Discovery Reconceived: Product Before Process[1]

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Motivated by the question, "What exactly about a mathematical concept should students discover, when they study it via discovery learning?", I present and demonstrate an interpretation of discovery pedagogy that attempts to sustain its ideology yet address its criticism. My approach hinges on decoupling the solution-procedure *process* (applying analytic algorithm to a situation under inquiry) from its resultant *product* (material displays or multimodal utterance, e.g., inscriptions, that experts interpret as bearing meanings pertaining to properties, relations, patterns, or structures in the situation). Whereas theories of learning often focus on process as the site of discovery, I propose to focus instead on product. Specifically, I view student discovery of mathematical concepts as guided heuristic–semiotic alignment of the product of mathematical analysis process with informal inference from naively seeing situations. I support my thesis with analyses of two vignettes, in which perception-driven design for intensive quantities was implemented as follows: (1) elicit students' perceptual judgment for a property of a situation created specifically so that the judgment agree with accepted theory; (2) guide students through enacting the analytic process for determining this property; and (3) help students see that the product of this process agrees with, and perhaps amplifies, the original judgment.

Discovery-based learning has been critiqued as an illogical, under-structured, ultimately unproductive pedagogical regime, whose rationale is incompatible with normative human practice (see Kirschner, Sweller, & Clark, 2006; Klahr, 2010; but cf. Schoenfeld, 2004). Yet whenever such critique is grounded in empirical studies demonstrating poor learning, the critique's validity necessarily hinges on the extent to which the research site indeed implemented the pedagogical principles under scrutiny (e.g., Cohen, 1990). That is, it could be that the fundamental notion of guiding learners to reinvent human knowledge is both psychologically powerful and culturally–historically viable, only that its actuation in the form of particular instructional designs has not always been conducive to sharing an appreciation for its ideology with the broader community of scholars, practitioners, and consumers. One might discern even within the bastions of discovery-based learning similar sentiments with respect to the more outspoken positions on what accounts for good instruction. Such acknowledgement is aired when educational philosophers temper their argumentation in an attempt to render their idealism convivial with school praxis and thus to negotiate continued access (Doorman et al., 2007; von Glasersfeld, 1992). However, a liability of apologetically patching up a staunch philosophy so as to meet pragmatic constraints is that we get a diluted ideology rendered effete: discovery-based learning becomes "not here and not there," with students not discovering knowledge and researchers not discovering why. Perhaps deeper questions should be asked.

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This presentation is motivated by commitment to the philosophical ideology underlying pedagogical methodology for discovery learning, namely the epistemological notion of grounded, meaningful, generative knowledge. Yet the essay is also motivated by concern that this ideology has been implicitly misinterpreted when instantiated in the form of curriculum and instruction, ultimately to the detriment of its very target population, the students. Accordingly, I will be offering and illustrating an alternative, empirically based, theoretical articulation of discovery pedagogy that attempts to sustain its ideology yet address its criticism. The research question framing this alternative approach is, "What exactly about a mathematical concept should students discover via discovery learning?"

My plan here is to pursue the research question by reflecting on two case studies of children who participated in implementations of pedagogical activities of my design. Empirical data from these and other studies have served me over the past decade as contexts for inquiry into the cognition and instruction of mathematical concepts, an inquiry that, in turn, keeps feeding back into further design and articulation of design principles. Here I will use these data to offer an empirically grounded "centrist" answer to the research question of what students should discover, at least with respect to a particular class of mathematical concepts (intensive quantities) as embodied in a particular type of design (perception-based learning). My choice of intensive-quantity concepts, such as geometrical similitude, density, chance, and slope, as the target notions of the designs explored in this paper is due to the human capacity to perceive and act on these phenomena as *a priori* perceptual gestalts (Gelman, 1998; Suzuki & Cavanagh, 1998; Xu & Garcia, 2008). In particular, my case studies involve designs for proportion and for probability, with participants aged 8.5 and 11, respectively.

The rationale of my proposal hinges on a common distinction between process and product in mathematical learning activities. Namely, by *process* I am referring to a general problem-solving sequence: (a) construing, parsing, and modeling a realistic situation along dimensions relevant to goal information; (b) determining targeted values within these dimensions (by enumeration and/or measurement); (c) manipulating these extracted values algorithmically with the aid of further mathematical instruments, tools, forms, and media, such as inscribing and developing an algebraic formula; and finally (d) reinterpreting obtained values or inferences in light of the source situation (e.g., Verschaffel, Greer, & De Corte, 2000). By *product* I refer primarily to any of the milestone mathematical displays created through engaging the process—I mean the objects themselves, irrespective of information that experts can glean from these artifacts. Yet products may also be embodied in multimodal utterance, such as speech or gesture. An example of a product would be the event space of a probability experiment, which is created through combinatorial analysis of a random generator—a novice can be guided to build this product, yet only an expert can infer from it anticipated outcome distributions. My thesis on discovery uses these terms as follows.

Unlike cognitive-developmental perspectives by which all conceptual learning is perforce subjective invention or construction (Piaget, 1968) and unlike pedagogical philosophies by which students are guided to reinvent solution processes (Freudenthal, 1986), here it is not the process that the child reinvents but a way of accepting its product as meaningful and valid, even though the product may initially have made no sense at all or even *because* it appeared to imply a conflicting inference. That is, guided reinvention of mathematical concepts may sometimes be guided heuristic–semiotic alignment of the product of mathematical analysis process with informal inference from naively seeing situations.

The objective of this presentation is emphatically *not* to suggest that all mathematical learning emulates or should transpire along product-before-process trajectories. Here I drew on a decade of design, in which our rationale was to ground students' mathematical meaning in *perception*-based tasks. Namely, student experiences in the activities discussed in this paper are grounded in their intuitive perceptual judgments for properties of a situation under scrutiny; the students then re-articulate these judgments via mathematical tools (Authors). Yet more recently we have been exploring designs that ground meaning in *action*-based tasks. Namely, the designs engage students in embodied-interaction problem-solving activities, in which they are guided to devise, rehearse, and articulate immersive

perceptuomotor solutions yet then encapsulate and re-articulate these solutions using available mathematical resources (Authors). In both design genres we witness students bootstrap pedagogically targeted ways of seeing, thinking, and speaking by virtue of appropriating symbolic artifacts as ad hoc means of better accomplishing a discursive or enactive goal yet, in so doing, surreptitiously reconfiguring their naïve perspectives in light of emergent affordances of these auspiciously available cultural resources.

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