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MATHEMATICAL REPRESENTATIONS AS CONCEPTUAL COMPOSITES: IMPLICATIONS FOR DESIGN

Dor Abrahamson
University of California, Berkeley

Positing that mathematical representations are covert conceptual composites*, i.e., they implicitly enfold coordination of two or more ideas, I propose a design framework for fostering deep conceptual understanding of standard mathematical representations. Working with* bridging tools, *students engage in situated problem-solving activities to recruit and insightfully* recompose *familiar representations into the standard representation. I demonstrate this framework through designs created for studies in three mathematical domains.*

This design-theory paper presents a framework that spells out intuitive aspects of the craft of design for mathematics education so as to formulate these aspects, giving designers tools for progressing from domain analysis and diagnosis of learning problems toward design, implementation, and data analysis. The proposed framework focuses on mathematical representations and attempts to provide specificity, a "design template," for implementing radical-constructivist philosophy of didactics in terms of actual objects, activities, and facilitation guidelines for mathematics learning environments. The paper emanates from reflection on a decade of design-based research my collaborators and I have conducted on students' mathematical learning, in three separate projects with designs for the content domains of fractions, ratio and proportion, and probability, respectively, with K-16 participants (Abrahamson, 2000; Abrahamson & Wilensky, in press; Fuson & Abrahamson, 2005).

The foundations of the proposed design framework are in the philosophies of constructivism and phenomenology (Freudenthal, 1986; Heidegger, 1962; Piaget & Inhelder, 1952). Also, I regard effective learning as acts of creativity, so I build on creativity studies (Steiner, 2001), which describe insight as the act of imaginatively combining ideas. Mathematical representations, I posit, are *conceptual composites*, i.e., they enfold a historical coordination of two or more ideas. For example, part-to-whole diagrams representing the idea of a fraction (see Figure 1) integrate the multiplicative relation between a part and a whole, e.g., 2-to-3, and the logical relation of inclusion, i.e., the part is integral to the whole. The composite nature of mathematical representations is often covert—one can use these representations without appreciating which ideas they enfold and how these ideas are coordinated. Consequently, learners who, at best, develop procedural fluency with these representations, may not experience a sense of understanding, because they lack opportunities to bridge the embedded ideas, even if these embedded ideas are each familiar and robust.

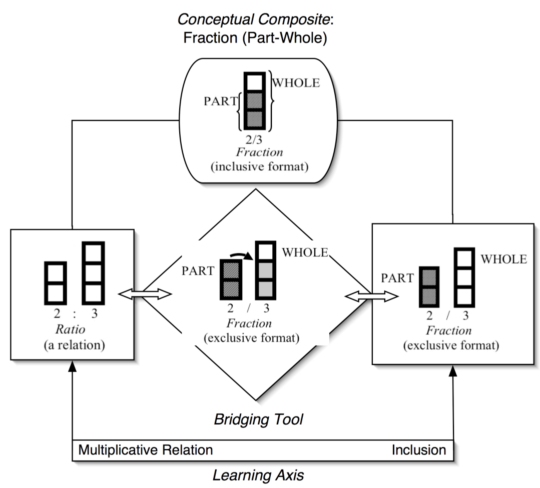

Figure 1. A part-to-whole fraction as a conceptual composite.

In the proposed framework, a designer creates a cluster of mathematical representations that *decompose* and "satellite" the target representation, highlighting its covert conceptual components. The teacher leads classroom discussion of situated problems to illuminate how the satellite representations are embedded in the target representation. Working with *bridging tools* (Abrahamson, 2004)—"ambiguous" representations interpretable as either of the complementary composite components—students *recompose* the components insightfully, as a reconciliation of the tension caused by the ambiguity, into the composite captured in the standard representation. Figures 1 and 2 demonstrate, for three designs, the standard representation interpreted as a conceptual composite and the bridging tools that help students build on their previous and emergent understandings and support students in seeing and coordinating these understandings.

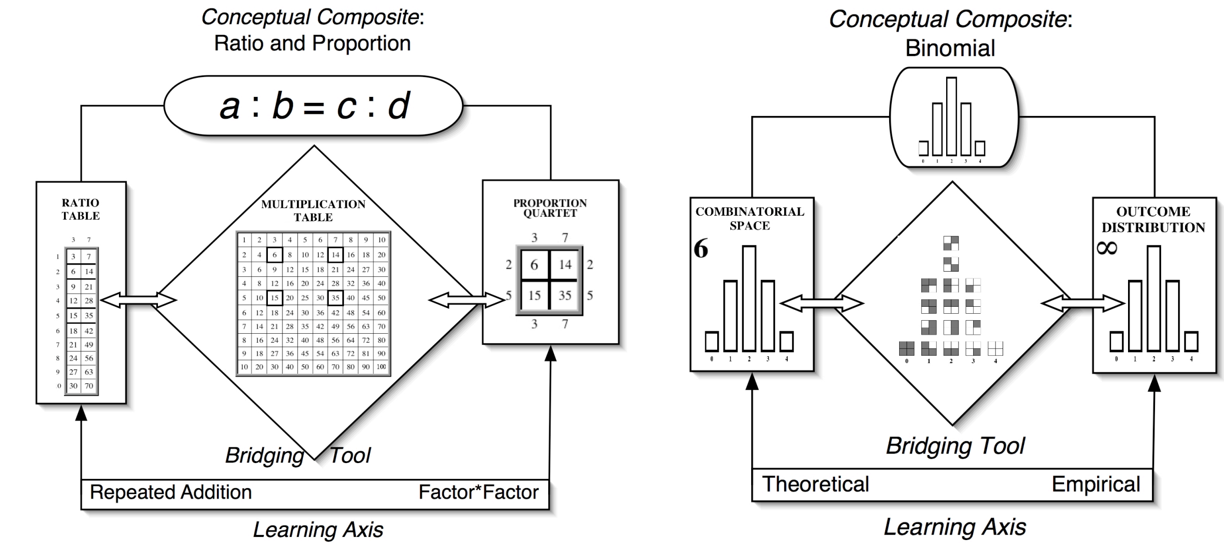

Figure 2. De-/re-compositions of ratio-and-proportion and probability representations.

The emergent framework may contribute to the work of researchers and practitioners: (a) For design-based researchers, the framework may guide both analyses and design in further mathematical domains; and (b) the framework may inform guidelines for professional development and, specifically, it may sensitize teachers to the possible opacity of some taken-as-shared mathematical constructs that are in fact historical composites—teachers informed by this framework may have new facilitation tools for seeing through the "smokescreen" of procedural fluency and helping students rebuild conceptual knowledge on their own robust understanding.

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