

## Dynamic Embodied Angles in a Simulated Restorative Planetarium

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### Objective: Embodied Restructuration of Euclidean Angles

In traditional geometry education, the dominant medium is paper (Papert, 2004). Thus, geometrical objects, e.g., triangles, are inscribed as static forms lying on a plane perpendicular to our line of vision. Whereas static forms are conducive to measurement and analysis, they implicitly entrain a static ontology of geometrical elements, such as angles (Thompson, 2013). A dynamic perspective on angles, grounded in embodied movement, introduces alternative geometrical ontologies, where angles are trans/formed by the body (Smith et al., 2014). This dynamic view presents angle measures as ratios out of the entire circle.

### Background: A Realistic Embodied Perspective on Angles, Incorporating Self and Environment

Freudenthal's (1971) pedagogical philosophy, Realistic Mathematics Education (RME), promotes an ecological approach to quantitative reasoning designed to narrow the prevailing epistemic gap between embodied know-how and formal subject matter (Gravemeijer, 1994). Situating mathematics education may, thus, reconcile naturalistic experience of angles as *dynamical-egocentric* and normative disciplinary representations of angles as *static-allocentric*. Such restructuring of angles could broaden access to the discipline by diversifying its prerequisite epistemic practices (Turtle & Papert, 1991; Wilensky & Papert, 2010). The pedagogical framework *embodied design* guides such mathematical restructuring by theorizing learning as dialogic negotiation between naturalistic perceptuomotor activity and cultural-historical forms (Abrahamson, 2019).

### Design: Geometry Planetarium – Enacting Angles in a Navajo Archeo-Astronomy Environment

*Geometry Planetarium* (GP) is a designed learning environment simulating essential perspectival qualities of Navajo archeo-astronomical practice in dialogic negotiation with Euclidean geometry. GP utilizes cultural-historical forms of *astrometrical* perception, which posits



Fig. 1: Outside view of the Geometry Planetarium



Fig. 2: Chaunese gauges the length of a shooting star by subtending its stretch.



Fig. 3: Dual pointer with affixed protractor.

spatial intervals between heavenly bodies as subtending two egocentric visual marks, whereby distance is gauged as an expanding egocentric angle. GP (Fig. 1) is an enchanted enclosure with other-worldly ambience. This environment creates opportunities for young students to reinstate Indigenous sensorimotor phenomenology of angle as egocentric dynamical enactment, replacing formal symbolic computation with realistic sensorimotor experience. In GP, the child becomes the vertex of a projected triangle, whose base is the gauged celestial interval. Young Chaunese compared the magnitudes of two objects in the GP sky (Fig. 2). She then explained her comparison by rotating her stretched arms outwards to embrace each imagined expanse. Finally, she was guided to repeat the double-pointing gauge by using an innovative device composed of two rotating dowels fastened at the base (Fig. 3). A protractor attached to the base enabled her to cite the angle measure of the expanse.

### Conclusions

Acknowledging and using the egocentric dynamical perspective on astronomical magnitude as embodied angle revitalizes Indigenous knowledge and integrates it with mathematical disciplinary knowledge to advance meaning in geometry education.

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