Constructing Movement in Mathematics and Dance: An Interdisciplinary Pedagogical Dialogue on Subjectivity and Awareness[1]

Dor Abrahamson, *University of California, Berkeley, USA, dor@berkeley.edu*
Ami Shulman, *Independent artist, movement educator, amishulman@gmail.com*

Abstract: A physical movement can be construed in many ways. For some researchers of mathematics education informed by embodiment theory this is important, as they perceive a mathematical concept as a polysemous structure grounded in multiple interrelated sensorimotor constructions. In this dance is no different. Similarly in both disciplines, the more ways one has of thinking about a movement and the more connections one builds across these different constructions, the deeper and richer one's understanding and proficiency in enacting the movement and the greater one's capacity to transpose the learning to new contexts. In both mathematics and dance, instructors thus seek to create conditions for students to develop diverse subjective constructions of the movements they are learning to enact and to explore relations across these different constructions. Any pedagogical discussion of movement, whether in dance or mathematics, must be a discussion of the individual's subjective phenomenology and increasing awareness. In reflection, the very possibility of the authors' interdisciplinary dialogue is testimony to the cohesive potential in systemic conceptualizations of human movement.

1. Introduction: Invitation to an Interdisciplinary Pas de Deux

Learning is the developing of new skills for responding effectively to situations relevant to survival and the accomplishment of natural and cultural objectives. Yet no two situations are ever quite the same. For skills to be useful, they should therefore embrace variation across situations. For a cognitive architecture to be adaptive, it should allow for embracing variation. It follows that learning is modifying what we know to do so as to cope with new situations; in so doing, we expand what we perceive as familiar situations. The perceived novelty of a situation, which is a subjective perception, can be anywhere from minor to major, with minor novelty going unnoticed but greater novelty requiring exploring the problem space in search of effective adaptation. The process is iterative, leading to growth in the individual's adaptiveness to its manifold ecology.

This generic description of learning is pervasive in psychology theory and discourse. The Swiss cognitive development psychologist Jean Piaget (1968) used the terms *assimilation* to describe the individual's adaptive endorsement of a novel situation as a variant on a familiar situation; and *accommodation* to describe the corresponding

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change in the individual's capacity to interact in this situation which thus effects its assimilation. In his systemic theory of genetic epistemology, Piaget conceptualized situations as subjective action-oriented perceptual constructions. Similarly, the movement scholar, therapist, and methodologist Moshe Feldenkrais underscored the importance of diversity in knowledge (Beringer, 2010; Buchanan & Ulrich, 2001; Feldenkrais, 1981). In order for someone to know to do something, Feldenkrais said, they must be able to do it in a hundred different ways. This essay draws on Piaget and Feldenkrais to argue that theorizing and working with diversity in students' subjective constructions of movement is important for teaching and learning in both dance and mathematics.

Building on this foundational interdisciplinary consensus respecting the ontology and epistemology of movement, we—a mathematics-education researcher inspired by embodiment theory (Abrahamson) and a dance educator, movement director, and Feldenkrais Method practitioner (Shulman)—will explore for common grounds through which educational scholars of mathematics and dance may benefit through conversation. That is, we will present results of our collaborative search for phenomena of common interest across our respective realms of inquiry, and we will demonstrate how our shared philosophical and theoretical perspectives on these phenomena enable a lingua franca for discussing human movement—its nature and pedagogy. Whereas this essay in and of itself may not offer either of our fields any new insight on their scholarship or practice, it is our hope that the essay will encourage and enable scholars from our own and other fields to enter in interdisciplinary dialogue on the phenomenology and pedagogy of human movement. At the very least, preparing this manuscript has compelled the two of us to share, debate, and refine central constructs pertaining to individual construction of movement and its potential affordances for instruction in our own disciplines and possibly beyond.

In a theoretical text below (Section 2), Abrahamson will present intellectual foundations for an enactivist conceptualization of mathematical activity as profoundly sensorimotor. These ideas are then anchored and contextualized through a brief description of an experimental application to mathematics pedagogy in the form an interactive technological device, the Mathematics Imagery Trainer. The device was designed so as to foster students' sensorimotor micro-movements prior to engaging in quantitative re-modeling of these movements in mathematical forms per the concept they are studying. Next, Shulman will respond by characterizing aspects of this proposed mathematics pedagogy as resonating with ideas and principles from movement direction inspired by Feldenkrais Method as well as phenomenological philosophy. Therein, Shulman will focus on: (1) *subjectivity* in the individual's orientation to, and experience of movement; and (2) *awareness* as a bridge from movement to reflection and utility (Section 3). We conclude with a summary of our argument as well as an invitation to expand the dialogue between scholars of dance and mathematics pedagogy (Section 4).

We wish to note that we are not the first scholars to point to a connection between the Feldenkrais Method and mathematics. Cole (2004) relates a personal growth story, in which a quest to rehabilitate his injured hands through the Method led him to new capacity for mathematical reasoning:

> As I gained in my ability to become aware of myself as a whole body, following the gesture of the movement instead of trying to keep track of the component

> parts, something in my mind specific to mathematics was changing…. The pieces I had lacked in my mathematical understanding I had lacked in my physical vocabulary as well. By improving my ability to experience and move within space I had discovered for myself a more accessible way to navigate among abstract mathematical concepts. (p. 17)

We, too, will be speaking about movement in space. Our focus will be on students' subjective discovery and cultivation of tacit schemes that mediate the enactment of movement; schemes that, once rising to consciousness through instructional intervention, lend personal meaning for these movements and empower us to move better, think better.

2. Theoretical Foundations: Learning as Adaptive Interaction

2.1 Embodiment Theory of Mathematics Learning

Per Piaget, the nature of a situation—what it is—is necessarily its contextual, *ad hoc*, *in situ* and subjective meaning for the specific individual engaged in some goal-oriented activity; the situation is constructed, in the sense that the individual attends to certain perceptual figures relevant to managing potential goal-oriented actions while ignoring other, irrelevant figures. Regularities in perceptually guided action give rise to sensorimotor schemes. It is these schemes that adapt to perceived variations in situations.

Thus all learning, at least pre-conceptual learning, consists of developing sensorimotor schemes, where the "sensori-" captures how the individual is organizing perception for action (what the figures are in the situation), and the "-motor" captures how the individual is organizing perceptually oriented motor action (neuromuscular coordinations). For example, if you are walking along a path and suddenly a pothole ahead draws your attention, it means you should adjust your gait temporarily so as to avoid falling in, and you may do so with little to no conscious attention. But perhaps you are encountering for the first time a pothole of this size, in which case adjusting your gait marks an accommodation of your sensorimotor scheme so as to assimilate this and prospective cases of potholes of similar magnitude. Notably, what we learn is not things per se but interactions with things. As Piaget (1971)) writes, "Knowing does not really imply making a copy of reality but, rather, reacting to it and transforming it (either apparently or effectively) in such a way as to include it functionally in the transformation systems with which these acts are linked" (p. 6).

Although his views of cognitive development have been cashiered over the past few decades, recent literature has been reviving Piaget's theory, vindicating it from mis-readings and bringing it back to the proscenium of research discourse (Abrahamson, Shayan, Bakker, & Van der Schaaf, 2016; Allen & Bickhard, 2013; Arsalidou & Pascual-Leone, 2016). Theoretical models of learning resonant with Piagetian views abound. We find them in dynamical systems theory (Thelen & Smith, 1994), enactivism (Varela, Thompson, & Rosch, 1991), coordination dynamics (Kelso, 1995, 2000), and various kinesiological theories derived from the work of Bernstein (1996) on dexterity and/or Gibson (1977)) on ecological psychology, such as constraints-based models (Newell, 1986, 1996; Newell & Ranganathan, 2010) and ecological dynamics (Araújo, Davids, Chow, Passos, & Raab, 2009; Chow, Davids, Button, & Renshaw, 2016).

Whereas these latter explications of learning have treated motor rather than conceptual learning, there is a growing sense among cognitive scientists that conceptual activity, too, is embodied in the sense that it is grounded in sensorimotor action that is either tacit, consciously simulated, or even externally manifest and materially engaged through various representational machinery and its manipulation routines (Anderson, 2003; Barsalou, 2010; Kirsh, 2013; Wilson, 2002). In fact, certain readings of Piaget (1971, p. 6), too, or, for that matter, the Belarus cultural–historical psychologist Lev Vygotsky (1997, pp. 161-163), implicate the sensorimotor quality of cognitive activity in mathematical reasoning. This evolving assertion that what we call the human mind is ecologically situated dynamical activity has been named variably as embodiment theory or the corporeal turn in the cognitive sciences, with certain important distinctions and nuances labeled as grounded, embodied, embedded, and extended cognition (Kiverstein & Clark, 2009).

Inspired by embodiment theories, our views on how people learn dance (Shulman) and mathematics (Abrahamson) generally agree with Piagetian constructivism even as we hold complementary Vygotskian perspectives on the social mediation of cultural forms (q.v., Abrahamson & Trninic, 2015). In particular, enactivist theory has informed Abrahamson's research program to implicate and leverage the action roots of mathematical reasoning. This research program is vested in educational design practice. Specifically, Abrahamson's design-research laboratory conceives, engineers, implements, and evaluates interactive environments where students learn mathematical concepts through: (a) solving movement problems; and only then (b) reflecting on and representing those solutions, first qualitatively in natural multimodal discourse and then, by way of appropriating and utilizing mathematical frames of references, quantitatively then symbolically (Abrahamson, 2014). This research team is treating phenomena of movement learning also from the perspectives of phenomenology (Merleau-Ponty, 1964) and Feldenkrais Method (Beringer, 2010; Feldenkrais, 1981). Importantly for this essay, Abrahamson's research team differentiates between movements as observable dynamical phenomena and the underlying subjective sensorimotor schemes by which individuals generate these movements (Abrahamson & Bakker, 2016). As we will soon elaborate, below, these latter views share essential precepts with the corporeal turn.

2.2 Open Questions for the Pedagogy of Dance and Mathematics

As educators as well as scholars of education, our questions around learning *ipso facto* carry questions about teaching. And certainly the question of learning cultural skills such as the choreography of movement in dance or mathematics compels us to inquire into the role and technique of instructors. Yet what might be the implications of all these theories of learning, which we have cited, for the practice of teaching?

Teaching is not the sheer communication of procedures. Teachers can teach neither movement nor mathematical concepts directly. Rather, they can create conditions for students to learn. These conditions may include a setting, a task, and means of accomplishing the task; in the course of attempting to accomplish the task, students bring to bear their skills. Along the way, the teacher influences how students perceive the situation and plan prospective action. But how exactly do teachers do this?

As we view movement teaching and learning within the cultural practices of dance and mathematics, we perceive strong convergence between pedagogical routines across these two domains, at least per the corporeal turn in the cognitive sciences, and in particular from the perspectives of phenomenology and Feldenkrais Method. From these mutual grounds we argue for the potential of dialogue between scholars and educators across the disciplines of dance and mathematics as informing theories of teaching and learning more generally.

2.3 Case Study of Embodied Mathematics Pedagogy: The Mathematics Imagery Trainer

Abrahamson has developed an instructional methodology in which students learn to move in new ways prior to signifying these movements mathematically (Howison, Trninic, Reinholz, & Abrahamson, 2011).

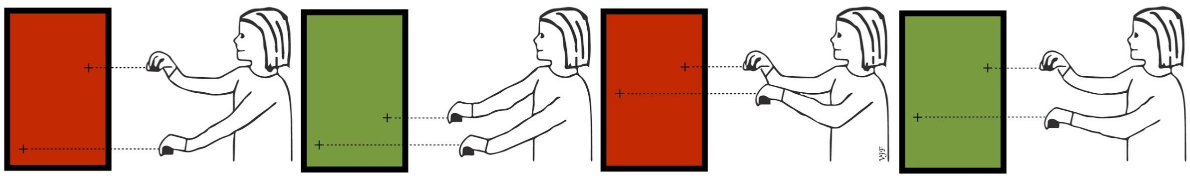
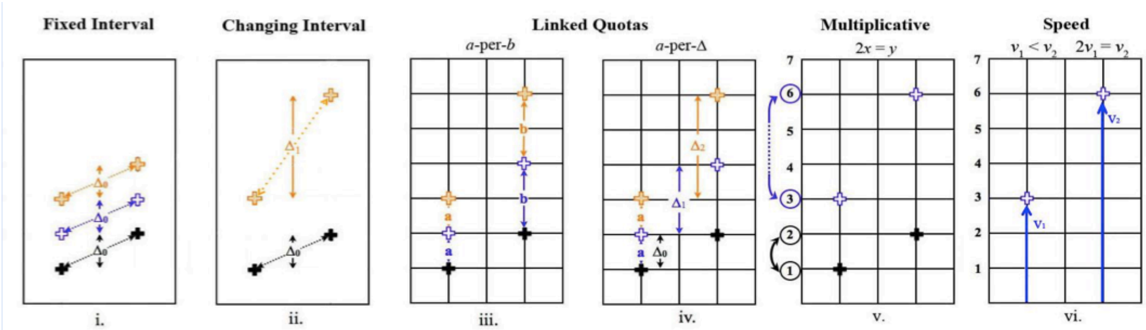


Figure 1. The Mathematics Imagery Trainer for Proportion: schematic activity sequence. The trainer is here set at a 1:2 ratio, so that the favorable sensory feedback (a green background) is activated only when the right hand is twice as high along the monitor as the left hand. Glossing over idiosyncratic variability, this figure sketches out our Grade 4 – 6 study participants' paradigmatic interaction sequence toward discovering one effective operatory scheme: (a) while exploring, the student first positions the hands incorrectly (red feedback); (b) stumbles upon a correct position (green); (c) raises hands maintaining a fixed interval between them (red); and (d) corrects position (green). Compare 1b and 1d, the two green configurations, to note the different vertical intervals between the cursors. The child might conclude that, "The higher my hands go, the bigger the interval." She learns to move in a new way centered on a new object.

Students work with a technological device called the Mathematics Imagery Trainer (see Figure 1) that senses and responds to the position of their hands in space. Students are tasked to discover a particular two-hand movement scheme that will effect the goal state of making a screen turn green. The device has been programmed so that the screen will be green only as long as the hands' respective heights above the monitor base relate by a specific ratio, for example 1:2. As the hands move in the case of the 1:2 ratio, keeping the screen consistently green, the right hand must at all times be double as high up along the monitor as compared to the left hand. This means that as the two hands rise simultaneously, the vertical interval between them increases (and vice versa for descent).



Legend: LC (left-hand cursor); RC (right-hand cursor); Δ (vertical & diagonal magnitude of interval between cursors); v (velocity).

Figure 2. Student generated solution strategies for the make-the-screen-green problem (the case of a 1:2 ratio): (i) Fixed Interval—maintaining Δ constant regardless of RC-and-LC elevation (incorrect solution); (ii) Changing Interval—modifying Δ correlative to RC-and-LC elevation (correct if proportion is preserved); With the introduction of the grid - Additive, either (iii) Co-Iterated Composite Units—both LC and RC either ascend or descend at respective constant values a and b (a-per-b), or (iv) LC rises by a (usually 1), RC by 1 box more than the previous Δ; (v) Multiplicative—relocating to a next "green" position as a function of the height of only one of the cursors (given LC at x and RC at y, 2x = y; x = ½ y), e.g., determining LC y-axis value, then doubling to find RC, or determining RC value, then halving for LC; and (vi) Speeds—LC and RC ascend/descend at different constant velocities (v1 < v2) or more specifically, RC velocity is double LC velocity (2v1 = v2; v1 = ½ v2).

Research has revealed multiple and diverse ways that individual students discovered for orienting productively toward the task of moving the hands while keeping the screen green. For example, students raise their hands ensuring that: (a) the interval between them keeps increasing; (b) one hand is always double as high as the other; (c) one hand moves faster as the other; (d) one hand rises in quotas that are double as large as the other hand (see Figure 2 for a technical elaboration of these and other strategies). Each of these orientations captures one essence of the mathematical notion of proportionality. Moreover, reflecting *across* these ways of moving appears to create opportunities conducive to deep conceptual understanding (Abrahamson, Lee, Negrete, & Gutiérrez, 2014). For example, switching between perceiving the movement as focused on the varying interval and as focused on relative heights supports students' conceptual transition from additive to multiplicative conceptual structures. Teachers play critical roles in steering students to reflect on the movement they enact and adopt mathematical framings on this movement.

We have now introduced the intellectual grounds of embodied mathematics pedagogy as well as the instructional activity that constituted the empirical context for Abrahamson's investigations of embodied mathematics learning. Next we turn to Shulman's response from the scholarship and practice of dance pedagogy, as contextualized by the above case of the Mathematics Imagery Trainer for Proportion.

3. Movement Pedagogy: Embracing Diversity, Fostering Awareness

Movement is an expression of intention through time and space. The enactment of movement is a subjective experience; awareness of this experience may lead to greater skill. Learning through the body provides not just the understanding of the movement itself—it offers direct insight into other possibilities implicit within the movement. Often, understanding movement requires deconstructing it into smaller increments, where each is not just a fragment of the whole but contains within it new potentials. Through awareness, the dancer can use those elements in infinite combinations and situations.

These and other principles of dance pedagogy, it turns out, are applicable also to mathematics pedagogy, or at least to enactivist mathematics pedagogy (Abrahamson & Trninic, 2015; Reid & Mgombelo, 2015). Children who study mathematics in Abrahamson's approach learn through movement to incorporate, spatialize, and conjure objects in the environment. By applying these skills across a range of situations bearing parametric variations, discussing their orientation to the movements, comparing and contrasting different orientations, and signifying the movement using formal frames of reference, vocabulary, and symbolic notation, the children come to understand how to transpose these concepts into new situations. In this section some key parallels between mathematics and dance pedagogy will be highlighted as they pertain to the Mathematics Trainer for Proportion.

The assertions and analyses offered in this section draw on Shulman's experience as a dancer and movement director whose pedagogical approach uses principles of the Feldenkrais Method. This method offers a systemic approach for individuals to differentiate, integrate, and diversify their motor co-ordination patterns, which in turn creates new possibilities in human functioning through awareness and movement.

3.1 Movement Enactment as Subjective Composition: Implications for Instruction

In dance, a movement or choreography is performed as a gestalt. There isn't just one way to do a movement or to think of a movement, no matter how precise the movement is. This is most evident when you reconstruct a piece of existing choreography. When you simply remount the movement itself, retrieving the dance, most often than not the dance doesn't resemble the initial choreography; the vision and intention of the original creation seems lost. This significant gap reveals that there is much more to movement than the shapes through which it passes. If asked, each individual dancer may have a completely different perspective on what the movement is or should be. This diversity in dancers' perspectives on a movement results not from their differing capability to recall the movement. Rather, this diversity is an expression of each dancer's unique somatic–kinesthetic organization and experience, which determines their idiosyncratic orientation to, and perspective on the common movement.

In like vein, Abrahamson's study participants arrive at a broad set of diverse sensorimotor solutions to the rather rudimentary movement of raising two hands at different speeds. (Even this expression, "raising two hands at different speeds," is not an objective description but itself is one of numerous particular ways of orienting toward the movement.) None of these solutions is better than another. Rather, the solution expresses a student's corporeal composition as implicated within a particular task at that moment.

Eliciting an individual response bears advantages for teaching. In a sense, telling a student what to do but not how to do it creates for that student an opportunity to tailor the movement to tacit nuances of their personal characteristics and particular aptitudes; to their own way of knowing and relating. This approach is a form of implicit instruction (see also Chow et al., 2016; Newell, 1986, 1996; Newell & Ranganathan, 2010).

Implicit instruction allows each student to learn through a process of self-discovery. Feldenkrais spoke of the potency of a learning that comes from this type of discovery and claimed that it is the only way to 'know' something. He also said that to know something you need to be able to do it in one hundred different ways, which promotes an integrated sense of self-use. In turn, multifarious knowing is conducive to prospective appropriation of the new knowledge in diverse settings. Only when a student understands a movement idea through action will you see it appearing in other contexts.

3.2 Subjective Self-Image as Obstacle or Opportunity for Learning Movement

Dancers respond to movement material through their individual experiences and most significantly through their self-image. The self-image is a fundamental principle of the Feldenkrais Method. It is the unconscious sense we have of ourselves that we project onto the world; the self-image is implicit in all our actions. Feldenkrais stated that a person will act more in accordance with how they perceive themselves to be than with how they actually are. In other words, we act in accordance to the resources we think and sense are available to us rather than those actually possible. Thus the self-image forms the basis of our habitual patterning in both thought and movement.

The self-image is a universal given. Feldenkrais Method instructors work with the self-image. They create conditions for students to diversify their habitual patterning with its unique perspective yet without contradicting the system. Lessons typically introduce a new task as a means of bringing attention to a familiar pattern and introducing a new pattern. For example, making someone walk backwards brings their attention to how they actually walk forwards. This awareness facilitates an integration of the new information through the subjective lens of the person. It is an unconscious process, albeit the person may later become conscious of a felt difference.

As students engage with Abrahamson's Mathematics Imagery Trainer task, initially they attempt to raise their hands keeping constant the spatial interval between the hands, and yet the device has been programmed such that the correct movement requires varying the interval between the hands—the interval should increase as the hands ascend and decrease as they descend (compare Figures 2i and 2ii). Notably, it is not the case that the children are biomechanically or psychologically incapable of the correct movement. Rather, in attempting to solve the task they initially bring to bear what they tacitly believe to be the reality of the situation. It is only once this tacit belief is refuted that the belief surfaces to consciousness, so that the student becomes aware of their belief and can then begin to modify it concordant with ongoing feedback from the technological system.

3.3 Embracing Diverse Subjectivity by Repositioning Movement as a Means to a Common Goal

Eliciting from dancers optimal performance requires acknowledging and incorporating their subjective perspective so as to preserve within their system a cohesive integrity. Yet this embodied subjectivity poses for the pedagogy of ensemble work the challenge of working with a plurality of individual perspectives whilst trying to achieve specificity within a collective goal. One pedagogical solution is to position the movement not as an end in and of itself but as a means to and end—some well-articulated common end.

This pedagogical strategy of repositioning movement as a means to an end is exercised in Abrahamson's Mathematics Imagery Trainer for Proportion, where the students are all attempting, in their diverse subjective ways, to achieve one and the same well-defined goal state of the technological system, namely to make the screen green.

For both mathematics and dance, perceiving movement as a means of achieving a well-defined goal may foster productive responses in prospective encounters within new yet apparently similar situations that would likely elicit the same movement as a response.

3.4 Movement as Polysemous Meanings: Potential for Learning Through Moving

When a person moves, they are orienting toward the movement in one particular way. They are immersed through movement, not looking at it. Yet when we stop to think about movement, we could potentially re-imagine it in a different way. This is just like an ambiguous visual figure. You are not seeing it *as* a duck or a rabbit—you are simply seeing duck or seeing rabbit (Wittgenstein, 1980). Yet when we stop to think about our seeing, those who saw duck only may be steered to see rabbit, and vice versa (Gopnik & Rosati, 2001). As such, one could speak of movement as "containing" a plethora of potential meanings. This potentiality of diverse perspectives all implicit within any given movement constitutes the key to an innovative pedagogical process. More broadly, developing capacity to decouple *how* we are seeing something from *what* we are looking at, that is, to disengage from one perspective on the world and consider another, is a powerful pan-domain skill that could support both richer reasoning on specific concepts and possibly greater social tolerance to others' views.

As Abrahamson points out with respect to students working with the Mathematics Imagery Trainer, perhaps the greatest pedagogical potential of movement polysemy lies in attempting to link up across different meanings for the same movement. Unlike the duck/rabbit ambiguous image, where the two meanings are mutually exclusive, in the case of movement the alternative sensorimotor orientations are biomechanically and conceptually complementary, so that shifting across these orientations creates opportunities for productive reasoning (Abrahamson et al., 2009; Abrahamson et al., 2014). In dance, these shifts make for more versatile coordinations and richer expressivity.

3.5 On the Ontology and Epistemology of Movement: Challenges and Solutions

Movement is ephemeral, dynamical, continuous. And yet when we reason and speak of movement, we are liable to reify it as static moments. Our discursive minds tend to "language" movement into a linear succession of elements that can be grasped and pinned down for scrutiny, measurement, description, instruction. Consequently, the pedagogy of movement is liable paradoxically to freeze it, ground it, undo it.

One way out of this quandary is to perceive movement for what it is—relational. Movement can be in relation to: gravity; the environment; spatial considerations; the biomechanical integrity of the body through which it is expressed along with the senses; the self; and others and their movement. By putting our attention on a select relational aspect of movement, we can sustain its dynamicism even as we thematize it.

Understanding movement in this regard made me reconsider my pedagogical approach in dance education: I stopped being interested at *what* I was teaching but rather in *how* I was teaching it. Similarly I focused on *how* a dancer approached the movement rather than *what* the movement was. I became more interested in the process of the movement and how the dancer got from one point to another, focusing not on the points themselves but through specific directives. This was a task-oriented approach with specificity: "How can your elbow reach your knee"; you become very clear about where those two parts are in order to *use* them. In this way it became a functional task.

The way a dancer would move within those points, would actually *familiarize* them with their instrument. I would take an exercise and each day reframe it from a different perspective. Gradually, the dancers would begin to understand the relational body—the body in action; the dancer understood how to think about movement and not about the shapes that they would pass through. This style of pedagogy set up unique conditions for learning, as it required of dancers a consciousness to their self-use. It's suggestive approach also allowed more developmental expansion in comparison to its alternative corrective approach. This implicit mode of instruction was permissive, inclusive and became exponential in developing an understanding of the body through movement. It *familiarized* the dancer with the territory of their body in a *functional reality*, and this had a major impact on their understanding of dynamic movement, which was also an implicit result of this methodology. I was teaching the dancer how to think, not what to think.

As Abrahamson reports, when students work in the Mathematics Imagery Trainer activity they may initially analyze their hand movements as traversing measurable quotas (see Figures 2iii and 2iv; doing so, students shift their bimanual movement from simultaneous (both hands at the same time) to sequential (one hand followed by the other). Eventually, however, by way of reasoning about the hands' respective speeds (Figure 2vi) the students can reclaim simultaneous movement even as they bear in mind the measured intervals traversed by each hand.

3.6 Awareness Through Movement as the Epistemic Bridge to Conceptual Understanding

Awareness of movement is a defining aspect in the learning process, as it actuates changes in the nervous system. Without awareness, we would likely just repeat our habitual impulses without the attention required to make the optimal choice for that particular situation. Abrahamson mentions how after his students had achieved the desired outcome, they reflected on what they had been doing in the attempt to describe it. This act of describing what one has done elicits a somatic awareness, which in turn integrates the learning into conceptual matter (see also Morgan & Abrahamson, 2016).

An essential principle in the Feldenkrais practice is to create guided opportunities for students to untangle their action complexes into simpler motor components, modify these components, and then selectively reintegrate into more salubrious complexes. Importantly, students must assume a degree of agency in achieving novel motion complexes. As Ginsburg (2010) clarifies: ''Learning itself is not conscious. The integration process itself is not conscious. Nevertheless, the process depends on conscious processes in feeling and detecting changes. The consequence is felt as difference'' (p. 185). This notion—that unconscious, subtle interactions drive adaptations to behavior, and that consciousness plays a post facto appraisal role in making sense of these changes—is crucial to our thesis of conceptual knowledge emerging from guided interaction through a felt sense of difference.

The gap between what we think is available to us versus what is actually possible is like a blindspot in the consciousness. The work of Feldenkrais helps to bridge this gap by clarifying the self-image so that there is more similarity between the desired action and the action itself. This is done through awareness, where information registers and somatic changes become possible.

3.7 On the Power of Gentle Touch: Toward a Systemic Movement Pedagogy

When introducing a new movement pattern to a student, if the information is too foreign or too threatening to the integrity of their system, it will be rejected. The information itself has to be accessible to the person through the current organization of their system. Functional Integration, a feature of the Feldenkrais Method, requires of the practitioner to attend closely to the systemic state of and individual student and use light touch so as to shift the student's systemic organization into a new dynamical configuration, ultimately improving the student's overall movement functioning. That is, smaller intervention may curiously generate greater effect on the system than the bigger, more global intervention. Though the global change may be more easily adopted as a whole, this would occur not as a gradual systemic shift (an accommodation) but as a break, similar to the arguments from dynamical systems theory cited earlier (Kostrubiec, Zanone, Fuchs, & Kelso, 2012; Smith & Thelen, 2003). By integrating some of the smaller variations within the whole, the person, while retaining a sense of agency, is better able to clarify their self-image and improve their self-use.

Functioning analogously, the Mathematics Imagery Trainer for Proportion guides students to shift gradually from additive to multiplicative reasoning: Students who initially believe the interval between their hands should remain invariant as they raise the hands are ushered to accommodate this reasoning so as to assimilate the target movement pattern. Thus students are led to conceptualize multiplicative structures (multiplication, division, fractions, ratio, and proportion) not as isolated from additive structures (counting, adding, subtracting) but as related variants on these structures.[2]

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[2] Resonance with this Feldenkrais Method technique of focused, nuanced intervention is found in discussions of coordination dynamics, wherein researchers transform the state of a system by manipulating the values of select attributes that thus serve as control parameters. Kelso (2000) explains the idea of *control parameters* as follows:

> These are analogous to what a social or behavioral scientist might call an independent variable. But the concept is entirely different, and the implications for experimental design in the social, behavioral, and cognitive sciences far reaching….In physical systems, control parameters refer to naturally occurring environmental variations or specific experimental manipulations that move the system through patterned states and cause them to change. (p. 65)

Complexity researchers are wont to reflect on the great efforts they invest in identifying a system's control parameter. Again, parallels to the work of Moshe Feldenkrais should not surprise us, given his training as a physicist during the dawn of cybernetics.

4. Conclusion

Within mundane sociocultural landscapes, mathematics and dance occupy dramatically disparate spaces. And yet the embodiment turn in the cognitive sciences is implicating these foreign disciplines as corporeally cognate—both transpire as sensorimotor activity, both avail from reflection. The interdisciplinary pas de deux presented herewith has only bolstered our growing conviction that our respective pedagogical worlds have much to share and debate. We have only scratched the surface.

When mathematical learning is conceptualized as sensorimotor exploration and entrainment, mathematics and dance appear to have similar pedagogical practices. Both center on students' subjective and idiosyncratic phenomenology of movement and both attempt to stimulate students' awareness of their action such that the students modify the action so as better to accord with a normative or desired cultural practice.

Movement as an objective construct is experienced and enacted via a myriad of subjective sensorimotor dispositions. Instructors should not shy away from this diversity of perspectives but rather leverage it by creating opportunities for students to bring to bear their own meanings, discover and assimilate new meanings, surface, reflect on, and accommodate existing meanings, and integrate these different meanings as conceptually complementary for the practice. One instructional methodology for achieving these results is to create conditions that reposition movement as a means of achieving an end, that is, as a tool for wielding environmental change. Within this framework, instructors should strive to maintain a dynamical construction of movement even as the analytic discursive formulations of movement are liable to stop and stave action. In all this, awareness of one's own movement is essential for bringing forth productive change. Instructors minded by this systemic approach to the phenomenology of movement stand a greater chance to foster critical and generative change through nuanced intervention.

As we step back to evaluate our collaboration in building this manuscript, we recognize an elephant dancing in the room: Through the prism of our mutual interest in movement we have witnessed an astonishing convergence of essential ideas from strange bedfellows—distant schools of philosophical, theoretical, and empirical thought and cultural practice that rarely converse yet agree on the systemic conceptualization of human behavior, reasoning, and learning. This gives us hope that the embodiment turn in the cognitive science will permeate and consolidate diverse disciplines into a cohesive doctrine.

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