdisciplinary collaborations." I was one of only two junior faculty, and the rest of the conference attendees were notable leaders of the field.

In summary, I am a design-based researcher of mathematical cognition and instruction. My scholarship is the investigation of mathematical learning, the development of methodology for design-based research, and the construction of researched principles for building effective instructional materials, activities, and facilitation guidelines. To date, I have presented and published on this work through 11 journal papers, 2 chapters, and many conference proceedings manuscripts. I believe that in my work to date I have helped to take on and expand upon core notions of embodied cognition, employing them in a theory-based way to produce robust instruction that helps students to grapple with difficult conceptual issues in mathematics learning. In the years to come I expect to expand this dialectic between theory and practice, contributing to enhanced theoretical descriptions of cognition and learning while at the same time exemplifying the theory with high quality instructional design.

Professional Activity

As the attached curriculum vitae details, my students and I continue to present papers at national and international conferences and publish widely in refereed conference proceedings. I have been invited to present papers at the International Congress on Mathematics Teaching, at the inaugural conference of the Mind, Brain, & Education society, the biennial meeting of the International Research Forum on Statistical Reasoning, Thinking, and Literacy (SRTL), and at the National Research Council / National Academies' Workshop on the Scope and Nature of Computational Thinking (where I presented to the panel my suggestion to analyze the development of computational literacy from a semiotics perspective).[4] I have recently been invited to speak at a Stanford departmental colloquium, to chair and introduce an AERA Presidential Session on complexity-science methodology in education research, to participate in a "think tank" meeting of the Research on Embodied Mathematics Cognition, Technology, and Learning project (Hall/Nemirovsky), and to participate in a Working Group on the future of the Learning Sciences at the biennial meeting of the International Conference on the Learning Sciences. Thrice have I been invited to publish in special issues of educational research journals:

- A student's synthesis of tacit and mathematical knowledge as a researcher's lens on bridging learning theory (2009). In M. Borovcnik & R. Kapadia (Eds.), Research and developments in probability education [Special issue]. *International Electronic Journal of Mathematics Education*;
- Try to see it my way: the discursive function of idiosyncratic mathematical metaphor (in press). In K. Makar & D. Ben-Zvi (Eds.), The role of context in developing students' reasoning about informal statistical inference [Special issue], *Mathematics Thinking and Learning*;
- Mathematical vision: perceptual reasoning as conceptual learning (revise/resubmit). In R. Hall & R. Nemirovsky (Eds.), Embodied cognition [Special issue]. *Journal of the Learning Sciences*. (Manuscript under review, revise/resubmit draft in preparation)

In addition to these invited presentations and publications, I continue to be a member of various professional organizations whose practice informs my research, such as the American Educational Research Association, the Jean Piaget Society, Cognitive Science Society, and the North-American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA, where I participate in the ongoing Working Group

[4] This report, which includes my contribution, is now a publication of the National Academies. http://www.nap.edu/openbook.php?record_id=12840

on probabilistic reasoning). It is my intention to become more centrally involved in these collaborative efforts so as to promote research on mathematical learning at a larger scale.

Teaching

During my first years as a new junior faculty still figuring out departmental needs and student interest, I built and taught a number of EDUC290C courses, including, "Design-Based Research in Mixed-Media Learning Environments," "Cognitive Ergonomics in STEM Education Research," "Principles for Embodied Design," "Modeling-Based Methodology for Design, Learning, & Research," and "Learning Chance: Computer-Supported Inquiry into Probability," all with full syllabi, including domain-specific reading lists, in- and out-of-class assignments, and term project specifications. Over the past five years, my instruction has fallen into a multi-annual regime that accommodates the GSE's programmatic demands. In the Fall term, I rotate annually between yet another new EDUC 290C course, "Paradigmatic Didactical Mathematical Situations," and one of the computer-modeling-oriented courses, above; In the Spring term, I teach the undergraduate EDUC130 (see below). In both terms, I teach "Design-Based Research in Mixed-Media Learning Environments," which is an ongoing forum for students conducting design-based research studies in the STEM disciplines (STEM = Science, Technology, Engineering, & Mathematics). Finally, throughout the year I hold my 223B Research Group section.

My courses have attracted students from many different units and programs across campus and from other UC universities (Davis), including the Berkeley Institute for Design as well as the departments of Architecture, Engineering, and Computer Sciences. My involvement in students' individual projects has led to several ongoing working relationships and mentorship through academic committees (as an external member on qualifying and prospectus examinations) and consultation on methodology and grant writing.

EDUC195c (now EDUC130). Three years ago, leaders of the UC Berkeley CalTeach initiative invited the Graduate School of Education to join the program by teaching "Knowing and Learning in Mathematics and Science," an undergraduate course created by the University of Texas (CalTeach was structured according to the UTeach model). Professor Schoenfeld and I volunteered to co-design and co-teach our version of the UTeach course, and two years ago we first taught it during Spring 2009 with the help of an able Physics graduate student (who subsequently switched to a GSE-based graduate program). The UTeach/CalTeach vision is to recruit among the undergraduate student body strong students in mathematics and the various sciences who are potentially interested in education but had never explicitly considered a career as a teacher. Our charge as designers of the Knowing and Learning course was to introduce these prospective educators to cognitive-science frameworks for making sense of students' reasoning. Our version of the course built on the UT "constitution" yet brought a strong focus on problem solving from Professor Schoenfeld and my respective graduate courses. We also enriched the content with our respective expertise. For example, I added a focus on computer-based modeling and a robotics workshop, and have been inviting experts –

both UCB-based and from other universities – to give invited lectures on a recurring basis. In Spring 2010, I was the sole instructor of record with two very able GSI's. I will be teaching the course in Spring 2011 and, hopefully, in many years to come.

EDUC130 has enrollments of several dozen students. The course is distributed over hands-on/reflection classroom activities, implementations in high-school classrooms, and a vibrant online discussion board, and the course project is a small empirical research study, in which our students teach at their high-school site one of the problems we had worked on in college, and then analyze their high-school students' artifacts as well as a video recording of the session in light of the literature we have read and discussed. Through periodic CalTeach meetings, and based on students' feedback, we have been iteratively modifying this new course toward its current form (see attached syllabus) and building a robust online infrastructure supporting the organization and facilitation of the course both for the instructors and students. The course has been visited and reviewed by CalTeach, UTeach, and their federal sponsors and supervisors, and we have received for the main much encouragement from all parties involved. Several students who have taken the course either became graduate students of education at the GSE or are otherwise active in GSE research groups and plan to combine a teaching career with academic research.

The EDUC130 course has been transformative for my teaching practice, because it was my first encounter with the UC Berkeley's undergraduate body and first massive encounter with future teachers, beyond the masters-and-certification courses at GSE. As such, I have been learning firsthand of both the challenges and promise of providing California with high quality teaching in the nationally vital STEM subjects. So doing, I have been gratified to witness the advantage of practicing, as teachers, what we preach as researchers of learning. Namely, high-demand/high-support instruction in a student-centered respectful, attentive, collaborative climate actually "works." As our evaluations demonstrate, students found the course engaging, informative, and useful for their future practice as teachers. Student comments reflect the academic diversity of the CalTeach student body: Some students from the natural sciences felt slightly overwhelmed by social- and cognitive-science readings, whereas others wished that we had devoted more class time—not only online discussion forums—to these same readings; some science majors wished we had more chemistry/biology work; and some students found the 3-hour course too long a stretch; some students wished we had mixed the groups more, whereas others wished we had not re-mixed them at all.[5]

Admittedly, my most gratifying mentorship experiences have been in my project-oriented research groups. In addition to my formal section of EDUC223B, I led two group

[5] Strewn within student feedback, especially early/mid-course feedback, are occasional comments regarding teachers at the high-school sites. It appears that some of our students may have been confused by the many bodies they were reporting to in this complex course. These misaddressed comments, whereas important in-an-of-themselves for CalTeach development, are easily recognizable through reference to particular names or settings that are patently unrelated to the EDUC130 university site.

meetings on a regular basis, one for each of my COR-sponsored projects, "Seeing Chance" and "Kinemathics." To the best of my ability, I have attempted in these auspices to model for my student–researchers the mixture of methodological rigor and generative confusion that are idiosyncratic hallmarks of working laboratories pursuing hypotheses through empirical studies; the excitement, the dead ends, and the wisdom to tell them apart. We read a wide scope of literature, all meticulously organized on Wiki's with detailed commentary on each and revised summaries of each meeting's minutes. These documents have led to a number of conference proceedings and journal papers. We periodically host scholars, both UCB resident researchers and visitors from other institutes, who comment on our work. My lab is open to UCB Undergraduate Research Apprenticeship Program (URAP) and New Experiences for Research & Diversity in Science (NERDS) students, summer interns from the greater UC system and beyond, and international students. (During a recent social gathering with seven of my lab members, we realized that only one was US born and we had seven non-overlapping languages among us.)

The advantages of accommodating diversity along multiple dimensions can sometimes bear its challenges. For example, a couple of UCB students from outside the GSE who participated in my EDUC290C course, "Design-Based Research in Mixed-Media Learning Environments" during Fall 2009 rated the course highly yet wished that I had a background in business (which I deplorably do not!), because their projects involved the development of services and products. In response, I plan to give future students a better introduction to the course, so as both to manage our expectations of each other and ensure that we focus on our strengths and seek the help we need where it is available.

In the near future, I will continue to develop my courses. My objective is to gain campus approval and course numbers for the two courses I will be teaching on a permanent basis. In addition, in collaboration with CalTeach, I hope to create frameworks and infrastructure for GSE graduate students interested in research on teacher preparation and professional development to become involved both in student-teacher mentorship and in empirical studies of preparation programs. Such a program of teaching and research could create for GSE doctoral students opportunities to build appealing resumés and, so doing, could increase the attractiveness of the GSE for prospective graduate students planning academic careers in schools of education.

Service

I continue to be an active reviewer. Since the last report, I reviewed 33 journal papers, some over two or more rounds. I am a member of the review board of the prominent *Journal of the Learning Sciences* as well as *International Journal of Computers for Mathematical Learning*, and I am invited occasionally to review for another fourteen journals, including *Science*. During the period in question, I also reviewed a total of 72 submissions for conference proceedings. In the academic year 2009 – 2010, I served on the Program Committee of the 2010 International Conference of the Learning Sciences (ICLS), the premier biennial gathering of the International Society of the Learning

Sciences, that convened in Chicago during summer 2010. In this capacity, I recruited reviewers, coordinated and oversaw several dozens of submissions, reviewed over a dozen of them personally, and then consolidated, summarized, and ranked the submissions under my charge and negotiated borderline cases with the other reviewers. As with journal reviewing, it has been a very useful experience to be behind the scenes, on the other side of the “submit” button, and appreciate how the scholarly community depends on so many quiet acts of leadership and mentorship. Reviewing has also enabled me to keep abreast with current research outside of my immediate tether. Finally, appreciating the good and the wanting in manuscripts has helped me continue to develop and sharpen a critical eye as I write my own papers and guide my graduate students.

I have also served on a National Science Foundation review panel (REESE), a very rewarding experience in terms of the mentorship it afforded into the criteria and procedures of grant committees, and have reviewed for international grant panels, such as the United States–Israel Binational Science Foundation. These experiences will inform my future efforts to support my research financially.

On campus, I volunteer on an ongoing basis as a Faculty Sponsor to Regents’ and Chancellor’s Scholars (an initiative of the Committee on Undergraduate Scholarships and Honors). In this capacity, I attend occasional meetings and interact with six undergraduate students, mainly through e-mail correspondence, by which I attempt to consult on all matters academic, logistical, and personal in response to their queries. On one occasion, a mentee visited my undergraduate CalTeach course and is now considering joining our graduate program.

In the future, I hope to contribute to the campus through committee work, such as on the board of the Berkeley Center for New Media.

Berkeley
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