EMBODIED MATHEMATICAL IMAGINATION AND COGNITION (EMIC) WORKING GROUP

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Embodied cognition is growing in theoretical importance and as a driving set of design principles for curriculum activities and technology innovations for mathematics education. The central aim of the EMIC (Embodied Mathematical Imagination and Cognition) Working Group is to attract engaged and inspired colleagues into a growing community of discourse around theoretical, technological, and methodological developments for advancing the study of embodied cognition for mathematics education. A thriving, informed, and interconnected community of scholars organized around embodied mathematical cognition will broaden the range of activities, practices, and emerging technologies that count as mathematical. EMIC builds upon our 2015 and 2016 working groups with a specific focus on examining the embodied nature of mathematical collaboration. In particular, we view collaboration as a type of crossroad that brings together people and artifacts, from which EMIC communication and activities can emerge.

Keywords: Classroom Discourse, Cognition, Informal Education, Learning Theory

Motivations for This Working Group

Recent empirical, theoretical and methodological developments in embodied cognition and gesture studies provide a solid and generative foundation for the establishment of **a regularly held Embodied Mathematical Imagination and Cognition (EMIC) Working Group** for PME-NA. The central aim of EMIC is to attract engaged and inspired colleagues into a growing community of discourse around theoretical, technological, and methodological developments for advancing the study of embodied cognition for mathematics education, including, but not limited to, studies of mathematical reasoning, instruction, the design and use of technological innovations, learning in and outside of formal educational settings, and across the lifespan.

The interplay of multiple perspectives and intellectual trajectories is vital for the study of embodied mathematical cognition to flourish. Partial confluences and differences have to be maintained throughout the conversations; this is because instead of being oriented towards a single and unified theory of mathematical cognition, EMIC strives to establish a philosophical/ educational “salon” in which entrenched dualisms, such as mind/body, language/materiality, or signifier/signified are subject to an ongoing and stirring criticism. A thriving, informed, and interconnected community of scholars organized around embodied mathematical cognition will broaden the range of activities and emerging technologies that count as mathematical, and envision alternative forms of engagement with mathematical ideas and practices (e.g., De Freitas & Sinclair, 2014). This broadening is particularly important at a time when schools and communities in North America face persistent achievement gaps between groups of students from many ethnic backgrounds, geographic regions, and socioeconomic circumstances (Ladson-Billings, 1995; Moses & Cobb, 2001; Rosebery, Warren, Ballenger & Ogonowski, 2005). There also is a need to articulate evidence-based findings and principles of embodied cognition to the research and development communities that are looking to generate and disseminate innovative programs for promoting mathematics learning through movement (e.g., Ottmar & Landy, 2016; Smith, King, & Hoyte, 2014). Generating, evaluating, and curating empirically validated and reliable methods for promoting mathematical development and effective instruction through embodied activities that are engaging and curricularly relevant is an urgent societal goal.

Past Meetings and Achievements of the EMIC Working Group

The first meeting of the EMIC working group took place in East Lansing, MI during PME-NA 2015, and has been continuing to expand ever since into a website (https://sites.google.com/site/emicpmena), a second EMIC working group in Tucson, AZ, at PME-NA 2016, and a sister pre-conference workshop event at the upcoming Computer Supported Collaborative Learning conference (Williams-Pierce et al., accepted). It has a somewhat longer origin, dating back to the 2007 AERA symposium, "Mathematics Learning and Embodied Cognition." By now, several research programs have formed to investigate the embodied nature of mathematics (e.g., Abrahamson 2014; Alibali & Nathan, 2012; Arzarello et al., 2009; De Freitas & Sinclair, 2014; Edwards, Ferrara, & Moore-Russo, 2014; Lakoff & Núñez, 2000; Ottmar & Landy, 2016; Radford 2009; Nathan, Walkington, Boncoddo, Pier, Williams, & Alibali, 2014; Soto-Johnson & Troup, 2014; Soto-Johnson, Hancock, & Oehrtman, 2016), demonstrating a "critical mass" of projects, findings, senior and junior investigators, and conceptual frameworks to support an on-going community of likeminded scholars within the mathematics education research community.

Since our first meeting at PME-NA 2015, some of **our collaborative accomplishments** include:

1. Creating a contact list with names and emails of attendees, and other interested scholars who could not attend PME-NA 2015 or 2016
2. Developing a group website using the Google Sites platform to support ongoing interactions throughout the year, and regularly adding additional resources/activities
3. Joint submission of an NSF DRK-12 by members who first met during the 2015 EMIC sessions
4. Some senior members joining a junior member's NSF ITEST and Cyberlearning grant proposals
5. Submission to the IES CASL program to study the role of action in pre-college proof performance in geometry (Funded 2016-2020 for Nathan & Walkington)
6. Submitting a proposal for the continuation of the EMIC WG to PME-NA 2016
7. Examining the potential for an NSF Research Coordination Network (RCN)
8. Application for a grant from Association for Psychological Science (APS) to develop a better website and offer stipends for contributors
9. Proposing a pre-conference workshop to CSCL 2017 on the embodied tools to promote STEM education, which was accepted as a full-day event (Williams-Pierce et al., accepted)

Current WorkingGroup Organizers

As the WorkingGroup has matured and expanded, we have a broadening set of organizers for the coming year that represent a range of institutions and theoretical perspectives (and is beyond the limit of six authors in the submission system). This, we believe, enriches the WorkingGroup experience and the long-term viability of the scholarly community. The current organizers for 2017 are (alphabetical by first name):

- Candace Walkington, Southern Methodist University
- Carmen J. Petrick Smith, University of Vermont
- Caro Williams-Pierce, University at Albany, SUNY
- David DeLiema, University of California, Berkeley
- David Landy, Indiana University
- Dor Abrahamson, University of California, Berkeley
- Erin Ottmar, Worcester Polytechnic Institute
- Hortensia Soto–Johnson, University of Northern Colorado
- Martha W. Alibali, University of Wisconsin-Madison
- Mitchell J. Nathan, University of Wisconsin-Madison
- Rebecca Boncoddo, Central Connecticut State University

Focal Issues in the Psychology of Mathematics Education

Emerging, yet influential, views of thinking and learning as embodied experiences have grown from several major intellectual developments in philosophy, psychology, anthropology, education, and the learning sciences that frame human communication as multi-modal interaction, and human thinking as multi-modal simulation of sensory-motor activity (Clark, 2008; Hostetter & Alibali, 2008; Lave, 1988; Nathan, 2014; Varela et al., 1992; Wilson, 2002). These views acknowledge the centrality of both unconscious and conscious motor and perceptual processes for influencing conscious awareness, and of embodied experience as following /producing pathways through social and cultural space. As Stevens (2012, p. 346) argues in his introduction to the *JLS* special issue on embodiment of mathematical reasoning,

> it will be hard to consign the body to the sidelines of mathematical cognition ever again if our goal is to make sense of how people make sense and take action with mathematical ideas, tools, and forms.

Four major ideas exemplify the plurality of ways that embodied cognition perspectives are relevant for the study of mathematical understanding: (1) Grounding of abstraction in perceptuo-motor activity as one alternative to representing concepts as purely amodal, abstract, arbitrary, and self-referential symbol systems. This conception shifts the locus of “thinking” from a central processor to a distributed web of perceptuo-motor activity situated within a physical and social setting. (2) Cognition emerges from perceptually guided action (Varela, Thompson, & Rosch, 1991). This tenet implies that things, including mathematical symbols and representations, are understood by the actions and practices we can perform with them, and by mentally simulating and imagining the actions and practices that underlie or constitute them. (3) Mathematics learning is always affective: There are no purely procedural or “neutral” forms of reasoning detached from the circulation of bodily-based feelings and interpretations surrounding our encounters with them. (4) Mathematical ideas are conveyed using rich, multimodal forms of communication, including gestures and tangible objects in the world.

Alongside these theoretical developments have been technical advances in multi-modal and spatial analysis, which allow scholars to collect new sources of evidence and subject them to powerful analytic procedures, from which they may propose new theories of embodied mathematical cognition and learning. Just as the “linguistic turn” in the social sciences was largely made possible by the innovation that enabled scholars to collect audio recordings of human speech and conversation *in situ*, growth of interest in multi-modal aspects of communication have been enabled by high quality video recording of human activity (e.g., Alibali et al., 2014; Levine & Scollon, 2004), motion capture technology (Hall, Ma, & Nemirovsky, 2014; Sinclair, 2014), developments in brain imaging (e.g., Barsalou, 2008; Gallese & Lakoff, 2005), multimodal learning analytics (Worsley & Blikstein, 2014), and data logs generated from embodied math learning technologies that interacts with touch and mouse-based interfaces (Manzo, Ottmar, & Landy, 2016).

Theme: The Crossroads of Collaboration

Inspired by the PME-NA 2017 theme, we will specifically focus on the ways in which people can influence one another. Examples that we will use during the Working Group include: the interactions of two middle school-aged friends playing a mathematics game together and using both physical and digital gesture to augment their spoken communication (Williams-Pierce, 2016); a teacher guiding the movements of a learner exploring ratios (Abrahamson & Sánchez-García, 2016); pre-service teachers using their distributed gestures to explore a mathematical conjecture and establish its truth and justification using embodied and extended cognition (Walkington, Woods & Nathan, under review); pre-service teachers interacting with designed dynamic algebraic notations as a means of engaging embodied aspects of mathematical derivation (Jacobson, Landy, & Ottmar, in prep); having students and teachers play and create embodied technology games to teach mathematics and computational thinking (Arroyo & Ottmar, 2016), and pairs of elementary students working together on a series of body-based tasks centered on angle concepts (Smith et al., 2016).

Through these examples, we will explore questions such as: when students meet within a common arena, how might an activity design motivate them to develop mathematical notions through making pragmatic ideas mutually intelligible, and how do they accomplish this feat? What are the roles of more knowledgeable members of the community in facilitating this process? During the conference, participants in our EMIC workshop will engage in dedicated activities and guided reflections as a basis for exploring the interpersonal and interactive crossroads of goals, action, and discourse as these play out in the emergence of mathematics learning. This investigative effort will be crafted so as to align with recent developments in embodiment literature, whereby scholars are struggling to model individual sensorimotor learning within established cultural practices, norms, and values.

Plan for Active Engagement of Participants

Our formula from PME-NA 2015 and 2016 proved to be effective: By inviting participants into math activities at the beginning of each session, we were rapidly drawn into those very aspects of mathematics that we find most rewarding. We plan to facilitate collaborative EMIC activities, followed by group discussions (and we now have many activities and members who can trade off in these roles!) that will help us all to “pull back” to the theoretical and methodological issues that are central to advancing math education research. Within this structure of beginning with mathematical activities and facilitated discussions, on **Day 1** we plan to begin with activities that forefront collaboration around EMIC activities, with four different groups engaging in different activities. These activities will serve as the foundation for a broad group discussion about the varied roles of collaboration in EMIC. See Figure 1 below for examples of collaborative activities from PME-NA 2016.

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Figure 1. Collaborative activities. Martha Alibali and two participants form a triangle together with their arms (left); a group just finished jointly assembling a large icosahedron in an activity facilitated by Dor Abrahamson and Leah Rosenbaum (right).

In previous years, we have found that the full first session will generally be taken up by introductions and and a round of activities followed by discussion. If there is additional time, we will begin brainstorming new collaborative EMIC activities - if there is not, then we will ask attendees to jot down any new activity ideas they have to share at the following session.

On **Day 2**, we will begin the session with technology-based collaborative activities, with four stations that pairs of participants rotate through. Examples of two of those stations are in Figure 2. Continuing with the routine established in Day 1, a full group discussion will follow, with a particular focus on designing EMIC digital contexts to support collaboration.

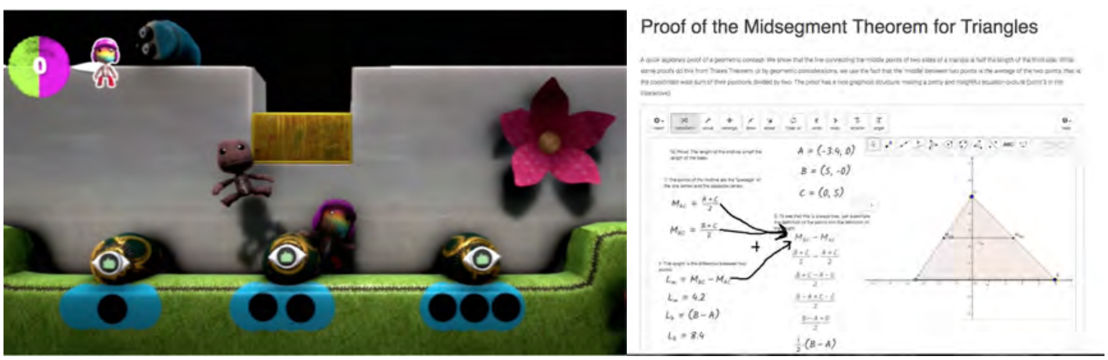

Figure 2. Digital collaborative activities. *Rolly's Adventure* by Caro Williams-Pierce (left); and *Graspable Math* by David Landy and Erin Ottmar (right).

After the discussion, we will discuss different EMIC activity ideas that participants began jotting down the day before, with the goal of developing additional collaborative activities that can be used in various research and learning contexts. The final activities will be shared on the EMIC website.

Day 3 is agenda-setting day, where we all discuss how we will keep the momentum going, such as developing an NSF Research Coordination Network (RCN), as a potential complement to the PME-NA Working Group. The RCN is not intended to promote any one particular research program, but rather to build the networked community of international scholars from which many fruitful lines of inquiry can emerge. Commensurate with the aims of the RCN, we will explore ways to

> share information and ideas, coordinate ongoing or planned research activities, foster synthesis and new collaborations, develop community standards, and in other ways advance science and education through communication and sharing of ideas.

Another example is to develop a proposal for a special issue of the *Journal of Research in Mathematics Education* that focuses on sharing the different theoretical perspectives, research activities, and operationalization of EMIC by the working group members.

In order to find common ground for the RCN submission and the JRME special issue, we may perform a live concept mapping activity that is displayed for all participants to explore the range of EMIC topics and identify common conceptual structure. We will discuss different general foci, such as teacher professional development with EMIC, designing EMIC games or museum exhibits, etc. Then, harkening back to the four major ideas that we developed earlier, sample seed topics for organizing this activity will be explored, such as:

1. Grounding Abstractions
    a. Conceptual blending (Tunner & Fauconnier, 1995) & metaphor (Lakoff & Núñez, 2000)
    b. Perceptuo-motor grounding of abstractions (Barsalou, 2008; Glenberg, 1997; Ottmar & Landy, 2016; Landy, Allen, & Zednik, 2014)
    c. Progressive formalization (Nathan, 2012; Romberg, 2001) & concreteness fading (Fyfe, McNeil, Son, & Goldstone, 2014)
    d. Use of manipulatives (Martin & Schwartz, 2005)
2. Cognition emerges from perceptually guided action: Designing interactive learning environments for EMIC
    a. Development of spatial reasoning (Uttal et al., 2009)
    b. Math cognition through action (Abrahamson, 2014; Nathan et al., 2014)
    c. Perceptual boundedness (Bieda & Nathan, 2009)
    d. Perceptuomotor integration (Ottmar, Landy, Goldstone, & Weitnauer, 2015; Nemirovsky, Kelton, & Rhodehamel, 2013)
    e. Attentional anchors and the emergence of mathematical objects (Abrahamson & Bakker, 2016; Abrahamson & Sánchez–García, 2016; Abrahamson et al., 2016; Duijzer et al., 2017)
    f. Mathematical imagination (Nemirovsky, Kelton, & Rhodehamel, 2012)
    g. Students' integer arithmetic learning depends on their actions (Nurnberger-Haag, 2015).
3. Affective Mathematics
    a. Modal engagements (Hall & Nemirovsky, 2012; Nathan et al., 2013)
    b. Sensuous cognition (Radford, 2009)
4. Gesture and Multimodality
    a. Gesture & multimodal instruction (Alibali & Nathan 2012; Cook et al., 2008; Edwards, 2009)
    b. Bodily activity of professional mathematicians (Nemirovsky & Smith, 2013; Soto-Johnson, Hancock, & Oehrtman, 2016)

c. Simulation of sensory-motor activity (Hostetter & Alibali, 2008; Nemirovsky & Ferrara, 2009)

We will also discuss the implications of this work and the different areas of the concept map for teaching, and discuss ideas for bridging the gap between research and practice.

Finally, we will introduce the EMIC website (publicly available at https://sites.google.com/site/emicpmena/home). On this website, we have a list of members with their emails and bios, information about our PME-NA presence, and short personal introduction videos. We've also created a space for members to share information about their research activities – particularly for videos of the complex gesture and action-based interactions that are difficult to express in text format. In addition, we have a common publications repository to share files or links (including to ResearchGate or Academia.edu publication profiles, so members don't have to upload their files in multiple places). At our 2015 working group, some junior members expressed particular interest in this literature support for their pending theses, while more senior members were eager to share and organize the emerging body of work on embodied math education. We've also linked the Google Sites platform directly to a Google Group, so members can participate in online forums (or the linked listserv), and discuss cutting edge topics, share in-progress working papers for review, or advertise for conferences, special issues, or other EMIC-relevant opportunities.

Follow-up Activities

We envision an emergent process for the specific follow-up activities based on participant input and our multi-day discussions. At a minimum, we will continue to develop a list of interested participants and grant them all access to our common discussion forum and literature compilation. Those that are interested in the NSF RCN plan will work to form the international set of collaborations and articulate the intellectual topics that will knit the network together; and those that are interested in the JRME special issue proposal will outline a specific timeline for progressing. One additional set of activities we hope to explore is to introduce educational practitioners at all levels of administration and across the lifespan to the power and utility of the EMIC perspective.

In the past three years, we have seen a great deal of progress. This is perhaps best exemplified by coming together of the EMIC website, the ongoing collaborations between members, and the proposals here and to CSCL, which each draws across multiple institutions. We thus will strive to explore ways to reach farther outside of our young group to continually make our work relevant, while also seeking to bolster and refine the theoretical underpinnings of an embodied view of mathematical thinking and teaching.

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