EMBODIED SPATIAL ARTICULATION: A GESTURE PERSPECTIVE ON STUDENT NEGOTIATION BETWEEN KINESTHETIC SCHEMAS AND EPISTEMIC FORMS IN LEARNING MATHEMATICS

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Two parallel strands in mathematics-education research—one that delineates students' embodied schemas supporting their mathematical cognition and the other that focuses on the mediation of cultural knowledge through mathematical tools—could converge through examining reciprocities between schemas and tools. Using a gesture-based methodology that attends to students' hand movements as they communicate their understanding, data examples from design research in two domains illustrate students' spontaneous spatial articulation of embodied cognition. Such embodied spatial articulation could be essential for deep understanding of content, because in performing these articulations, students may be negotiating between their dynamic image-based intuitive understanding of a concept and the static formal mathematical formats of representing the concept. Implications for mathematics education are drawn.

The growing body of literature on 'situated cognition' and 'cognition in context' (e.g., Lave & Wenger, 1991; Hutchins & Palen, 1998) is informing research in mathematics education. In particular, we are challenged to think of mathematical cognition not as "abstract" in-the-head processes devoid of concrete grounding, but as phenomenologically, intrinsically, and necessarily dwelling in student interactions with objects in their environment (Heidegger, 1962; Freudenthal, 1986; Varela, Thompson, & Rosch, 1991), such as mathematical representations. Some scholars maintain that mathematics is possible at all as a human endeavor, because the cognition of mathematics leverages and elaborates on embodied schemas that underpin thinking, such as 'containment,' 'repetition,' or 'extension' (Lakoff & Nuñez, 2000). Other scholars focus on the role that mathematical tools, such as representations and calculation devices, play in the interpersonal mediation of mathematical reasoning, such as Stigler (1984), who studied the "mental abacus." The plausibility of these parallel strands of research—the former possibly more "Piagetian" and the latter more "Vygotskiian"—invites the question of how individuals learn to use cultural tools. Specifically, if the scope of mathematical cognition is largely dictated by a repertory of embodied schema and if mathematical reasoning simulates the internalized operation of mechanical tools, how do these ends meet? Do existing schemas accommodate structures inherent in new tools? Do new tools foster the development of new schemas? Answers to these questions, and in particular a framework and terminology for describing what transpires in student–tool interactions, should be of interest to constructivist-education practitioners: Designers who strive to create learning tools supporting intuitive understanding of mathematical concepts often do not have a language to articulate what it is they are doing when they create tools that "work," and so it is difficult to evaluate and teach effective design; Teachers informed by a framework articulating types of student–tool interactions that are important for deep understanding of the concepts inherent in the tools may be encouraged to create classroom opportunities for such types of interactions. Addressing the issue of schema-driven learning versus tool-driven learning, this paper takes a position that the truth may lie somewhere in between. The objective of this paper is to spell out a theoretical position and to outline, through examples, a methodology for gathering data towards supporting this position. Future work will elaborate and expand on this theory and apply the methodology to support the position.

Learning mathematics with understanding involves students' ongoing negotiation between their embodied schemas and the cultural tools students engage with when participating in classroom activities. This theoretical position evolved through observations of students' discourse pertaining to innovative mathematical representations that were introduced into their classroom as part of design-research studies (Abrahamson, 2003, 2004a, 2004b; Fuson & Abrahamson, 2004; Abrahamson & Wilensky, 2004a, 2004b, 2004c). To support this position, it is necessary first to explain why this position has not been stated up to now. For that, we begin by focusing on the mathematical representations or, more broadly, the 'bridging tools' (Abrahamson, 2004a) that were designed for these studies. A design perspective in the study of student learning is helpful, because the agenda of designers is to create tools that "work," and this agenda informs—at least tacitly—the designers' search for mathematical representations that resonate with students' intuitions. Such resonance may be indexed by the extent of fitness between students' embodied schemas and the structures inherent in the designed tools. Following are pedagogical motivations for designing bridging tools and a discussion of gesture-based methodological lenses on student discourse. These lenses afford a distinction between evidence of students' schema-driven and tool-driven learning. Using examples from classroom interactions, we will demonstrate how embodied schemas and cultural forms are separate yet reciprocally related resources in students' learning, and how this learning can be articulated in terms of students' reconciliation between the schemas and the forms.

Bridging Tools

Abrahamson (2004a) discusses *bridging tools*, pedagogical mathematical representations that are designed to foreground and ground processes underlying a domain. The position of bridging tools between simple visual contexts and formal mathematical notation is designed to resonate with constructs, perceptual mechanisms, and schemas that are taken to be universal for the target population of students. Working with such tools, students can engage their experience and mathematical knowledge towards developing an informed fluency with more advanced concepts.

The term 'bridging tools,' although coined in the context of current design work, applies also to traditional mathematical representations that are effective in fostering learning with understanding. That is, mathematical representations that foster deep understanding are those that, either through historical "natural selection" or intentional design, are a priori tuned to accommodate learners' resources, such as their embodied cognition. Such bridging tools may paradoxically encumber the study of students' embodied cognition, because students' interactions with these tools do not easily reveal embodied cognition as a phenomenon that merits a standalone construct. All you see is "kids working with stuff"—the students attend selectively to elements within the mathematical tools, move objects from place to place, write numbers in appropriate locations, and so on. That is, the nature and order of students' attentive glances, manual operations, and solution procedures appear governed entirely by the implicit protocols afforded by the tools and by the classroom facilitator who models for students the conventional use of the tools. Thus, any putative 'embodied cognition' underpinning students' attentive operations remains a hypothetical construct of tenuous theoretical or practical standing. A theoretical decoupling of students' embodied cognition from their hands-on operating on the tools becomes more plausible upon closely examining students as they communicate what they see in the tools and what they are doing with them, such as in classroom discussions. In order to examine schemas, forms, and student negotiations between them, I use lenses from the theory and methodologies of gesture studies. These lenses purportedly reveal embodied cognition as decoupled from operations on tools yet reciprocally related to these tools.

Gesture Studies as Lenses on Embodied Cognition

The study of student gesturing, formerly a disparate intellectual pursuit associated mostly with psycholinguistics (e.g., McNeill & Duncan, 2002) and anthropology (e.g., Urton, 1997), has become a growing research effort within the community of scholars of mathematics education (e.g., Alibali, Bassok, Olseth, Syc, & Goldin-Meadow, 1999). Gesturing plays a crucial role in establishing shared meanings for new artifacts (Hutchins & Palen, 1998; Roth & Welzel, 2001). Through extended operating on new artifacts, learners develop skills of imaging these artifacts and operating on these images even in the absence of the physical embodiment of the artifacts and without any observable gesturing on these images (see Stigler, 1984, on the "mental abacus"; see Nemirovsky, Noble, Ramos–Oliveira, & DiMattia, 2003; see Urton, 1997, on tacit cultural images; see Goodwin, 1994, on how a professional culture mediates ways of seeing).

Whereas students' internalizations of mathematical learning tools do not subsume all that students learn in mathematics classrooms, it could be that these internalized spatial–dynamic images are vehicles of mathematical reasoning upon which hinge and cohere other aspects of effective domain-specific mathematical practice, such as the modeling of situations and solution procedures (Abrahamson, 2003, 2004a; Fuson & Abrahamson, 2004). Also, there does not necessarily exist a monotonous relation between gesturing and learning, and so the extent of gesturing in communicating a mathematical idea cannot index learning in any simple way. For instance, gesturing may wax towards a moment of clarity ("aha!") and then wane once the novelty of the new insight subsides (Goldin-Meadow, 2003) and is constituted as no longer warranting explication. Finally, bringing into classrooms innovative mathematical representations does not necessarily imply that students will not have had any prior experience with other representations that incorporate similar features. Therefore, it is probably not warranted to infer from the innovativeness of the tools that they incorporate innovative forms. On the contrary, the design principles of bridging tools together with the assumed historical reciprocity between embodied schemas and structures inherent in cultural forms means that students' classroom negotiations are informed by prior exposures to similar forms in other contexts. That is, the embodied schemas are probably not innate and the symbolic forms are not innovative, but rather both are woven into learners' "interconnected patterns of activity in which they [the symbols] are embedded" (Dreyfus, 1994, cited in McNeill & Duncan, 2000). This said, in the next section, we will focus on cases in which students appear to be coordinating between a way of seeing a mathematical object—a way of seeing that was not explicitly or at least not consciously conveyed by the facilitator who was the first to model the use of the object—and the format of the consensual mathematical notation associated with the concept in question.

Examples of Negotiation Between Embodied Knowledge and Mathematics Learning Tools

Students' negotiations between their kinesthetic schemas and the forms of mathematical representations are yet uncharted territory in mathematics education. Following are two examples of this phenomenon. In both examples, students use their body so as to articulate an idea within their body space (the 'peri-personal space'). The examples differ both in that they are taken from design studies in different mathematical domains and in that, whereas the first demonstrates a student spatially articulating a mathematical operation, the second shows a student reorganizing real objects in her immediate space towards expressing them as a mathematical relation. Such analyses and the questions that they raise inform ongoing work on a particular design, but they also can set an agenda for a research program that detects and classifies students' embodied metaphors of mathematics as observed in classrooms. A classification of students' embodied mathematics as it relates to mathematical representations may constitute a resource for design in mathematics education. The following data are presented in the *translipction* format that combines transcription, clips, and superimposed diagrams (Abrhamson, 2004; Fuson & Abrahamson, 2004).

Ratio and Proportion: Negotiating Between Schemas of Growth through Rhythmic Repetition and Constant Increments Between Products Going Down Multiplication-Table Columns

In a design for ratio and proportion (Abrahamson, 2003, 2004a; Fuson & Abrahamson, 2004), M'Buto used the multiplication table to solve a ratio-and-proportion word problem (see Figure 1, below). In the problem, an agent was advancing by increments of 5 units per some fixed time unit, and a sub goal towards the solution of the problem was for students to find out how far the agent advances in 11 increments. M'Buto added an 11th row at the bottom of his multiplication table (in front of him on his desk) that had 10 rows. In this 11th row, he wrote '55' in Column 5 (the column that has a '5' at the top; see Figure 1, on left). Ms. Winningham asks M'Buto how he knew to write '55.' In his oral response, M'Buto connects between a model of multiplication as repeated addition and the use of the multiplication table to retrieve multiplication cross products. In his gestures that complement the oral communication, M'Buto first reveals an image of multiplication as a rhythmic progression along a straight trajectory beginning at his torso and extending diagonally away, remaining in the plain of his torso (Figure 1, center). Immediately after, (Figure 1, on right) M'Buto scallops vertically down a column of a large multiplication table he is apparently imaging as if positioned directly in front of his face.

M'Buto' multiplication table with an 11th row.	"Every time you times 5 by another number	it goes up five."

Figure 1: Negotiating between an embodied spatial–dynamic topology of multiplication (center) and an understanding of a column in the multiplication table as growing by a constant increment (on right), in explaining a strategy for determining the product of 11 and 5 (on left).

M'Buto's scalloping motion down the present–absent multiplication table that accompanies the utterance "it goes up five" (see Figure 1, above, on right) corresponds to the gesture employed by the researcher–teacher (the author), the classroom teacher, and students participating in classroom discussions on several occasions on the intervention days prior to this moment (Abrahamson, 2004a, 2004b; Fuson & Abrahamson, 2004). Also, the sweeps and scope of this gesture correspond to the physical size of the classroom multiplication table used extensively in this unit (the scope is much larger than the multiplication table that is in front of M'Buto on his desk). However, the gesture accompanying M'Buto's first words (Figure 1, above, in the center) was never employed by the teachers, at least not explicitly. Where did this gesture come from, how does it correspond to the multiplication table, and what does all this reveal about M'Buto's learning process?

There appears to be a separation, for M'Buto, at this point, between an inner sense of multiplying and how it may plot onto the mathematical representation introduced in the design. M'Buto is negotiating personal and classroom resources on several levels. He is: (a) deploying an embodied model of the multiplication operation pragmatically in problem solving and obtaining a numerical solution; (b) plotting an embodied sense of "timesing" onto the concrete multiplication-table column, perhaps mediated by the imaged multiplication table; (c) interfacing the concrete multiplication table upon which he added the 11th row with the imaged multiplication table; and (d) articulating a kinesthetic theorem-in-action within the linear constraints of the spoken communication medium. Where and how did M'Buto form or internalize the embodied spatial–dynamic image of multiplication? If this image is shared by other students, what does this mean in terms of helping students link the image with the multiplication table? What can we make of the fact that M'Buto successfully speaks of the numbers "going up" while his hand is patently going down?

Probability and Statistics: Using Embodied Hemispheres to Link to an a:b Symbolic Form

In a design for probability and statistics (Abrahamson & Wilensky, 2004a, 2004b, 2004c), Carry responds to a student–leader's prompt to explain a sample taken out of a population of thousands of green and blue squares on a computer interface (see Figure 2, below). Looking away from the computer, Carry gesturally extracts the green squares, placing them on the left "hemisphere" of her body, and then places the blue squares on her right hemisphere. Within 2.5 minutes of discussion, six students engaged the same embodied mechanism to parse and structure their seeing of the visual stimulus. This uniformity in students' gesture patterns suggests that a shared mathematical vision is being co-constructed in the classroom. This vision is design driven: The computer-based bridging tool is aimed to assist students in negotiating between, on the one hand, proportional reasoning and enumeration, and, on the other hand, the formal notation of multiplicative constructs involving proportionality, such as density (of green in the population). It could be that whether or not a gesturing person does so consciously, these gestures help other students see the tools in like ways that are conducive to similar understanding.

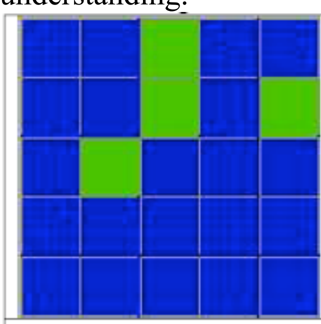			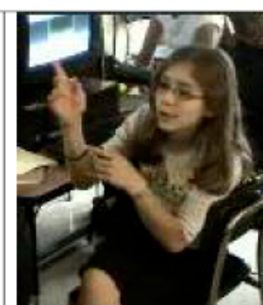
Fragment from the SAMPLER interface.	"Out of 25, 4 are green	and	21 are blue."

Figure 2: Negotiating between a visual metaphor of density and the formal *a:b* ratio symbol.

Embodied Spatial Articulation

Embodied spatial articulation is an individual's design-facilitated negotiation between personal and cultural resources pertaining to the visuo-spatiality of mathematical situations and representations. The personal resources are proto-mathematical action-based images and the cultural resources are the appropriate seeing-in-using of classroom spatial–numerical artifacts. I wager that embodied spatial articulation underpins human interacting with epistemic artifacts historically, developmentally, in the designer's workshop, and in classroom space–time (see Abrahamson, 2004a, 2004b, for references supporting this contention). The roles of gesturing in the teaching and learning of mathematics are in supporting the students' intra/inter-personal engagement of tacit body-based strategies for spatial modeling of mathematical concepts. This modeling serves in the co-constitution of domain-specific *epistemic forms* (Collins & Ferguson, 1993) that come from and respond to artifacts.

Educational Significance

If students achieve deep understanding of mathematical concepts by negotiating between embodied resources and cultural artifacts, then learning environments should ideally foster such negotiations—the environments should include historical and innovative representations that readily afford relevant embodied schemas as well as activities, such as individual work and group discussion, designed to create space and time for these negotiations. An embodied-cognition approach together with the gesture-based lenses on student discourse afford a methodology for obtaining nuanced descriptions of students' learning processes. This perspective responds to Confrey's (1991) call to attend to students' voice, only here we are listening to a body-based voice that has been historically neglected.

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