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Leveling Algebra Transparency: Giant Steps Towards a New Approach to Learning?

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Abstract:
Despite historical re-design efforts, algebra continues to challenge many students. This design-based research study explored the conjecture that learners can mentally construct the technical properties of algebraic systems by manually constructing models of word problems. Drawing on the theory of subjective transparency, the design seeks to "level transparency" by creating a technological environment where students must discover each technical property, e.g., the uniform size of variable quantities, and only then the environment automatizes that property (the "reverse-scaffolding" principle). In a comparison study with forty Grade 4 and 9 participants, the study group outperformed a control group, for whom the technical features were fully pre-automatized. The emerging framework stands to inform the design of activities for algebra and other gate-keeper concepts.

1. Objective:

Algebra is a *praxis cogitans* (Radford, 2003)—a particular epistemic orientation toward situations, in which the magnitudes of selected objects as well as quantitative relations among them are scrutinized, determined, systematized, and elaborated. Algebra is most powerful when the emergent system of logico–mathematical relations inherent to a situation is modeled, converted toward, and encoded as a set of propositions in the symbolic register, because these propositions can be manipulated toward determining information that is embedded in the situation (Duval, 2006; Nathan, 2012).

As educators, we make choices regarding the situations that students investigate algebraically as well as the modeling forms or formats that mediate the converting of these situations to symbolic register. Our choices, in turn, are informed by a theory of learning as well as a pedagogical framework. Design-based research studies attempt to evaluate a conjecture pertaining both to a proposed instructional methodology and the merit of its motivating theory and framework (Confrey, 2005). This paper reports on findings from a design-based research study of algebraic cognition. The methodology consists of a technological environment for modeling-based algebra learning, and the pedagogical approach draws on the theory of transparency (Hancock, 1995; Meira, 1998).

2. Theoretical Framework

The theoretical construct of *transparency* captures relations between, on the one hand, artifacts inherent to a cultural practice, and, on the other hand, a social agent's understanding of how features of these artifacts mediate the accomplishment of particular practices (Meira, 1998). In general, artifacts bear information structures, logical relations, and activity constraints that offload intentionality onto external features (Kirsh, 2010;

Martin, 2009). However, artifacts that are used to foster content learning should be transparent, because figuring out how they work is tantamount to understanding content (e.g., compare a calculator to an abacus), a process Meira (1998) calls developing subjective transparency of an artifact.

The theory of transparency is important for education, because we must make detailed choices regarding which specific logico–mathematical operations to "blackbox" vs. "glassbox." A motivation for this study was a design team's practical deliberations over transparency, as we converted an activity from a mechanical device to a virtual environment. Eventually, we conceptualized algebra as a conceptual system deployed virtually as a modeling activity. We conjectured that individual students would achieve subjective transparency of the conceptual system by tinkering with the construction resources in attempt to reach parity, or functional fidelity, with a given problem situation. Thus learning algebra would be tantamount to developing construction know-how, that is, a set of situated heuristics and technical criteria for evaluating whether the virtual model preserves information structures implicit to the problem situation.

In order to formalize our design approach, we articulated the construct of "SILO," *situated intermediary learning objective* (Authors, ICLS 2014), which bears affinity to earlier work on the relation between tinkering and learning (Papert, 1980; Pratt & Noss, 2010; Wilensky & Reisman, 2006).

This dissertation project has now matured to the stage that all the empirical data have been gathered and currently under analysis. Two research questions have been formulated:

- *Design.* Can a technology-based learning environment foster student development of algebra transparency? What tasks, resources, and constraints could enable this process?
- *Theory.* What general instructional methodology might best inform the design of an environment fostering algebra transparency? In particular, is the generic notion of "scaffolding" useful in conceptualizing this methodology? If not, then might scaffolding be re-conceptualized?

3. Methods and Data Sources [please note that we have compiled these two sections]

3.1 Design-Based Research

Design-Based Research is characterized as an iterative approach. We embody a theoretically driven pedagogical conjecture in the form of an artifact (materials + activities). By implementing these products and collecting empirical records, we evaluate our conjecture. Typically, this cycle is iterated several times, leading to three types of products: refined theoretical models of teaching and learning, new artifacts that may serve in instructional units, and insights about design process (Authors, 2007).

3.2 Giant Steps for Algebra

The design of Giant Steps for Algebra (GS4A) seeks to investigate the potential of a new pedagogical approach to constructing algebraic transparency. GS4A draws on Dickinson and Eade (2004), who proposed the double-number-line algebra model (See Figure 1).

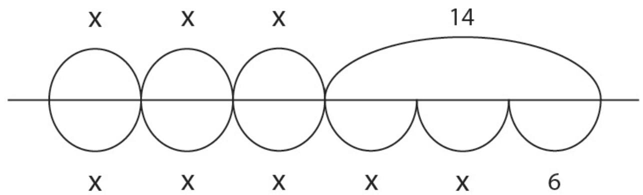


Figure 1. Number-line instantiation of “$3x + 14 = 5x + 6$”

This visualization of algebraic equivalence appears to facilitate an offloading of source information onto a diagram’s inherent logico-figural constraints. In terms of affordances for transparency, the number-line model renders highly salient—as compared to the classical balance-scale model—the logical relations between variable and integers, both within - and between expressions, in the form of its spatial features.

We used the number-line visualization in designing our learning activity. GS4A is a situation-based model (Walkington, Petrosino, & Sherman, 2013). Per the embodied-design framework (Abrahamson, 2009, 2012) GS4A seeks to engage and leverage students’ tacit knowledge about simple ambulatory motion and spatial relations.

A Giant walked 3 steps and then another 2 meters. She buried the treasure. On the next day, she wanted to bury more treasure in exactly the same place, but she was not sure where that place was. She walked 4 steps and then, feeling she’d gone too far, she walked back one meter. Yes! She found the treasure!

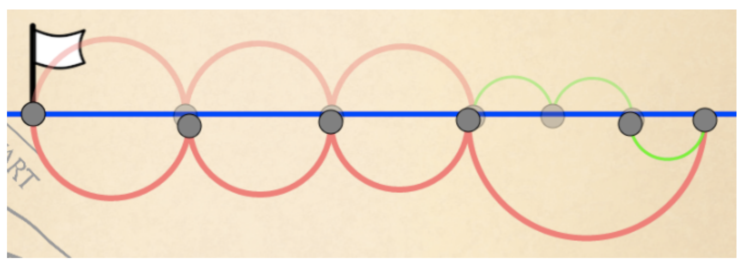
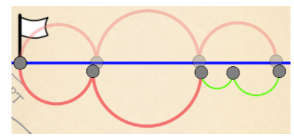
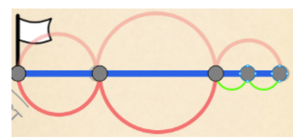
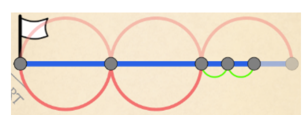
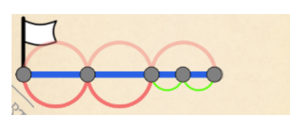


Figure 2. GS4A narrative and model. On both Day 1 and Day 2, the giant travels from the flag on the left, toward the right. Red loops represent giant steps, green loops represent meters. Day 1 is marked above the line, Day 2 is marked below the line.

Based on pilot work with a range of mechanical devices using a variety of concrete media (Authors, IDC 2013), we realized that constructing models within GS4A was composed of constructing transparency for finer-grained phenomena (Abrahamson, Chase, Kumar, & Jain, 2014) that we named SILOs (*situated, intermediary learning objectives*). As we turned to redeploy GS4A in a scalable technological format, the SILOs proved instrumental as a blueprint for the activity architecture. Therein, students transition from each interaction phase to the next upon demonstrating, via their electronic actions, mastery over one of the SILOs. The idea is thus to step learners through an activity while enabling them to build subjective transparency of the emerging model. Borrowing the notion of “levels” from popular computer games—the gradual rewarding of manifest competency with increased power that is linked to increased skill—in GS4A we *level transparency* (see Table 1 for the specific set of SILOs in relation to the interaction features). At each new level, the technology offloads the SILO from the user—it automatically generates and maintains the specific technical details and functional relations that the user had just discovered.

Table 1.

Leveling Transparency: Matched SILOs and Levels in the Giant Steps for Algebra Technological Design

SILO	Level	System Constraints, User Activity, and Behavior Criterion	Interface
1. Consistent Measures	1. Free Form	System offers no support in coordinating units or expressions.	
	Activity	User builds all parts of the model manually.	
	Criterion	User expresses frustration in equalizing units.	
2. Equivalent Expressions	2. Fixed Meters	System generates meter units in predetermined size and maintains uniform size automatically.	
	Activity	User builds variables manually.	
	Criterion	User expresses frustration with managing uniform variable units the lengths of Days 1 & 2.	
3. Shared Frame of Reference	3. Stretchy	System monitors for manual adjustment to the size of *any* of the variable units and accordingly adjusts the size of *all* variable units.	
	Activity	User adjusts the variable size to equalize the two propositions.	 
	Criterion	User reads off the value of a variable unit in terms of the number of known units (meters) it subtends, e.g., one giant step is 2 meters long.	

3.3 Research Design and General Methods Information

The rationale of the experimental design was to measure and then compare learning under two conditions: *with* the leveling-transparency functionality (study group) and *without* it (control group). In the control group, users would not need to "earn" the automatic functionalities—they would receive *ab initiio* a fully-fledged technological application.

We hypothesized that the control-group students would thus experience reduced opportunity to develop subjective transparency of the conceptual system.

Forty Grade 4 and 9 students (18 male, 21 female) from an independent elementary and public high school voluntarily participated individually in task-based semi-structured interviews (Clement, 2000; Ginsburg, 1997)—20 in the study group ("Leveled"), 20 in the control group ("Automatic"). The interviews took place in a quiet room on the school sites, and were all videotaped for subsequent analysis.

3.5 Analysis

Using micro-genetic analysis techniques (Siegler, 2006), we have been working to assess the participants' development of subjective transparency of the algebra conceptual system. The emerging coding structure is designed to identify and characterize moments when the participants articulate understanding of each SILO, that is, each structural feature of the model.

4. Results and conclusions

4.1 Main Effect

From preliminary (pre-quantitative) analysis, we have tentatively inferred that study-group participants developed greater subjective transparency of the algebraic system as compared to the control-group participants. At this point, we can demonstrate the gist of the main effect by offering vignettes from our empirical records, as follows below.

4.2 Data Samples

Susan (pseudonym) is working in the Leveled condition (study group). She is at Level #1, working on a narrative corresponding to the formal proposition "$4x = 3x + 2$." She has completed the Day 1 travel diagram (see in Figure 3 the four red loops above the horizontal line) and is now working on the Day 2 travel diagram below the line.

Res.: Ok. So she goes…
Susan: 3 giant steps and…..
Res.: …and then....
Susan: 2 meters. (switches computer's interactive feature to "meters," draws 2 equivalent meters that subtend the 4th giant step immediately above it)
Res.: So she goes 2 meters and then she finds the right spot
Susan: Yeah
Res.: So in your drawing did she find the right spot?
Susan: Hmmm well yeah.

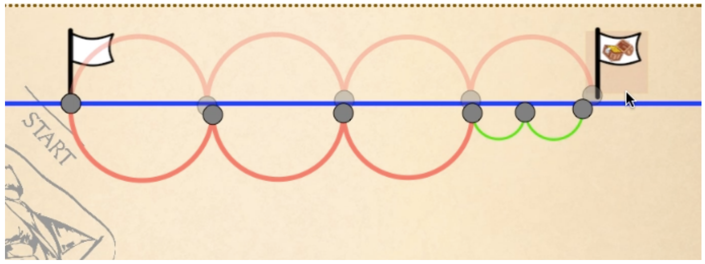


Figure 3. Susan's construction for a Giant Steps story corresponding to $4x = 3x + 2$.

Immediately, Susan has identified that the end point for both days is in the same screen location (see treasure flag in Figure 3, on the right) and that, consequently, the 2 meters on Day 2 will subtend the same distance as the 4th giant step on Day 1.

We now turn to Karrie (pseudonym), a participant in the control group, who is working on the same item.

<u>Karrie</u>: It says she walks 2 steps further ahead and finds the treasure. But that doesn't make sense because it is more back than the other treasure. (Karrie has drawn a model in which the giant steps are too large so that the respective ends of Day 1 and Day 2 are not aligned).

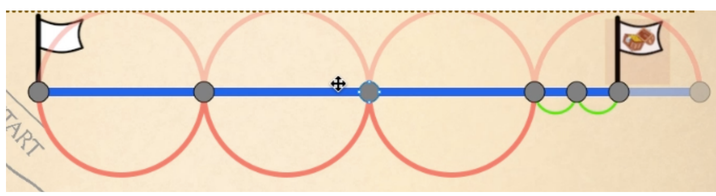


Figure 4. Karrie's construction for a Giant Steps story corresponding to $4x = 3x + 2$.

Concerned by this misalignment between the end points of Days 1 and 2, Karrie suggests inserting additional meters. The interviewer responds by stating that doing so would change the information in the story.

<u>Karrie</u>: I can change the size of the giant steps.

In an attempt to stretch the Day 2 travel diagram so that it reaches the treasure flag, Karrie stretches the giant steps in Day 2 (the red loops below the blue line, see Figure 4). Recall that in the Automatic mode the variable distances (all the red loops) are interlinked, both within - and between days. Consequently, the variables in both Day 2 *and* Day 1 all stretched uniformly, and the two misaligned ends only become farther apart. Karrie then attempted the same maneuver by decreasing the step size in Day 1, but still she could not make ends meet.

<u>Karrie</u>: It moves the whole thing.

Karrie has not achieved the SILO of equivalent quantities. Consequently she is also having difficulty in appreciating the implication of uniform variable size for the intactness of her story line. To Karrie, neither algebraic feature is transparent.

Based on this and many other preliminary observations, we are inclined to assert tentatively that GS4A indeed fosters algebraic transparency, and the leveling transparency activity architecture enables this process (Research Question 1).

Furthermore, the generic notion of "scaffolding"—that is, that educators should facilitate learning via co-enacting aspects of complex practice—does not foster algebra transparency. We, therefore, propose that an alternate methodology should be considered. The Leveling Transparency architecture offers just this, in that it can be conceptualized as *reverse scaffolding*—the technological system co-constructs the model with the student only once the student understands the necessity and functionality of each specific property of the model, thus relieving the user of what they *know* to do rather than what they *do not know* to do.

4.3 Future Work

By the time of the conference, a fully-fledged coding system will have been devised and implemented, so as to evaluate the proposed activity quantitatively. In parallel, qualitative analyses of the data corpus will continue to inform our emerging understandings of relations between theory of learning, pedagogical framework, artifacts, and interaction.

5. Educational or scientific importance of the study

Students persistently struggle in algebra courses, so much so that algebra is referred to as the "gate-keeper" course for college acceptance and completion. Identifying and empirically testing learning artifacts that support the construction of algebraic transparency is essential for solidifying the success of students in mathematics moving forward. Additionally, ubiquitous access to computerized devices must be matched by effective pedagogical frameworks. Leveling transparency, as a theoretical principle for designing technology-based constructivist learning activities could stand to significantly inform the design of pedagogical tools in many domains, and for all students. It is thus that educational research can contribute *toward justice*, per the AERA 2015 conference theme.

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