Embodied Geometry: signs and gestures used in the deaf mathematics classroom – the case of symmetry

Christina M. Krause

University of Duisburg-Essen

Cite as

Krause, C.M. (2018). Embodied Geometry: Signs and gestures used in the deaf mathematics classroom – the case of symmetry. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 171-193). Rotterdam: Sense.

Embodied Geometry: signs and gestures used in the deaf mathematics classroom – the case of symmetry

> By identifying the differences and similarities in the practices of those whose knowledge of the world is mediated through different sensory channels, we might not only become better able to respond to their particular needs, but also build more robust understandings of the relationships between experience and cognition more generally. (Healy, 2015, p. 289)

Research in the field of deaf studies shows that learning mathematics appears to be more difficult for deaf children than it is for those who can hear. Deaf students' basic math skills are found to lag several years on average behind those of hearing learners of same age (e.g. Kelly, Lang, & Pagliaro, 2003; Nunes, 2004; Pagliaro, 2006; Traxler, 2000). This is partly explained by a lack of informal mathematical knowledge typically gained by hearing children implicitly through everyday interactions in early childhood (Nunes & Moreno, 1998). Furthermore, deaf students struggle with reading, understanding and processing word problems (Hyde, Zevenbergen, & Power, 2003) since they have not had sensory access to the language in which the problems are written so that this language is a foreign language for them.

These previous studies mainly compare the learning *products* of deaf students with those of hearing students but rarely focus on the learning *processes*. If we assume that mathematical knowledge becomes shaped by processes of meaning making and that mathematical thinking is influenced by our interaction with the world, I suggest that it might be naïve to assume that the learning product will be the same for deaf students, considering the circumstances of learning are not. One major difference between social learning processes in regular and deaf classrooms concerns the modality of language, with spoken language being used in the first case and sign language in the latter, each with its specific characteristics.

Building on a Vygotskian approach, Healy (2015) claimed that the sensory channels by which we perceive information deeply influence the structure and process of thinking so that the substitution of the ear by the eye when interacting with others in the mathematics classroom may also influence what kind of mathematical knowledge is constructed. So, "rather than seeing difference as equated to a state of deficiency, difference can be treated as just that, difference" (Healy, 2015, p. 291). Taking this difference into account may help us get a better understanding of learning processes also more generally.

This chapter points out some of the differences in the way learning is experienced when sound is no longer a primary sensory channel and discusses possible consequences for the learning of mathematics in the deaf classroom. In doing so, it aims at drawing attention to the importance of taking a more thorough look at the specific situation that is faced by deaf learners (and their teachers), focusing on the influence of sign language as an important component of the process of mathematical meaning making in social interaction.

To approach this goal, theory gained from non-mathematics-specific studies on sign language will be introduced, namely the consideration of iconicity as one feature of sign languages that may influence conceptualization (Grote, 2013).

In this chapter, I present a case study from a fifth grade geometry classroom with all students as well as the teacher being deaf and communicating in German sign Language. I will draw on three examples to reconstruct how two 'mathematical signs'[1] develop to their use in the classroom together with the mathematical ideas they refer to and how this forms processes of iconization, that is processes in which iconic relationships between the signs and their respective referenced idea become established.

This investigation is especially important considering that there are rarely conventions about 'mathematical signs' to refer to a mathematical idea. They are often more or less idiosyncratic to the teacher, especially when it comes to more abstract concepts in upper grades. It is therefore important to shed light on the signs that are used, 'where they come from', and what might be implied by the iconicity between these signs and the mathematical idea.

Based on the analyses, I will discuss possible theoretical implications for the learning of mathematics within a theoretical framework that sees social, semiotic, and individual approaches to learning as being deeply intertwined, described in the next section.

Learning mathematics between the social, the semiotic, and the individual

In the mathematics classroom, mathematical ideas and objects are mainly encountered and discussed in interaction among students and the teacher. Learning mathematics can therefore be considered a social phenomenon in which individuals co-construct mathematical meaning. But what influences this construction? One aspect may concern the semiotic nature of the social

[1] It should be noted that the term 'sign' can be used generically, in the semiotic sense, and specifically to refer to morphological units of signed languages which are roughly the equivalents of words or short phrases. While the meaning in any particular case should also be clear from the context I point out that it is only referred to 'signs' in a semiotic sense in the section dealing with theoretical assumptions on learning mathematics, "Learning mathematics between the social, the semiotic, and the individual".

learning process, such that mathematical objects cannot be accessed directly but only mediated through (semiotic) signs (Seeger, 2006). These signs may be of spoken, written or gestural form, or may be multimodal in their nature and processed through different sensory channels (Arzarello, 2006). When interacting by means of these (semiotic) signs, they need to be interpreted in order to react to them, which makes interaction a constant process of interpreting and responding to each other based on this interpretation (Bikner-Ahsbahs, 2006; Krause, 2016). Gestures form a specific kind of these signs – “idiosyncratic spontaneous movement[s] of the hands and arms accompanying speech” (McNeill, 1992, p. 37) that are not performing any physical action such as writing, scratching and so on. On the social side, they are part of the orchestration of (semiotic) signs that form the multimodal utterances that shape social interaction and can play an active part in epistemic processes (Dreyfus, Sabena, Kidron, & Arzarello, 2014; Krause, 2016). On the individual side, they are an embodied means of expression in which thinking can become manifest (Goldin-Meadow, 2003). Furthermore, they are considered to contribute to the formation of “the embodied mind” (Varela, Thompson & Rosch 1991), under the assumption that

> [m]eaning is in many ways socially constructed, but, it is *not arbitrary*. It is subject to constraints which arise from biological embodied processes that take place in the ongoing interaction between mutually constituted sensemakers and the medium in which they exist. (Núñez, Edwards, & Matos, 1999, p. 53, italics in the original)

In recent years, increasing attention has been turned toward the theory of embodied cognition, thus valuing the role of the body in mathematical thinking and learning (see Edwards, Ferrara, & Moore-Russo, 2014 for a comprehensive overview and ‘emerging perspectives’). Following this approach, our (mathematical) thinking is crucially influenced by our physical being in the world, in which bodily experience is considered a core source for all conceptual understanding (Lakoff & Núñez, 2000; Nemirovsky, 2003) and this thinking, vice versa, becomes embodied (Arzarello 2006; Edwards, 2009; Goldin-Meadow, 2003). Embodied signs may thus be seen as a meeting point for social and individual learning, being shaped by and themselves shaping mathematical thought and social interaction.

For deaf learners, the embodied approach and the role of bodily means of expression takes on even greater significance than for hearing learners. On the one hand, the learning process within social interaction is highly shaped by visual signs since the language used is a visual one. Mathematics literally needs to become visual in the deaf mathematics classroom. On the other hand, the bodily experiences that shape conceptual thinking are different for deaf learners. This has already been considered by Healy and colleagues (Healy, 2015; Healy, Ramos, Fernandes, & Botelho Peixoto, 2016), who understand the sensory channels as instrument in the sense of Vygotsky. According to Vygotsky “the eye, like the ear, is an instrument that can be substituted by another” (Vygotsky, 1997, p. 83, in Healy, 2015, p. 291), but this substitution also involves a change in the process and structure of thinking (Vygotsky, 1981, in Healy, 2015). Healy and colleagues claim that

> To better understand the deaf mathematics learners, we need to better understand what it means to practice mathematics in the medium of sign language and how those whose cognitive processes are mediated by a visual-gestural-somatic language as opposed to a sequential-auditory language come to think mathematically. (Healy et al., 2016, p. 145)

Approaching this issue, I apply theory from studies on sign language, to achieve a more comprehensive understanding of how the use of sign language may influence mathematical conceptualization as shaped in and by the interactions with the world.

Iconicity as a feature of sign languages

Sign languages are not mere word to word translations of spoken language into gestural signs. They are languages with their own syntactic rules, steadily and naturally growing in the community they are used in. However, due to their partly local evolution and also to the fact that the use of sign languages was discouraged in favor of oral education in many parts of the world until the last century (Healy et al., 2016; see also Sacks, 2000), many different sign languages and dialects have developed in different parts of the world. Although these different sign languages differ in their vocabulary just as spoken languages do, there are several features that all sign languages seem to have in common. I will refer to these common features when using the singular, ‘sign language’, in this chapter.

Grote (2010; 2013) focuses on two of these features of sign language when investigating the effects of language modality on conceptual categorization, namely *simultaneity* as an aspect of articulation, and *iconicity* as an aspect concerning the signified-signifier relationship.

While the feature of simultaneity and its possible implications for learning mathematics in the modality of sign language is discussed in Krause (*in press*), this chapter will focus on the feature of iconicity:

Spoken words can bear an iconic relationship to their referents when creating an auditory correspondence to them. An example of such an auditory iconic, or onomatopoetic word is given by the “flipflop,”-sound that made its way into the dictionary to refer to a specific type of sandals. While such an iconic relationship is rare in spoken languages, the gestural modality facilitates the provision of iconicity when linguistic signs resemble qualities of the signed concept, for example in hand shape or movement. This iconicity can concern the form of a physical referent, but also the performance of an action. Such similarity is often found in signs. Grote claimed that “those features that become reflected in the iconic moment of the sign language gain specific relevance for the whole semantic concept” (Grote, 2010, p. 316, translated by the author) and provided evidence from an experiment conducted with deaf signers, hearing signers and hearing non-signers. In verification tests, she asked the subjects to decide whether given images matched a given concept. For the signers, both hearing and non-hearing, the reaction times turned out to be significantly shorter for those images representing the features suggested iconically in their respective signs, while for the non-signers, no significant difference has been observed.

But what does this mean for the learning of mathematics? Iconicity of sign language may have major implications for mathematical learning processes understood within the framework described earlier in this chapter, in which I embrace individual, social and semiotic aspects of learning. Given that iconic aspects may be of specific relevance for the mathematical idea, developing a better comprehension of the iconic nature of gestures and signs used may also support comprehension of how deaf learners construct mathematical knowledge.

In the learning process, the students have to make sense of the mathematical ideas that are encountered, the signs and gestures used to refer to these ideas, the linguistic terms and symbols which refer to the idea in the written modality, and the mouthing or viseme (the movement of the mouth) and facial expression. All four aspects have to be coordinated in order to interact with respect to the idea. This coordination is established in discourse and social interaction more or less explicitly. It is thus important to clarify the relationships between these aspects and, since sign language is the natural mode of expression, to observe which aspects of the mathematical idea become reflected in the gestural modality, considering the process of establishing an iconic relationship – the process of iconization – and the epistemic process as going hand in hand. As Grote claims, “assuming that epistemic processes are processes inherently mediated by signs, the similarity that forms the relationship between icon and referential object is constituted actively” (Grote 2010, p. 312, translated by the author). So which aspects of the mathematical idea are reflected in this iconicity? Which similarity forming the relationship between iconic sign and mathematical referential idea is constituted in the interaction? How is meaning made of signs that ought to be used in mathematical meaning making? ‘Mathematical signs’ are rarely conventionalized but often more or less idiosyncratic to the teachers. It is thus key to have a look into the classrooms in which these signs are used in the interaction so that we may answer these questions with reference to specific cases. Investigating the dialectic in the process of iconization of 'mathematical signs' will thus be the focus of this chapter.

Approaching Embodied Geometry

The data used is taken from a larger project dealing with the influence of sign language on learning mathematics (see Krause, *in press*). In this project, I collaborate with a school for deaf and hearing-impaired children, covering Grades 5 to 10 with students from age 10 to 16. The students of this school are considered quite well-achieving in comparison to other special-needs schools. Many students graduating from this school proceed with one of the very few special needs schools in Germany that offer graduation with the German 'Abitur', the final exams required to enroll in university. For the purpose of this chapter, we will focus on a Grade 5 Geometry classroom, in which all of the students, as well as the teacher, are deaf and communicate through German sign Language.

A series of mathematics lessons has been videotaped from three perspectives—one from the front, one from the front left, and a third from the front right—to best capture the signs and gestures used while interacting in the classroom. The students sit in a semi-circle so they can see each other and the teacher. The videos have been subtitled by two deaf coworkers almost 'literally' in the sense that the signs were transcribed in the order in which they occur.

The analysis is conducted directly on the video data, identifying scenes of 'mathematical interaction'; that is, interaction regarding a mathematical idea. The signs and gestures are interpreted within the context provided by the larger discourse and by considering their synchronic and diachronic relationships to other signs and gestures and to inscriptions (written signs); synchronic relationships in this sense concern signs, gestures and inscriptions as they are used simultaneously, for diachronic relationships also signs, gestures and inscriptions used earlier in the process are taken into account. While this methodological approach is similar to an analysis within developing semiotic bundles (Arzarello, 2006), there needs to be an adjustment of the semiotic bundle model to the specificity of the interaction. This model considers speech in the oral modality and not linguistic signs more generally. Based on this interpretation, I describe which aspects of the mathematical idea are embodied in the visual-gestural approach.

When filming in the fifth grade geometry classroom started, the students dealt with line segments. During the following four weeks of filming, the students learned about straight lines, half lines/rays, intersections, parallelism and orthogonality of lines, coordinate systems, and axial and point symmetry.

In the following examples, we will take a closer look at the ideas of axial symmetry and point symmetry and their respective signs, and how the iconic relationships between signs and mathematical ideas are constituted – mostly by the teacher – within the social interaction in the mathematics classroom. We will see how this constitution also includes some non- or pre-conventional visual-gestural approaches to the mathematical ideas that may influence the process of iconization.[2]

Case studies: Iconic aspects in visual-gestural approaches to geometry

Example 1**:** As*sign*ing meaning to "axial symmetry"

To introduce the idea of axial symmetry, the teacher tells the students to fold a piece of paper, cut something out, unfold it and tell what they recognize and what the folding line stands for by pointing along the line and asking MEANING?[3] ("What does this line mean/stand for?"; see Fig. 1a). One of the students offers an answer by signing "TOPIC MIRROR"[4] and the teacher agrees and repeats the idea for the class by lifting his flat hand in front of his face and signing MIRROR, by slightly rotating his hand as illustrated in Fig. 1b. Then, he positions his hand to line up with the folded line on the paper and repeats an inverted form of the sign MIRROR (Fig 1c).

[2] While gestures are idiosyncratic and non-conventionalized, the Signs used in sign languages follow certain conventions just as the words used in spoken languages. However, signers use non-conventionalized gestures in addition to the Signs. Following McNeill's definition of gestures given before as "accompanying speech", the gestures in this case can be understood as "idiosyncratic spontaneous movements of the hands and arms" *accompanying the signed discourse.* Both types of gestural expression can hardly be distinguished (see also Healy, Ramos, Fernandes, & Botelho Peixoto, 2016). Being performed in the same visual-gestural modality, Signs and gestures are deeply intertwined in their use and in their interpretation, probably even more intertwined than are gestures and spoken language (Liddell & Metzger, 1998).

[3] As customary in sign language study, the transcriptions of the Signs as translated from German to English are presented in capitalized words. An interpretation in context is added in brackets.

[4] That means she performs the Signs "topic" and "mirror". A corresponding picture can not be displayed since the Signs are performed to close to her face to make anonymization possible.

Fig. 1: Axis of reflection as MIRROR (b)

Right after, he takes another folding product from a student to show where there is an axis of symmetry and where there is none. The piece of paper is almost symmetric along two axes, but the accurate symmetry can only be identified by means of the folding line. The teacher turns the piece such that the folding line is oriented vertically, locates the hand on the folding line (Fig. 2a) and then moves it in the same way as is done when signing MIRROR (Fig. 2b); at the same time, he shapes his mouth as if saying “Spiegel” (German for “mirror”). A bit later, he folds a piece of paper twice to create two axes, and the idea is extended to holding the mirror horizontally (Fig. 2c).

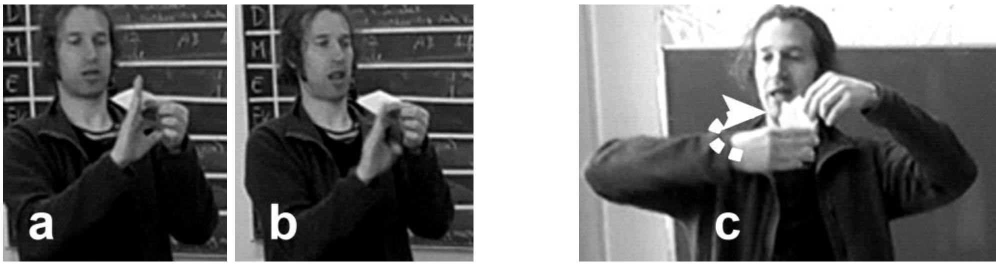

Fig. 2a & 2b: Indicating the vertical axis of reflection on a folded piece of paper
Fig. 2c: Extending the idea of reflection to a horizontal axis of reflection

The mathematical terminology is introduced later in a literally multimodal way: On the board, the definition is written, headlined by “Axially symmetric figures: A figure with at least one <u>axis of reflection</u> is called <u>axially symmetric.</u>” This description is supplemented by images of a quadrilateral with one axis of reflection and a square (see Fig. 3).

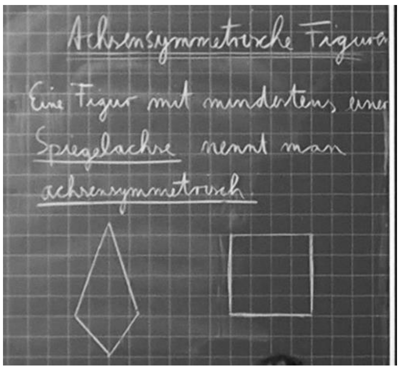

Fig. 3: Inscription on the board: Axially Symmetric Figures

The teacher then introduces the sign explicitly by pointing at the written word and signing SIGNING AXIAL SYMMETRIC (Fig. 4):

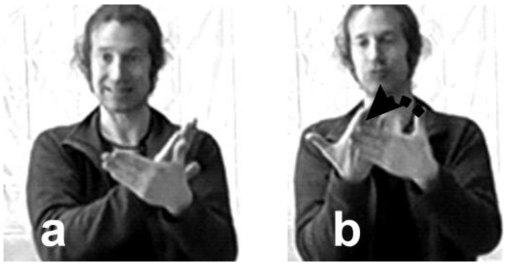

Fig. 4: AXIAL SYMMETRIC

(Mouthing: a: “<u>A</u>chsen” (axial), b: “symmetri<u>sch</u>” (symmetric))

This is followed by fingerspelling the first part, “A-C-H-S-E” (axis) and clarifying what is meant by the word “Achse”, then repeating the sign and going through the written definition word by word, again repeating the sign several times, and discussing the two examples on the board.

The first approach to the idea of axial symmetry is hence given by creating a symmetric figure by means of folding and cutting and associating it with the idea of reflecting. This approach is not unusual also in the regular classroom, with the verbal mathematical term “axis of reflection” even referring to this idea. However, while the spoken term may recall the processes of reflecting, the sign introduced for “axial symmetry” may reflect two approaches and not only one as they have been combined in the process of iconization: On the one hand, the sign is a variation of another sign, MIRROR/REFLECTION, used in the explanation, which is itself iconic (with the vertical flat hand rotated in front of the face, see Fig, 1b). On the other hand, the sign imitates iconically the process of folding the paper and producing a folded line as axis of symmetry, a quite common didactic approach to the idea of axial symmetry that was used to introduce the concept in class.

The sign offers what I call *innerlanguage (or innerlinguistic) iconicity*[5] to the signs MIRROR and FOLDING: its performance differs from the performance of these signs that are not primarily related to a mathematical idea only in the mouthing. It therefore links the mathematical idea to the concepts of reflecting and folding through recalling the related signs in addition to recalling the idea of axial symmetry as accessed through the activity of folding.

[5] The translation from the German “innersprachliche Ikonizität” proves itself tricky since there is no suitable translation to the German “sprachlich”. However, it is referred here to a certain iconicity within the same language.

***Example 2*:** From axial symmetry to point symmetry

In spoken language, the terms “axial symmetry” and “point symmetry” are clearly connected to each other as specific kinds of symmetries. This connection does not become as clear from the signs used in this classroom for the two mathematical ideas. In the former example, we already saw how axial symmetry was introduced and how the actions of reflecting and folding are iconically reflected in the respective sign. When the teacher introduces the idea of point symmetry some days later, he builds on that by recalling the sign for “axial symmetry”, highlighting the component AXIAL as part of the compound sign AXIAL-SYMMETRY, which he had not done before (see Fig. 5a. for AXIAL, succeeded by the sign for AXIAL-SYMMETRY shown in Fig. 5b and 5c).

Fig. 5: Explicit decomposition of the sign AXIAL-SYMMETRY

On the board, he draws a rectangle (Fig. 6), which is now used to demonstrate how it is not axially symmetric along the diagonal, but is symmetric in another sense.

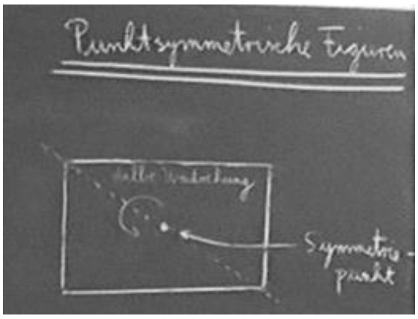

Fig. 6: Inscription on the board, with arrow indicating the “point of symmetry”, titled with “Point Symmetrically Figures” and “semi rotation” written in the upper part of the rectangle

After indicating a diagonal line as a potential axis of reflection (Fig. 7a), he suggests a “folding” of the upper right corner across the diagonal (Fig. 7b and c) as he did before in the case of axial symmetry. He then indicates the point that would result from reflecting the upper right corner in that way (Fig. 8a and b). He holds this indication while turning halfway around to the students, slightly shaking his head (Fig. 8b).

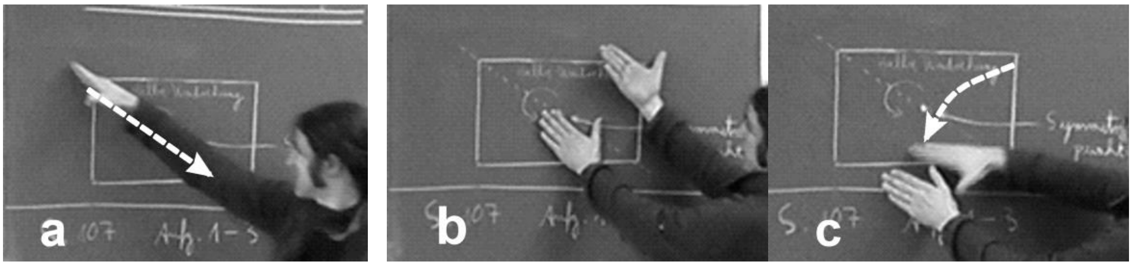

Fig. 7: Indicating the reflection mapping on the diagonal line

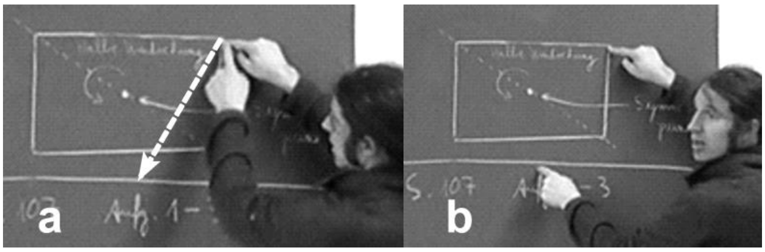

Fig. 8: Indicating the potential mapping of a point under the reflection mapping on the diagonal line

Following that, he comes back to the point he already marked as important in his image ("point of symmetry") (Fig. 9) and indicates it with his hand shaped like the sign POINT (which is conventionally performed in the palm of the hand).

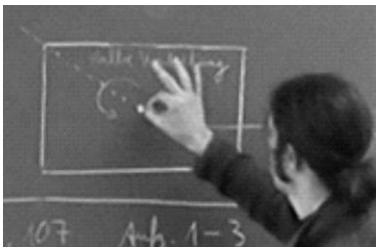

Fig. 9: Indication of the "point of symmetry" with the hand shaped like signing POINT

Then, he indicates with both hands the lower left part, with respect to the diagonal (Fig. 10) followed by "rotating" the upper right part into the lower left part by turning the hand configuration in a semicircle (Fig. 11). The teacher then explicates the concrete extent of the rotation as "half circling" (Fig. 12; the respective sign CIRCLING can be better seen in Fig. 13). Note that "circling" already constitutes a part of what later will be the sign for "point symmetry".

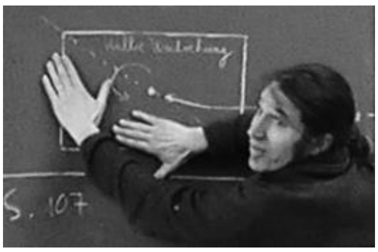

Fig. 10: Indicating the lower left part of the rectangle

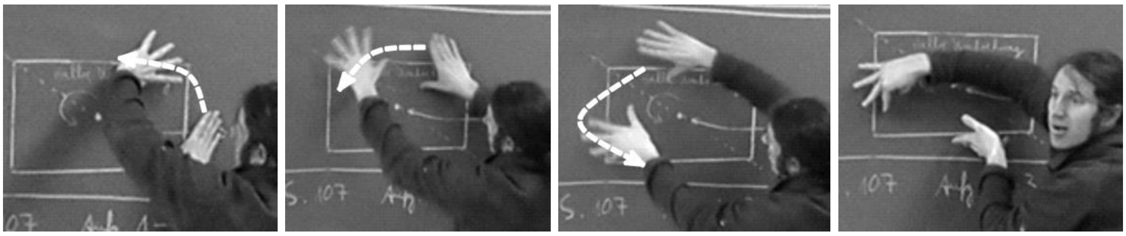

Fig. 11: Suggesting a virtual rotation of the upper right part of the rectangle into the lower left part

Fig. 12: signing HALF CIRCLING. For HALF (a), his right hand moves as if cutting the flat left palm in half. For CIRCLING (b), he sets the right thumb on the palm of the straight left hand and moves his right hand once around the wrist, index finger straight.

After several questions about the importance of direction and extent of the rotation, the teacher makes it clear that the direction of rotating does not matter, but that "half circling" is important (Fig. 13).

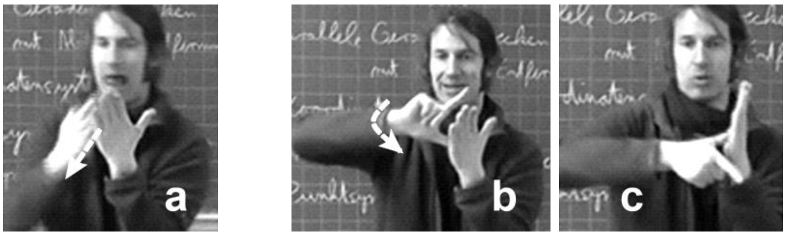

Fig. 13: HALF - CIRCLING

To foster the idea of point symmetry, the teacher discusses another example with the students, projecting the images of a Queen and a Jack from a deck of playing cards onto the wall (Fig. 14.

Fig. 14: Picture projected on the wall (Queen and Jack)

The students are asked to imagine the centre. The teacher imaginatively rotates the image again, this time with one hand only, first with the right hand (Fig. 15), then with the left hand (Fig. 16).

Fig. 15: Suggesting a virtual rotation of the image of the Queen by 180° with his right hand

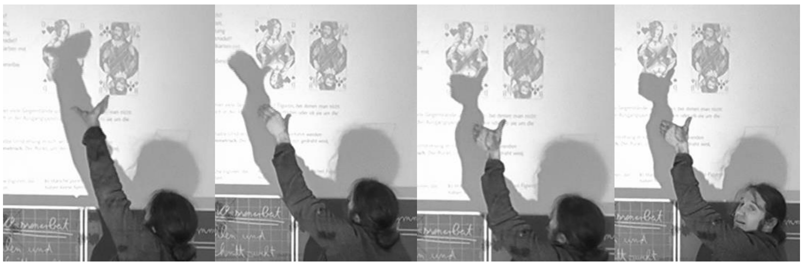

Fig. 16: Suggesting a virtual rotation of the image of the Queen by 180° with his left hand

Following this, he introduces explicitly the sign indicating the idea of "point symmetry". For this, he highlights that the upper part matches the lower part when being rotated and adds THEREFORE POINT SYMMETRY (Fig. 17), the sign compound by POINT (Fig. 17a) and a rotation of the hand with the thumb located in the palm of the other hand (Fig. 17b).

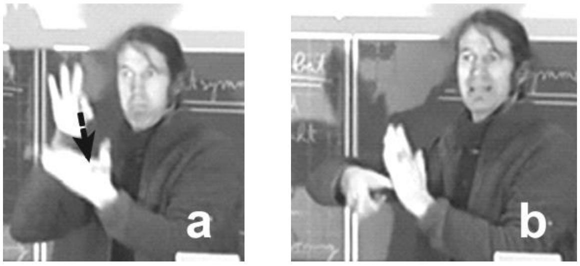

Fig. 17: POINT SYMMETRY

The rotations in Fig. 15 and Fig. 16 are repeated several times, first by virtually rotating the Jack as done previously with the Queen, before the teacher introduces a new example (Fig. 18).

Fig. 18: Picture projected on the wall (second example for point symmetry)

Fig. 19: Suggesting potential horizontal reflection

Fig. 20: “point symmetry”/rotating

The figure on the left is found to be axially symmetric along the diagonals, but, as the teacher highlights, also point symmetric. He indicates the axial symmetry by imitating the folding with direct reference to the image (Fig. 19), then refers to the point symmetry first in language, then in action. For this, he uses only the sign, detached from the image (Fig. 20), before he illustrates the point symmetry by suggesting the virtual rotation on the image as done earlier for the Queen and the Jack.

This process of iconization shows how the sign for “point symmetry” becomes grounded by the teacher in the action of virtually rotating a figure around a central point. First, he performs this rotation with direct indexical reference to images, then detaches from the image to perform the rotation around a point in the palm of his hand. With this, he introduces the intended sign.

However, he repeatedly explicates that it is important to rotate in a semi-circle, not having introduced rotation mappings and angles more generally so far. This important feature, making the point symmetry a specific case of radial symmetry, is not accurately reflected in the rotation movement of the sign POINT SYMMETRY.

Example 3: Same but different

In the second part of the same lesson, another teacher stands in to supervise the students. This teacher is hearing but also signing and teaches mathematics outside her subject area.

In contrast to the signs the deaf teacher uses, her signs reflect a grounding in the defining parameter of the specific kind of symmetry (*point* in Fig. 21 and *line* in Fig. 22)[6], combined with the sign she uses for symmetry more generally, which features two parts “matching” each other (being congruent). While the sign used by the deaf teacher developed with reference to the *process* of rotating – a congruence mapping in a mathematical sense – this sign emphasizes properties of the figure in a *product*-related way.

[6] Different from the deaf-signing teacher, the hearing-signing teacher did not teach the class on a regular basis, so I do not have her statement of agreement for being shown without being anonymized in print publications.

Fig. 21: POINT + “match” (not a conventional sign), referring to point symmetry

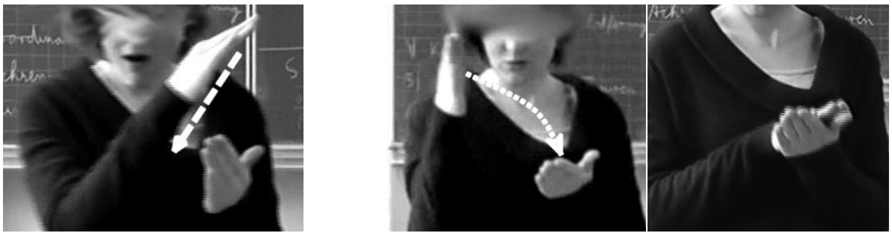

Fig. 22: LINE + “match” (not a conventional sign), referring to axial symmetry

This does not mean that her *explanations* do not emphasize the process aspect of rotation, but that her *sign* does not reflect this aspect. While supervising the students in the second part of the lesson, she occasionally explains the idea to single students that ask for help with the exercises they are asked to work on. Her signs become explicated when she clarifies the second component of the sign as referring to the idea of two parts matching when folding or when rotating respectively. However, given that this teacher did not introduce the concepts of symmetry in the class, the iconic relationships between her signs and the respective mathematical ideas is not constituted actively. Her signs reflect another approach than the dynamic one used by the deaf teacher. While the deaf teacher grounds the symmetries and the respective signs in actions that may be seen as corresponding to the *congruence mappings*, the hearing teacher emphasizes the *congruency* of the parts of the figure in her signs. Those who have already developed an idea of the concept of point symmetry may be able to make sense of this new sign on their own but for those who still struggle with the idea it may cause interference, since it may become confused with the idea of folding as it was introduced before due to the movement by which the “matching” is iconized.

While the first sign referred to the rotation but did not reflect its specific extent, the second sign refers to the congruency of areas but does not take into account the inversion of orientation we find in point symmetry with respect to axial symmetry. They show, however, two perspectives on the same concept.

Example 4: The defining parameter

Being confronted with two signs for the same idea, the students are left to decide how they will refer to this concept.

In the second part of the lesson, one student helps two of his classmates solve a task. In his explanation, he uses his own sign for point symmetry, picking up the one aspect the signs of the two teachers have in common; i.e., the one providing information about the defining parameter, or the "point" (Fig. 23 and b) combined with fingerspelling a shortened form of the written word "symmetry" (S-Y-M).

Fig. 23: sign used by a student to refer to point symmetry

The student does not seem to favour one sign over the other at that point in time, although he refers to rotating the sheet to check for point symmetry in his further explanation.

Summary: The iconicity in a gestural approach to symmetry

The examples given provide an analysis of the signs used to refer to the mathematical ideas of "axial symmetry" and "point symmetry" with respect to the iconicity in which they reflect aspects of these mathematical ideas. The three questions posed related to the iconicity concerned (1), what one might call the 'mathematical content as recognizable from a higher standpoint', (2) the ideas from which this mathematical content is approached as grounded in the social interaction, and (3) the way in which the mathematical content is connected to the 'mathematical sign' by establishing an iconic relationship between them. Let us sum up how these three aspects played into the cases we observed:

(1) Since there are no standard conventions concerning the 'mathematical signs' as there are for mathematical terms in spoken language, the teachers often develop their own signs that are most probably influenced by their conceptualization 'on a higher standpoint'.

The signs of the deaf teacher (see Fig. 4/5 for 'axial symmetry' and Fig. 17 for 'point symmetry') reflect a *constructive, dynamic conception* of the mathematical ideas that appears to be mainly related to the *mappings* that preserve the congruency of the figures considered. The signs consist of signing the defining parameters—axis and point respectively—and a component that may be understood as reference to the respective mapping.[7]

The signs of the hearing teacher (see Fig. 21 for ‘point symmetry’ and Fig. 22 for ‘axial symmetry’) reflect, in contrast, an *analytic, static* conception that is focused on the *feature of the figures* being considered congruent. While the first part of her signs give the defining parameter—just like the signs of the deaf teacher do—the second part is the same for ‘axial symmetry’ and ‘point symmetry’, a generic sign reflecting the idea of ‘matching’ one on top of the other. This might not only be shaped by her conception of the mathematical ideas, but also by the terms in spoken language which are constituted by ‘defining parameter’ + “symmetry”.

(2) The mathematical idea of ‘axial symmetry’ is grounded in the activity of folding and, in a more abstract way, in the idea of reflecting. With the latter, the teacher takes up an approach suggested by one of the students referring to “TOPIC MIRROR” and combines it with the initial folding-approach.

The idea of point symmetry can be seen as built up from the idea of axial symmetry in so far as it is introduced in explicit distinction from it, while having in common the idea of moving one part of a figure to match another part of it (see example 2). This movement was actively constituted for axial symmetry by folding. The rotation by 180°—the respective movement concerning point symmetry—is only performed virtually with direct indexical reference to images inscribed on the board or reflected on the wall.

(3) The deaf teacher integrates the introduction of the mathematical signs in the process of encountering the mathematical idea such that “the similarity that forms the relationship between icon and [mathematical] referential object is constituted actively” (Grote 2010, p. 312) as grounded in the same (actual or virtual) actions as the mathematical contents themselves. For axial symmetry, this concerns the actual action of folding and the imagined action of reflecting. While the iconicity to the idea of the axis as a mirror is established actively in the process of iconization within the classroom discourse, the connection between the act of folding is grounded in the didactic approach taken and becomes reflected in the sign for AXIAL SYMMETRY as this sign shared components

[7] Note that the teacher shortened the Sign for ‘axial symmetry’ to the second component, but not the Sign for ‘point symmetry’.

with FOLDING in German sign language. However, the sign MIRROR also shares several features, i.e., the shaping of the straight hand and the slight turn of the hand around its long axis, providing an *innerlanguage iconicity*. The coordination of the ideas of “axial symmetry”, “reflecting” and “folding” may not only connect the three ideas on a conceptual level but may also help the students to make sense of the linguistic mathematical term “axis of reflection” to which meaning must also be assigned. For the point symmetry, the sign captures the hypothetical action of rotating the images around one point.

The hearing teacher had to constitute the iconic relationship between her signs and the underlying mathematical ideas explicitly. They did not develop together with the mathematical idea in a process of iconization, and the signs are thus not grounded in the process for the students. The explication of the mathematical content is an additional factor causing effort for the hearing teacher and also for the students.

Discussion

I started this chapter with a quote that indicates the value of taking a more comprehensive look at the “practices of those whose knowledge of the world is mediated through different sensory channels” (Healy, 2015, p. 289). One of these practices concerns the practice of discourse or, more generally, the social interaction in the mathematics classroom, which is shaped differently for deaf learners. For them, signs and gestures play a crucial, if not central, role in this social interaction and probably also in the learning of mathematics.

Starting from this premise, this chapter aimed to better understand how this special kind of discourse shapes the learning of mathematics. It surveyed a landscape in which the use of sign language as gestural language is considered an important component in the conceptualization both from a social and also from an individual perspective.

Mathematical discourse needs mathematical terms or, in the case of deaf students, mathematical signs. Just as in the mainstream classroom, a common language has to be found to communicate about mathematics, to present mathematical ideas and results and to reason about them. The theoretical framework presented in this chapter provides an idea how mathematical discourse carried out in a gestural – that is visual-spatial – language may influence the learning of mathematics and the conceptualization of mathematical ideas. The empirical analysis provided some insight into the complexity of coordinating the learning about a mathematical idea and introducing a sign that reflects aspects of this idea, both at the same time. So far, there is no didactic approach that helps teachers to prepare for this challenge.

The empirical investigation within this chapter focuses on the iconicity of signs with respect to the mathematical content and its role in mathematical meaning making in the context of introducing the ideas of axial symmetry and point symmetry. The development of the gestural approach to the mathematical ideas is reminiscent of the development of *associated gestures* as observed in social processes of constructing mathematical knowledge carried out by hearing students (Krause 2016, pp. 140-158). While for the hearing learners these associated gestures can become *situationally conventionalized* to serve as a non-verbal term while solving a task, the signs in the deaf math classroom become associated with mathematical meaning in the process of iconization and may then serve as a *conventionalized* term beyond the concrete situation. Another difference lies in the way meaning is assigned to the associated gestures and signs. For the latter, the interplay with speech and inscription plays a crucial role since the co-expressive verbal utterance provides information influencing the interpretation of the gesture and with that, also the meaning of the gesture as it develops over time. The meaning of the gesture within the social interaction is, however, rarely made explicit. The signs, on the other hand, need to be explicated as conventionalized signs at a certain point to distinguish them from, but also to connect them to, the non-conventionalized gestures used in the process of iconization.

Looking at the signs used by the teachers to refer to the ideas of axial symmetry and point symmetry also provides a view on their own conceptualization of these ideas. Their signs highlight different facets of the concepts—a dynamic one that focuses on the movement and a static one that focuses on the congruency—which tell us different things about the nature of the mathematical idea. Following an embodied approach, the mathematical thinking of the teachers is also influenced by their physical experiences with the world. Is there hence a connection to their dynamical spatial language? It may be too bold to hypothesize about a preference for dynamic approaches to mathematical ideas, but a further investigation of these preferences may play a part in helping us to “build more robust understandings of the relationships between experience and cognition more generally” (Healy, 2015, p. 289).

Knowledge about how the use of the gestural modality of sign language influences the learning processes in the mathematics classroom may not only be useful to provide a didactic approach for the deaf mathematics classroom. The visual component offered by gestures in the mathematics classroom has been found to be an important resource when it comes to the learning of mathematics (Arzarello, Robutti, Paola, & Sabena, 2009; Arzarello & Paola, 2007;

Sabena, 2007). Research in the deaf classroom can help to look beyond what is said to shed a gestural perspective on mathematical discourse such that the investigation of the diversity of deaf students' learning can be seen as a chance to break barriers, not only in the inclusive classroom.

The landscape that has been surveyed in this chapter is far from being fully explored; this can only be the tip of the iceberg. More research will provide a better understanding of the relationship between the role of the gestural modality of sign language in the learning of mathematics and mathematical cognition. This chapter shall therefore be understood as an invitation.

Acknowledgments

I am grateful for the funding received from the presidency of the University of Duisburg-Essen, providing a start-up for making first steps towards a research program focusing on the role of signs and gestures in the learning of mathematics of deaf and hearing-impaired students. Also, I want to thank the teachers, the students and the students' parents for their collaboration in this project.

Furthermore, I appreciate the helpful comments and suggestions given by the internal reviewers, and by Leland McCleary, who took great part in improving this chapter by providing suggestions and critique from the perspective of (cognitive) linguistics and the study of gestures and sign languages.

Literature

Arzarello, F. (2006). Semiosis as a multimodal process. *Revista Latinoamericana de Investigacion en Matemática Educativa, Numero Especial*, 267-299.

Arzarello, F., & Paola, D. (2007). Semiotic game: The role of the teacher. In J. Woo, H. Lew, K. Park, & D. Seo (eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, 2* (pp. 17-24). Seoul, Korea: PME.

Arzarello, F., Robutti, O., Paola, D., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics, 70*, 97-109.

Bikner-Ahsbahs, A. (2006). Semiotic sequence analysis – Constructing epistemic types. In J. Novotná, H. Moraovsá, M. Krátká, & N. Stehliková (eds.): Mathematics in the centre.

Proceedings of the 30th Conference of the International group for the Psychology of Mathematics Education, vol. 2, (pp. 161-168). Prague, Czech Republic: Charles University, Faculty of Education.

Dreyfus, T., Sabena, C., Kidron, I., & Arzarello, F. (2014). The Epistemic Role of Gestures. In A. Bikner-Ahsbahs, & S. Prediger (eds.), *Networking of theories as a research practice in mathematics education* (pp. 127-152). New York: Springer.

Edwards, L. D. (2009). Gestures and conceptual integration in mathematical talk. *Educational Studies in Mathematics, 70*, 127-141.

Edwards, L. D., Ferrara, F., & Moore-Russo, D. (2014). *Emerging perspectives of gesture and embodiment in mathematics* (eds.). Charlotte, NC: Information Age Publishing.

Grote, K. (2010). Denken Gehörlose anders? Auswirkungen der gestisch-visuellen Gebärdensprache auf die Begriffsbildung. [Do signers think differently? Influence of the gestural-visual sign language on conceptualization.] *Das Zeichen - Zeitschrift für Sprache und Kultur Gehörloser 85*, 310-319.

Grote, K. (2013). *'Modality relativity?' The influence of sign language and spoken language on semantic categorization*. Dissertation. Retrieved January 10, 2017, from darwin.bth.rwth-aachen.de/opus3/volltexte/2013/4546/

Goldin-Meadow, S. (2003). *Hearing gesture: how our hands help us think*. Cambridge, MA: Harvard University Press.

Healy, L. (2015). Hands that see, hands that speak: Investigating relationships between sensory activity, forms of communicating and mathematical cognition. In S. J. Cho (ed.), *Selected regular lectures from the 12th International Congress on Mathematical Education* (pp. 298–316). New York: Springer.

Healy, L., Ramos, E. B., Fernandes, S. H. A. A., & Botelho Peixoto, J. L. (2016). Mathematics in the Hands of Deaf Learners and Blind Learners: Visual–Gestural–Somatic Means of Doing and Expressing Mathematics. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, M. Setati-Phakeng, P. Valero, & M. V. Ubillús (eds.), *Mathematics education and language diversity (The 21st ICMI Study).* Springer International Publishing.

Hyde, M., Zevenbergen, R., & Power, D. (2003). Deaf and hard of hearing students´ performance on arithmetic word problems. *American Annals of the Deaf, 148,* 56-64.

Kelly, R. R., Lang, H. G., & Pagliaro, C. M. (2003). Mathematics word problem solving for deaf students: A survey of perception and practices. *Journal of Deaf Studies and Deaf Education, 8,* 104-119.

Krause, C. M. (2016). *The mathematics in our hands. How gestures contribute to constructing mathematical knowledge.* Heidelberg, Germany: Springer Spektrum.

Krause, C. M. (in press). DeafMath: Exploring the influence of sign language on mathematical conceptualization. *Proceedings of the 10th Congress of the European Society for Research in Mathematics Education* (CERME 10 in Dublin 2017).

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being.* New York: Basic Books.

Liddell, S. & Metzger, M. (1998). Gesture in sign language discourse. *Journal of Pragmatics, 30,* 657-697.

McNeill, D. (1992). *Hand and mind: what gestures reveal about thought.* Chicago: University of Chicago Press.

Nemirovsky, R. (2003). Three conjectures concerning the relationship between body activity and understanding mathematics. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 113-120). Hawaii, USA: PME.

Nunes, T. & Moreno, C. (1998). Is hearing impairment a cause of difficulties in learning mathematics? In C. Donlan (ed.), *The development of mathematical skills* (pp. 227-254). Hove, UK: Psychology Press.

Nunes, T. (2004). *Teaching mathematics to deaf children*. London, U.K.: Whurr Publishers.

Núñez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics, 39,* 45-65.

Pagliaro, C. M. (2006). Mathematics education and the deaf learner. In D. F. Moores & D. S. Martin (eds.), *Deaf learners: Development in curriculum and instruction* (pp. 29-40). Washington, DC: Gallaudet University Press.

Sabena, C. (2007). *Body and signs: a multimodal semiotic approach to teaching–learning processes in early Calculus.* PhD-Dissertation. Turin: Turin University.

Sacks, O. (2000). *Seeing voices*. New York: Vintage Books.

Seeger, F. (2006). Ein semiotischer Blick auf die Psychologie des Mathematiklernens [A semiotic perspective on the learning of mathematics]. *Journal für Mathematikdidaktik (3/4)*, 265-284.

Traxler, C. B. (2000). Measuring up to performance standards in reading and mathematics: Achievement of selected deaf and hard-of-hearing students in the national norming of the ninth edition Stanford Achievement Test. *Journal of Deaf Studies and Deaf Education, 5*, 337–348.

Varela, F. J., Thompson, E. T., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience.* Cambridge, MA: MIT Press.

Vygotsky, L.S. (1981). The instrumental method in psychology. In J.V. Wertsch, (ed.), *The concept of activity in Soviet psychology*. (pp. 134-143). Armonk, NY: M.E. Sharpe.