Abrahamson, D. (2016). The ecological dynamics of mathematics education: The emergence of proportional reasoning in fields of promoted action (Invited keynote in W. van Dooren & G. Wiiliams [Co-Chairs], Topic Study Group 27: Learning and cognition in mathematics). In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education (ICME 13).* Hamburg: University of Hamburg.

THE ECOLOGICAL DYNAMICS OF MATHEMATICS EDUCATION: THE EMERGENCE OF PROPORTIONAL REASONING IN FIELDS OF PROMOTED ACTION

Dor Abrahamson

University of California, Berkeley, USA

Originating in the fields of kinesiology and sports science, the theory of ecological dynamics draws on ecological psychology and dynamical-systems theory to explain processes of motor-action learning in sociocultural contexts. From this systemic view, learning is the achievement of new task-oriented dynamical stability between an agent and its environment, and teaching is the sociocultural orchestration of constraints that steer the agent toward developing new motor-action coordination. Ecological dynamics could therefore frame empirical investigations into students' multimodal behaviors as they engage in solving physical-interaction problems designed to ground mathematical concepts. I summarize analyses of integrated clinical, action-logging, and eye-tracking data from a design-research project centered on the Mathematical Imagery Trainer. I propose that recent technological innovations for fostering as well as analyzing multimodal action may revive Piagetian scholarship and design.

CONSTRUCTIVISM REDUX: LOOKING BACK, LOOKING FORWARD

What if Piaget was right?

What if the core phenomenology of understanding and using mathematical concepts differs from the operations of representing and elaborating them as formal symbolic notation (Piaget, 1952)? What if, in particular, learning a new mathematical concept is necessarily the construction of a new sensorimotor coordination oriented on a new phenomenal category (Piaget, 1968)? And what if mathematical operations "are still actions although they are carried out mentally" (Piaget, 1971, p. 6)? What, moreover, if there are no representations in the mind, so that any would-be psychological 'structure' is not encoded as symbolical propositions but rather just epiphenomenal to engaged perceptual construction? What if knowledge is borne as a web of dynamically enacted complex relations between individuals and their environments, so that human cognition is inherently situated (Piaget, 1970)? What, indeed, if even expert mathematical modeling, reasoning, and problem solving still tacitly drew on sensorimotor operatory schemes? Because if all this is true, then we are not teaching mathematics as best we could nor are we best researching mathematics education.

Are we, like Piaget, centering our theory of learning on embodied activity? Or rather, are we only paying lip service to the embodiment perspective yet assimilating it into cognitivist epistemology (Gallagher, 2015)? Early in their careers, educational researchers all study Piaget. Later, many will avow constructivist or even radical-constructivist perspectives and apply Piagetian methods in their research studies. But in fact how Piagetian are activities that well-intending educational design researchers commonly build for children to learn mathematical concepts? Are these activities geared to create opportunities for students to develop new sensorimotor coordination? And what about our empirical instruments and analytical methods? How far have we progressed beyond Piaget's semi-structured clinical interviews and micro-ethnographic analysis of videography? Are we, for example, capturing evidence of reflective abstraction? Would it not be interesting if we could see—literally see—the ontogenesis of a mathematical concept? Would it not be worthwhile to know if there are better ways to teach a foundational concept in mathematics, such as proportion?

Recent empirical findings lead us to believe that we can observe empirically the ontogenesis of mathematical concepts—we can literally see, in real time, the situated psychological process of inventing and utilizing new higher-order objects that give rise to mathematical reasoning (Shayan, Abrahamson, Bakker, Duijzer, & van der Schaaf, 2015). And yet clearly this construction of objects and affordances is culturally designed and mediated. To make sense of these data, we have thus sought an expansive theoretical perspective, by which genetic epistemology (Piaget, 1968) could be viewed as complementary both to sociocultural (Vygotsky, 1926) and ecological-psychology theory (Gibson, 1977). That expansive theoretical perspective is ecological dynamics, a sports-sciences view on learning and teaching complex motor action (Vilar, Araújo, Davids, & Renshaw, 2012).

We now are prepared—theoretically, methodologically, and technologically—to take Piagetian scholarship to the next level, where we design learning environments with the explicit objective of creating opportunities for students to develop new sensorimotor schemes from which mathematical meanings emerge. And we are keen to conduct and collaborate on design projects serving as empirical contexts by which further to study the pedagogical implications of ecological dynamics.

This brief paper argues for theoretical and practical utilities of an ecological-dynamics view on mathematics learning as constructivist–sociocultural emergent process. We do so by means of demonstrating the contribution of this view to a thematic succession of design-research projects.

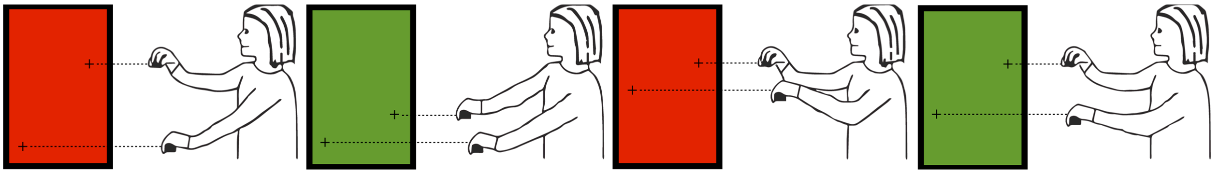
TRACKING LEARNING: EVOLUTION OF A DESIGN-BASED RESEARCH PROGRAM

The Embodied Design Research Laboratory (EDRL, UC Berkeley) has been collaborating with researchers from Utrecht University to study the emergence of mathematical concepts from guided sensorimotor activity (Shayan et al., 2015). This effort builds on and extends EDRL's earlier design-based research project, *Kinemathics* (Abrahamson & Howison, 2008), that contributed insights into the embodied and enactive nature of mathematics learning and teaching (Abrahamson et al., 2011, 2012, 2014; Abrahamson & Trninic, 2015; Howison et. al, 2011; Hutto, Kirchhoff, & Abrahamson, 2015). Below, we first review EDRL's initial design rationale, artifacts, and empirical work and then discuss findings from the current collaboration. We will propose the significance of our work for honing ecological-dynamics perspectives and revisiting Piaget's genetic epistemology.

The Mathematical Imagery Trainer for Proportion: Hypothesizing Attentional Anchors

Our investigation of mathematical ontogenesis as constructivist–sociocultural embodied process uses technological platforms with the capacity to carry educational activities by which students develop new sensorimotor schemes as situated solutions to interaction problems (Abrahamson & Lindgren, 2014; Lindgren & Johnson-Glenberg, 2013). We have been attracted to NUI (natural-user-interaction platforms), by which we attempt to launch mathematical learning as direct physical actions on virtual objects, unmediated by verbal utterance or symbolic notation. Our didactical design is conceptually oriented and informed by educational-research literature on target content domain. As design-based researchers (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), we view the practice of envisioning, engineering, and evaluating didactical activities as creating intellectual and empirical context wherein to pursue emergent problems germane to the learning sciences.

The *Kinemathics* project took on the design problem of students' enduring conceptual challenges with the target content domain of proportional relations. We assumed that students have scarce sense of what proportional equivalence is, feels, or looks like. We began by choreographing a bimanual motor-action scheme that enacts proportional equivalence. We then created technological conditions for students to move in a new way that emulates this physical–dynamical scheme. Our two-step activity plan was for students to: (1) develop a target motor-action scheme as a dynamical solution to a situated problem bearing no mathematical symbolism; and (2) describe these schemes mathematically, using semiotic means we then interpolate into the action problem space.

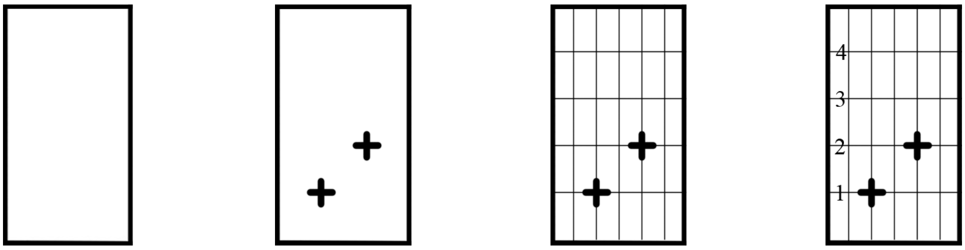


a. b. c. d.

Figure 1. The Mathematical Imagery Trainer for Proportion (MIT-P) set at a 1:2 ratio, so that the favorable sensory feedback (a green background) is activated only when the right hand is twice as high along the monitor as the left hand. This figure sketches out our Grade 4 – 6 study participants' paradigmatic interaction sequence toward discovering an effective operatory scheme: (a) while exploring, the student first positions the hands incorrectly (red feedback); (b) stumbles upon a correct position (green); (c) raises hands maintaining a fixed interval between them (red); and (d) corrects position (green). Compare 1b and 1d, the two green configurations, to note the different vertical intervals between the cursors. (Art acknowledgment: Virginia J. Flood)

Our design solution was the Mathematical Imagery Trainer for Proportion (MIT-P, Fig. 1). We seat a student at a desk in front of a large, red-colored screen and ask the student to "make the screen green." We do not disclose the target concept or mathematical context. The screen will be green only if the cursors' heights along the screen relate by the correct ratio (e.g., 1:2). Participants are tasked first to make the screen green, then to maintain a green screen while they move their hands.

The activity advances along a sequence of stages, each launched when the instructor introduces a new display overlay immediately after the student has satisfied a protocol criterion (Fig. 2). The full design includes a ratio table for students to control the cursors indirectly via inserting numbers. (For an iPad version that we have used in classrooms, see www.tinyurl.com/FreeMITP).



a. b. c. d.

Figure 2. MIT-P display schematics, beginning with (a) a blank screen, and then featuring the virtual objects (symbolic artifacts) that the facilitator incrementally overlays onto the display: (b) cursors; (c) a grid; and (d) numerals along the y-axis of the grid. For the purposes of this paper, these screen schematics have been simplified and not drawn to scale nor is the ratio table displayed.

We implemented the MIT-P design in the form of a tutorial task-based clinical interview with 22 Grade 4 – 6 students. We explored both individual and paired work, and these sessions were audio–video recorded for subsequent analysis (Reinholz, Trninic, Howison, & Abrahamson, 2010). Our primary methodological approach is for the laboratory's researchers to engage in collaborative ethnographic micro-analysis of selected brief episodes from the entire data corpus (Siegler, 2006), where we focus on the study participants' and tutors' range of physical actions and multimodal utterance around the available media (Ferrara, 2014). This analytical process is iterative and in dialogue with the learning-sciences literature, leading to the progressive identification, labeling, and refinement of emergent categories (Strauss & Corbin, 1990). Patterned behaviors that cannot be explained by existing literature may generalize as ontological innovations (diSessa & Cobb, 2004).

One recurring behavioral pattern in our empirical data that drew our attention was students' spontaneous invention of new objects or gestalts in the interactive space, such as manipulating the interval between their hands (e.g., see Fig. 1). A spatial interval is an immaterial yet bonafide phenomenological object. Its subjective invention, as suggested by students' actions and discourse, appears to have promoted both performance and reflection. Manipulating the interval, students first developed dexterity in solving the interaction problem and then adopted the interpolated frames of reference, such as the grid, to mathematize the solution scheme. For example, students would say and demonstrate, "The higher the hands go, the bigger the distance between them"; once the grid was introduced (see Fig. 2) they would shift to saying, "For every 1 unit I go up on the left, I go up 2 units on the right." Thus the interval popped into students' consciousness as an artifact wedged between self and environment, a hybrid instrument serving both to organize new motor-action coordination and enable articulation, representation, and formalization of this situated activity.

Though we were witnessing conceptual ontogenesis from sensorimotor activity, à la Piaget, initially we did not realize this. Instead, we approached our empirical data from the perspective of recent developments in cognitive science that emphasize the embodied, enacted, extended aspects of human cognition. Broadly I refer to a manifold of converging positions that all mental activity is necessarily the enactment—overt or covert—of sensorimotor interactions with the natural and sociocultural environment (Clark, 1999; Kiverstein, 2012). This post-cognitivist approach rejects dualist theorizations of conceptual knowledge as disembodied symbolic propositions in an isolated brain (Barsalou, 2010). Instead, learning and reasoning are grounded in situated, goal-oriented sensorimotor schemes; and formal knowing is the discursive signification and representational elaboration of these schemes (de Freitas & Sinclair, 2014; Hutto et al., 2015; Nemirovsky, 2003; Roth, 2014; Varela, Thompson, & Rosch, 1991). As we considered these dynamical and systemic views on individual learning in the cultural context, we realized an auspicious opportunity to reconsider and integrate historical educational theory, in particular Piaget's genetic epistemology in dialogue with Vygotsky's cultural-historical psychology (see also diSessa, Levin, & Brown, in press). In particular, we came to view the MIT-P as a *field of promoted action* (Reed & Bril, 1996): a constructed social setting for enacting evolved cultural practices that engage novices in discovery-based development of new motor-action coordination. Still, what role does the interval between the hands play in the MIT-P field of promoted action? We view the interval as an attentional anchor:

Borrowed from radical-enactivist and ecological-dynamics perspectives on sports cognition and performance, *attentional anchors* are real or projected environmental elements, features, or aspects that mediate teaching, learning, or enactment of motor-action skill (Abrahamson & Sánchez-García, in press; Hutto et al., 2015; Hutto & Sánchez-García, 2015). A table-tennis coach may ask a novice to project an imaginary right-angled triangle onto her field of action: Two sides are inscribed by a ball's bounce onto her table, the third side by her paddle striking the ball (Liao & Masters, 2001). Attentional anchors invoke into the perceptual manifold what kinesiologists call order parameters or "steering wheels," pivotal action structures that simplify complex physical tasks (Kostrubiec et al., 2012; Newell & Ranganathan, 2010). As such, discovering the interval collapses an overwhelming task of controlling independent manipulations of two cursors into a manageable bimanual task of steering a single focal object. Thus the attentional anchor productively constrained the agent's grip on the world. It moreover then became an objectified entity mobilizing mathematical discourse.

Still, our hypothesis regarding the psychological constitution of attentional anchors was based only upon clinical data. Furthermore, we did not know whether students arrive at attentional anchors via "bottom up" discovery or "top down" inference. This is where eye-tracking technology entered.

Eye-Tracking the Emergence of Attentional Anchors

Eye-tracking technology gathers streaming data of a person's foveal vision point—the running "where" of visual perception. These data, for example 60 logs per second, are later visualized for analysts investigating perceptual behavior in specific contexts of interest, such as game developers studying children's interaction with tablet-based applications. Some devices (e.g., Tobii, Fig. 3, on left) videotape a user's manual actions on a focal device while tracking their concurrent eye gaze. These data can be combined by overlaying eye-gaze scan-path traces on the videography (Fig. 3).

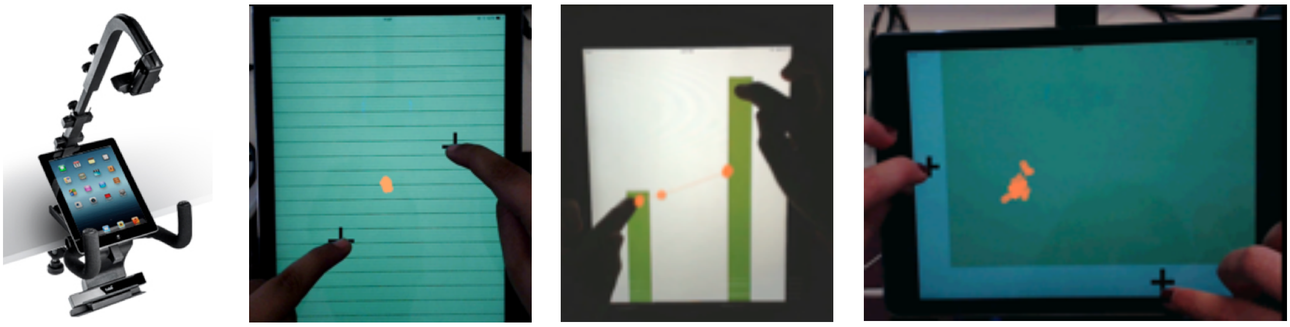


Figure 3. Integrated visualization of multimodal interaction: eye-gaze tracking overlaid on hand-action videography. Concurrent dynamical patterns in gaze scan-path (orange spots & lines) and bimanual manipulation suggest the emergence of new motor-action coordinations oriented on new attentional anchors; task effectiveness improves, and clinical self-reports of new strategies ensue.

Empirical data from eye-tracking studies with the Utrecht build of the Mathematical Imagery Trainer have corroborated the finding that students invent attentional anchors as their spontaneous solution to challenging bimanual-coordination problems (see Fig. 3, Shayan et al., 2015). Our dynamical integrated visualizations, along with audio recordings of verbal utterance, enable us to reconstruct the subjective psychological evolution of new objects or constellations of two or more objects in the student's sensorimotor interaction space: We literally see unmarked features of the environment foreground and coalesce into dynamically stable steering wheels that students handle as utensils for operating on the environment in accord with the task objective (keeping the screen green). Moreover, we see immediate impact of tutor intervention on student forms of engagement. For example, as each student manipulated two cursors—one in the left hand, one in the right—they began spontaneously to gaze at locations on the screen and track invisible moving targets, even though these "objects" bore no contoured optical stimuli (Fig. 3). Responding to the tutor's prompt, students referred to properties of these emergent objects—size, elevation, angularity, etc., even as their performance improved. Attentional anchors were instrumental to solving interaction problems.

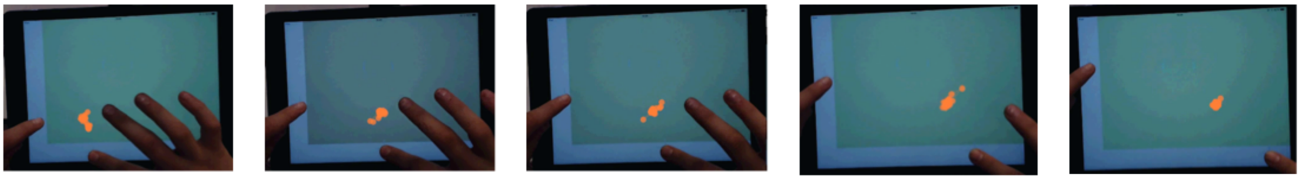


Figure 4. A student uses a spontaneous attentional anchor—an imaginary line between his two index fingertips—to guide his bimanual coordination of proportional (1:2) motion along orthogonal axes. Note how the gaze is always one step ahead of the line, anticipating the line's next placement.

Figure 4 presents an image sequence from the dynamical integrated-data visualization of a tutorial trial with Lars (pseudonym), a 14 years-old low-track vocational-education student. Here Lars is playing in the orthogonal mode, where one tries to keep the screen green while sliding the left- and right-index fingertips along the y and x axes, respectively. Prior to this episode, Lars's gaze had alternated between the left and right fingertips. He then began gazing at a moving point *between* the fingertips, where there were no explicit stimuli. To explain his technique, Lars gestured an imaginary diagonal line subtended by his fingertips—attending to this line, he slid his fingertips along their axes so as to shift the line rightward, all the while maintaining constant the line's angle relative to the x-axis as well as correcting per the screen color. Other students tackling this puzzle organized their orthogonal bimanual action by gazing at their fingertips' Cartesian point $[x, y]$. They thus completed with their eyes a rectangle configured also from the origin and two fingertips. Similar to Lars, as they slid their fingers their gaze ran along a diagonal trajectory. Those students were reinventing the Cartesian field as a means of solving an embodied-interaction problem.

CONCLUSION: EYE-TRACKING PIAGET?

Originating in kinesiology and sports science, the theory of ecological dynamics bears relevance for the scholarship and practice of mathematics education. In particular, the construct of attentional anchors—emergent ontologies serving better grips on the world—illuminates processes of situated learning. Learning via engaging in embodied-interaction tasks is a tentative process: Students construct new motor-action coordinations oriented on attentional anchors, then entify these in multimodal discourse and by appropriating forms introduced by teachers. We presented empirical data as evidence for Piaget's reflective abstraction, specifically coordination (a new sensorimotor scheme oriented on a new category) and encapsulation (objectification of said scheme/category). Evidence of attentional anchors promotes a research agenda looking to understand conceptual ontogenesis from a reconciled constructivist–sociocultural perspective. Namely, the phenomenon of students spontaneously re-inventing mathematically bound sensorimotor schemes within culturally designed/facilitated fields of promoted action underscores the complementary roles of individual and cultural vectors in achieving new dynamical equilibria that ground conceptual meanings.

These are early, exciting days for the research method of multimodal learning analytics (Worsley & Blikstein, 2014), especially when applied to pedagogical design for multimodal learning. New theoretical and practical questions emerge as we iterate through our study cycles. In particular, now that we can see how students construct and manipulate subjective objects in the environment, we are more aware of gaps between naïve and formal visualizations. For example, the dynamical eye-tracking images of Lars operating an imaginary diagonal line to make the screen green alert us to a problematic discrepancy between his idiosyncratic dexterity and the incompatible semiotic potential latent to the particular symbolic artifact (the grid) we interpolated into his interaction space. We must mind these gaps so as to offer students more effective relations between embodied-interaction solutions and normative forms. Lars's diagonal, for example, could cue the tutor, whether human or artificial, to steer Lars toward measuring geometrically similar triangles as his entry into proportional concepts. Another child whose eye tracking suggests she is inventing a Cartesian attentional anchor would be steered to examine similar rectangles. And how would Lars respond if we overlaid his gaze path in real time onto his screen? Would he appropriate its linear trajectory?

We speculate that attentional anchors are the pivotal constructions of mathematical concepts: While in ecological-dynamics terms attentional anchors are emergent, self-imposed constraints affording subjective development of new motor-action coordinations, Piaget might concur that these are goal-oriented higher-order categories mediating operational conservation of environmental properties. Having rediscovered Piaget, we aim to extend his research by addressing a classical "chicken-or-egg" problem: What comes first, a new motor-action coordination or the new object it operates upon/through? Could we demonstrate empirically a dialectical evolution of sensorimotor schemes and their target categories? Is it important for students to discover attentional anchors themselves? What new conceptual domains should we explore next with our multimodal research paradigm?

References

Abrahamson, D., Gutiérrez, J. F., Charoenying, T., Negrete, A. G., & Bumbacher, E. (2012). Fostering hooks and shifts: Tutorial tactics for guided mathematical discovery. *Technology, Knowledge, and Learning, 17*(1-2), 61-86.

Abrahamson, D., & Howison, M. (2008). *Kinemathics: Kinetically induced mathematical learning*. Paper presented at the UC Berkeley Gesture Study Group (E. Sweetser, Organizer). Accessed September 27, 2015 at http://edrl.berkeley.edu/sites/default/files/Abrahamson-Howison-2008_kinemathics.pdf.

Abrahamson, D., Lee, R. G., Negrete, A. G., & Gutiérrez, J. F. (2014). Coordinating visualizations of polysemous action: Values added for grounding proportion. *ZDM–Mathematics Education, 46*(1), 79-93.

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences (2nd ed.)* (pp. 358-376). Cambridge, UK: CUP.

Abrahamson, D., & Sánchez-García, R. (in press). A call to action: Towards an ecological-dynamics theory of mathematics learning, teaching, and design. In T. Bartell & K. Bieda (Eds.), *Proc. of PME-NA 37*. East Lansing: PME-NA.

Abrahamson, D., & Trninic, D. (2015). Bringing forth mathematical concepts: Signifying sensorimotor enactment in fields of promoted action. *ZDM Mathematics Education, 47*(2), 295–306.

Abrahamson, D., Trninic, D., Gutiérrez, J. F., Huth, J., & Lee, R. G. (2011). Hooks and shifts: A dialectical study of mediated discovery. *Technology, Knowledge, and Learning, 16*(1), 55-85.

Barsalou, L. W. (2010). Grounded cognition: Past, present, and future. *Topics in Cog. Sci., 2*(4), 716-724.

Clark, A. (1999). An embodied cognitive science? *Trends in Cognitive Sciences, 3*(9), 345-351.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

de Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom.* New York: Cambridge University Press.

diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *The Journal of the Learning Sciences, 13*(1), 77-103.

diSessa, A. A., Levin, M., & Brown, N. J. S. (Eds.). (in press). *Knowledge and interaction: A synthetic agenda for the learning sciences*. New York: Routledge.

Ferrara, F. (2014). How multimodality works in mathematical activity: young children graphing motion. *International Journal of Science and Mathematics Education, 12*(4), 917-939.

Gallagher, S. (2015). Invasion of the body snatchers: How embodied cognition is being disembodied. *The Philosophers' Magazine, April*, 96-102.

Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting and knowing: Toward an ecological psychology* (pp. 67-82). Hillsdale, NJ: LEA.

Howison, M., Trninic, D., Reinholz, D., & Abrahamson, D. (2011). The Mathematical Imagery Trainer: From embodied interaction to conceptual learning. In G. Fitzpatrick, C. Gutwin, B. Begole, W. A. Kellogg, & D. Tan (Eds.), *Proceedings of CHI* (Vol. "Full Papers," pp. 1989-1998). NYC: ACM Press.

Hutto, D. D., Kirchhoff, M. D., & Abrahamson, D. (2015). The enactive roots of STEM: Rethinking educational design in mathematics. *Educational Psych. Review, 27*(3), 371-389.

Hutto, D. D., & Sánchez-García, R. (2015). Choking RECtified: Embodied expertise beyond Dreyfus. *Phenomenology and the Cognitive Sciences, 14*(2), 309-331.

Kiverstein, J. (2012). The meaning of embodiment. *Topics in Cognitive Science, 4*(4), 740-758.

Kostrubiec, V., Zanone, P.-G., Fuchs, A., & Kelso, J. A. S. (2012). Beyond the blank slate: Routes to learning new coordination patterns depend on the intrinsic dynamics of the learner—experimental evidence and theoretical model. *Frontiers in Human Neuroscience, 6*.

Liao, C., & Masters, R. S. (2001). Analogy learning: A means to implicit motor learning. *Journal of Sports Sciences, 19*, 307-319.

Lindgren, R., & Johnson-Glenberg, M. (2013). Emboldened by embodiment: Six precepts for research on embodied learning and mixed reality. *Educational Researcher, 42*(8), 445-452.

Nemirovsky, R. (2003). Three conjectures concerning the relationship between body activity and understanding mathematics. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings PME 27* (Vol. 1, pp. 105-109). Columbus, OH: Eric.

Newell, K. M., & Ranganathan, R. (2010). Instructions as constraints in motor skill acquisition. In I. Renshaw, K. Davids, & G. J. P. Savelsbergh (Eds.), *Motor learning in practice: A constraints-led approach* (pp. 17-32). Florence, KY: Routledge.

Piaget, J. (1952). *The child's conception of number*. London: Routledge.

Piaget, J. (1968). *Genetic epistemology* (E. Duckworth, Trans.). New York: Columbia U. Press.

Piaget, J. (1970). *Structuralism* (C. Maschler, Trans.). New York: Basic Books.

Piaget, J. (1971). *Biology and knowledge: An essay on the relations between organic regulations and cognitive processes* (B. Walsh, Trans.). Chicago, IL: The University of Chicago Press.

Reed, E. S., & Bril, B. (1996). The primacy of action in development. In M. L. Latash & M. T. Turvey (Eds.), *Dexterity and its development* (pp. 431-451). Mahwah, NJ: LEA.

Reinholz, D., Trninic, D., Howison, M., & Abrahamson, D. (2010). It's not easy being green: Embodied artifacts and the guided emergence of mathematical meaning. In P. Brosnan, D. Erchick, & L. Flevares (Eds.), *Proceedings of PME-NA 32* (Vol. VI, pp. 1488-1496). Columbus, OH: PME-NA.

Roth, W.-M. (2014). On the pregnance of bodily movement and geometrical objects: A post-constructivist account of the origin of mathematical knowledge. *Journal of Pedagogy, 5*(1), 65-89.

Shayan, S., Abrahamson, D., Bakker, A., Duijzer, C., & van der Schaaf, M. (2015). The emergence of proportional reasoning from embodied interaction with a tablet application: An eye-tracking study. In L. Gómez Chova, A. López Martínez, & I. Candel Torres (Eds.), *Proceedings of the 9th International Technology, Education, and Development Conference (INTED 2015)* (pp. 5732-5741). Madrid: IATED.

Siegler, R. S. (2006). Microgenetic analyses of learning. In D. Kuhn & R. S. Siegler (Eds.), *Handbook of child psychology* (6 ed., Vol. 2, pp. 464-510). Hoboken, NJ: Wiley.

Strauss, A. L., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, CA: Sage Publications.

Worsley, M., & Blikstein, P. (2014). Using multimodal learning analytics to study learning mechanisms. In J. Stamper, Z. Pardos, M. Mavrikis, & B. M. McLaren (Eds.), *Proc. of the 7th Int. Conference on Educational Data Mining* (pp. 431-432). London, UK: Institute of Education.

Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. Cambridge, MA: M.I.T. Press.

Vilar, L., Araújo, D., Davids, K., & Renshaw, I. (2012). The need for 'representative task design' in evaluating efficacy of skills tests in sport. *Journal of Sports Sciences, 30*(16), 1727-1730.

Vygotsky, L. S. (1926/1997). *Educational psychology* (R. H. Silverman, *Trans.*). Boca Raton: CRC Press.