

SUPPORTING DISCURSIVE SHIFTS TOWARD MATHEMATICAL DEFINING

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The practice of defining differs across academic and non-academic discursive genres. As a result, students often have trouble formulating, interpreting, and using definitions in mathematical proof and problem solving. This design-based research project evaluated the conjecture that under appropriate conditions students could be steered to leverage naturalistic discursive practices in generating mathematical definitions. Specifically, I examined for learning effects of a dyadic game-based activity requiring the resolution of ambiguity in reference.

Theoretical Framework

Mathematical definitions, I submit, are not inherent truisms but rather meaning-relations built to fulfill particular purposes, often via an iterative process of formulating and evaluating a definition with respect to its utility, conformity to intuition, or other criteria. A definition, I further submit, could be viewed as an *epistemic form* (a target structure guiding inquiry), the end product of an *epistemic game* (Collins & Ferguson, 1993). The Specifications Game (SG) was designed to prompt students to reinvent one such epistemic game. The outcomes of SG were interpreted through the lens of commognition as a theoretical framework (Sfard, 2007).

Methods and Data Sources

Four dyads (ages 11, 15, & 22) participated in a pilot implementation of SG. One student attempted to specify a missing shape (e.g., triangle), while the other provide counter-examples (Fig. 1). I then prompted them to examine and modify their list of requisite properties into a conjectured definition and compare and evaluate alternative definitions for the object. Specifications given (e.g., “tall,” “straight sides”) were analyzed for discursive shifts in the course of game play. All data was examined for participants’ meta-discursive rules of defining.



Figure 1. A game board (left) and its pool of game pieces (right). Define the missing shape!

Conclusions

As they apprentice into the field of mathematics, students gradually move away from a view of mathematical definitions as arbitrary systems of constraints couched in non-normative language, towards a view of definitions as purposeful epistemic forms satisfying specific properties. By drawing on familiar cultural practices, setting up mathematical language as a potential solution to ambiguity, distributing the process over two participants, and encouraging reflection, SG supports students in enacting an epistemic game of formulating definitions. In this process, students become sensitized to properties of definitions such as specificity and minimality, resulting in a discursive shift away from mundane practices of defining.

References

- Collins, A., & Ferguson, W. (1993). Epistemic forms and epistemic games: Structures and strategies to guide inquiry. *Educational Psychologist, 28*(1), 25-42.
- Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.