The Three M's: Imagination, Embodiment, and Mathematics

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The objective of this paper is to call for research into the mechanisms and potential agency of imagination in mathematical reasoning and to propose an agenda for such research. Drawing on a broad spectrum of resources in philosophy, the cognitive sciences, embodiment theory, and mathematics-education research, I conjecture that by learning mathematics in environments that support engagement of imagination, students could tap this powerful cognitive tool to support the construction and effective application of concepts. The research objectives are to: (a) investigate the roles of imagination in mathematical creativity, learning, and problem solving, e.g. exploring whether mathematicians' images are idiosyncratic, culturally mediated, or some combination thereof; (b) ground an understanding of the roles of imagination in current cognitive-science theories and pedagogical perspectives; (c) develop methodology for evaluating students' access to imagination as a cognitive resource and their imaginative engagement in learning activities; (d) outline principles for the design of objects and activities that encourage students both to engage in imaginative mathematical reasoning and, *inter alia*, to embrace imagination as an accessible cognitive resource; (e) design and build mathematical objects and create activities that encourage and guide utilization of imagination; and (f) research students' learning in these designed activities.

Overview

The objective of this paper is to call for research into the mechanisms and potential agency of imagination in mathematical reasoning and to propose an agenda for such research. By 'imagination,' I refer to the representative power to evoke, decompose, and recombine materials furnished by direct apprehension (Webster, 1913). The conjecture motivating this call is that by learning mathematics in environments that support engagement of imagination, students could tap this powerful cognitive tool to support the construction and effective application of concepts. The suggested research builds on work in phenomenology (Heidegger, 1962; Merleau-Ponty, 1992), critical theory and media studies (e.g., Benjamin, 1986; Taussig, 1993), psychotherapy (Gendlin, 1982), research in play, creativity, and intuition (e.g., Csikszentmihalyi, 1991; Huizinga, 1955; Piaget, 1962), cognitive science (e.g., Barsalou, 1999; Gibson, 1977), psycholinguistic studies of gesture (e.g., McNeill, 1992; McNeill & Duncan, 2000), the study of imagery and perception (e.g., Damasio, 2000; Decety, 1996; Kosslyn, 2005; Schwartz & Black, 1999), and embodiment theory (e.g., Abram, 1996; Fauconnier & Turner, 2002; Johnson, 1987; Lakoff & Núñez, 2000; Varela, Thompson, & Rosch, 1991), as well as on studies of the roles of spatial reasoning, visualization, and imagery in situations where mathematics, in particular, is used, learned, and researched (Arnheim, 1969; Davis, 1993; Fuson & Abrahamson, 2005; Urton, 1997). The JPS conference theme, *Art and Human Development*, appears as an auspicious, timely, opportunity to call for leveraging funds of thematic knowledge accruing outside of educational-research circles toward interdisciplinary, mutually fruitful, discourse on imagination, embodiment, creativity, and knowledge. Insight coming from such collaboration may be beneficial for mathematics-education researchers engaged in designing learning environments. Specifically, understanding the dynamic interplay of imagination, objects (e.g., mathematical representations), and content would contribute toward the development of theoretical frameworks for designing and facilitating mathematics learning environments as well as toward creating appropriate professional-development guidelines. The paper lays out foundational tenets of this call, explores some potentially useful resources, and suggests avenues of research.

Background: Why 3Ms?

Imagination plays a role in reasoning about objects (Shepard & Metzler, 1971), and it is pivotal in the emotive mechanism underlying decision making (Damasio, 2000). Imagery and perception draw on shared cognitive resources (Decety, 1996). Moreover, imagination has been anecdotally implicated as key to the creativity of mathematicians and scientists who generate, manipulate, and re-combine images (Hadamard, 1945; Poincaré, 2003; Polanyi, 1958; Steiner, 2001). Yet, although imagination is apparently important for mathematical practice, by and large mathematical content is not taught in ways that foster engagement of imagination.

Mathematicians manipulate images, students manipulate symbols. At least, in traditional curricula, students are not encouraged to engage in mathematical reasoning but, mostly, in rote solution of arithmetical problems (Schoenfeld, 1985). Yet, it could be that *deep learning* (what students should be doing) draws on the same metacognitive strategies and underlying cognitive faculties as *mathematical inventiveness* (what mathematicians do). If this is true, then even though students are learning 'ready-made' knowledge, they should be as deeply engaged and invested as were the mathematicians who first articulated this knowledge (see Wilensky, 1997).

Furthermore, if images are indeed pivotal in mathematical reasoning, we need theoretical models that explain the roles of imagination in learning. Where do images come from? How might *personal* images enable *interpersonal* communication-based mediation of mathematical understanding? How can mathematics-education practitioners foster students' construction and manipulation of mental images? What constitutes a "good" image for addition, for multiplication, for distribution? How does the goodness of such images relate to the canonical mathematical representations such as equations and graphs? How do "concrete" mathematical representations become these alleged invisible images? In turn, how do students incorporealize these images in problem solving? Above all, is mathematics not "abstract," amodal?

Drawing on embodiment theory, my premise is that mathematical concepts are not amodal but, in fact, *multi*-modal images -- rich spatial–dynamic simulations engaging different senses and different blends of these senses -- upon which "ride" mathematical reasoning, procedures, and vocabulary (Fuson & Abrahamson, 2005). When students begin learning a new concept, they already have embodied images of previously studied mathematical operations and relations. The teacher mediates new concepts that are embedded in static mathematical representations ("crystallized templates of action," Stetsenko, 2002) by modeling ways of interacting with these representations, thus mobilizing these representations, "lifting" them into the lived, contextualized, semiotic space (see also Taussig, 1993). Students learn by negotiating between their previously embodied images and these guided interpretations of the new mathematical representations (see also Abrahamson, 2004; Johnson, 1987). Subsequently, newly perceived situational contexts cue these images back into the embodied space to support mathematical reasoning by evoking relevant sensations or imagined perceptions of body movement and feeling (Nemirovsky, Noble, Ramos-Oliveira, & DiMattia, 2003; Root-Bernstein & Root-Bernstein, 1999). Imagination, then, is mental pantomime -- it may be the "missing link" that elucidates relations between interaction design and learning; it is the agent of *reflection* (mimesis, thought).

Finally, an "image perspective" on mathematical learning establishes relevance for the close study of student and teacher hand gesturing (Goldin-Meadow, 2003) as potentially indicative of tacit images at play in the classroom negotiation of mathematical meaning. The proposed project will investigate gesture as online articulating of imagination (see also McNeill & Duncan, 2000). More generally, the project will examine mathematics education as the process of *handing down* images.

Future Research

The *Three M's* project objectives are to: (a) investigate the roles of imagination in mathematical creativity, learning, and problem solving, e.g., to explore whether mathematicians' images are idiosyncratic, culturally mediated, or some combination thereof; (b) ground an understanding of the roles of imagination in current cognitive-science theories and pedagogical perspectives; (c) develop methodology for evaluating students' access to imagination as a cognitive resource and their imaginative engagement in learning activities; (d) outline principles for the design of objects and activities that encourage students both to engage in imaginative mathematical reasoning and, *inter alia*, to embrace imagination as an accessible cognitive resource; (e) design and build mathematical objects and create activities that encourage and guide utilization of imagination; and (f) research students' learning in these designed activities.

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