ICONICITY IN SIGNED FRACTION TALK OF HEARING-IMPAIRED SIXTH GRADERS

Christina M. Krause

University of Duisburg-Essen, Germany

This paper reports from a project on the investigation of the influences of sign language on the development of conceptualizations of mathematical ideas. Following research in Deaf Studies, iconic aspects of an idea represented in the related sign are considered one factor impacting the understanding of the signed concept. This paper adopts this approach and proposes a categorization of the diverse types of iconic references made by the students when signing about fractions, based on interviews with deaf and hard-of-hearing students using sign language as natural everyday-language.

INTRODUCTION AND THEORETICAL BACKGROUND

How do deaf students learn mathematics? How do they think about mathematical ideas? And how can answering these questions help us gain more comprehensive insights, not only about how to better respond to the specific needs of deaf students, but also about what influences learning and conceptualisation in general? While these questions are inspired by Healy (2015), research from the field of Deaf Studies suggests approaches towards answering them by considering specific features of sign language that have been found to influence the conceptualization of the signed idea.

This contribution presents part of a larger study that aims at understanding the influence of sign language on mathematical learning and establishing sensitizing concepts to foreground the impact of sign language in mathematical discourse. Specifically, this current report focuses on examining how students sign about fractions and how this might influence their understanding of fractions. Therefore, the objective of this paper is to provide first categories to describe how students sign in fraction talk.

Assuming that knowledge is constructed by individuals through co-construction in social interaction, communication as it is carried out in the gestural-somatic mode of sign language is considered to have a non-trivial impact on this learning process from two perspectives. On the one hand, visual aspects represented in the sign might influence 'what is actually talked about' and how the signed utterance may be interpreted as a whole, similar to as it is already considered for the case of gestures accompanying speech in learning processes of hearing children (Krause, 2016). On the other hand, following the theory of embodied cognition we can assume that bodily existence and the being in and experiencing the physical world impacts how we construct meaning and what kind of meaning we construct (Núñez, Edwards, &

2017. In Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education,* Vol. 3, pp. 89-96. Singapore: PME.

Matos, 1999, p. 53). With respect to the role of body in cognition, Wilson and Foglia state in their *embodiment thesis* more specifically:

> Many features of cognition are embodied in that they are deeply dependent upon characteristics of the physical body of an agent, such that the agent's beyond-the-brain body plays a significant causal role, or a physically constitutive role, in that agent's cognitive processing. (Wilson & Foglia, 2016, paragraph 3)

One aspect they highlight with respect to the body’s role in cognition concerns the “body as constraint”, which implies that

- Some forms of cognition will be easier, and will come more naturally, because of an agent's bodily characteristics; likewise, some kinds of cognition will be difficult or even impossible because of the body that a cognitive agent has.
- Cognitive variation is sometimes explained by an appeal to bodily variation. (Wilson & Foglia, 2016, paragraph 3)

Therefore, the conditions for deaf students with respect to cognitive processing can be considered being different to those of their hearing peers due to bodily variation.
Furthermore, from a socio-cultural perspective, mathematics is mediated semiotically and the way we come in touch with mathematics – whether it is through auditive signs or mainly through visual signs – alters the structure and the flow of how we think mathematically (Healy, 2015, referring to Vygotsky, 1917). In accordance with this, it is not the question *if* deaf students can develop mathematical skills just as their hearing counterparts, but rather *how* these skills develop and how the “profound restructuration of the intellect” (Healy, 2015, p. 299) caused by the substitution of the bodily tool in semiotic mediation changes how the mathematical thinking and knowledge becomes structured.

Influence of sign language on conceptualization

Research in the field of Deaf Studies points out that certain features of sign language influence the conceptualization of the corresponding signed ideas (Grote, 2013). One of these features concerns the *iconicity* of a sign, that is, the relationship between a sign and the aspects of the idea or object that can become reflected in this sign as evoked by some kind of similarity, e.g. to an action or object. According to Grote (2013), the iconicity of the sign influences which ideas become marked as distinctively linked to the concept. While in this study, only German Sign Language (DGS) is considered, the feature of iconicity encompasses sign languages in general (see Grote, 2013).

Sign languages are naturally growing languages and as such, they have been acknowledged as languages only since the last century. While for many mathematical concepts there is no common consensus about corresponding ‘mathematical signs’, these signs often develop in the discourse in the mathematics classroom (Fernandes & Healy, 2014). Investigating which aspects are reflected iconically in the signs used is thus key to getting a better understanding of how this idea becomes encountered and which aspects become considered important to ‘stand for’ the mathematical idea.

METHODS

This study was carried out in cooperation with a German school for special educational needs that focuses on ‘Hearing and Communication”. Ten deaf or hard-of-hearing students from a grade six class, were invited to participate in the interviews. German Sign Language was the primary language of each of the students. In the mathematics lessons, the hearing mathematics teacher used sign language as well as spoken language. The topic focused on in the interviews – fraction numbers – was covered in class two months earlier.

Interview methodology

One purpose of the interview is to investigate the students’ fraction talk, that is, to find out more about *how* the students talk about fractions and ideas related to fraction numbers. Therefore, two aspects become key in the methodological approach to the interviews:

- The students have to be encouraged to talk in their natural language, that is, they need to feel free to use sign language.
- The interviewer themselves shall not provide signs to refer to mathematical ideas that stand in the focus of investigation to not influence how the students talk about these ideas.

The first issue is encountered by having the interviews carried out by a deaf assistant that already contributes in the project by subtitling video data gathered in the classroom (see also Krause, *in press*). The interviewer has neither a research nor a specific mathematical background, which required to design an interview guideline and introducing her thoroughly to the purpose and the aims of the interview. While this proceeding provides good conditions for the first of the two aspects mentioned above, it obstructed the researcher to intervene in cases where further questioning may have helped assessing the students’ ideas of the mathematical concepts.

The second methodological aspect underlying the planning of the interviews was encountered by a specifically geared interview design that made use of ‘term cards’ and ‘fraction cards’. In the course of the interview, cards have been presented to the students, each labelled with a fraction term. The fraction terms given to the students were (English translation provided in brackets): ‘Bruch’ (*fraction*), ‘Zähler’ (*numerator*), ‘Nenner’ (*denominator*), ‘kürzen’ (*simplifying/reducing*), ‘erweitern’ (*expanding*), ‘Bruchrechnung’ (*fractional arithmetic*), and ‘Brüche vergleichen’ (*comparing fractions*).

The students are asked to talk about one term after the other, initiated by the interviewer asking “*I will give you some words. How would you explain the meaning?”* (signed as “words give-to-you content meaning explain-to-me (what?)”) after a first introduction to the interview situation. Subsequently, the interviewer asks the students “what fits together what?”, lets them regroup the cards on the table and asks for an explanation for the grouping they made. This slimmed down version of a concept map is trialled to gather further insights about the aspects considered significant for the students with respect to the mathematical ideas.

Following this, two fraction cards are given to the students, one labelled with the fraction $\frac{4}{2}$, the other one with the fraction $\frac{6}{4}$. The final task consists of students comparing these two fractions and deciding which one is bigger. The students' explanations ought to provide a further perspective on how the students talk about fractions in the specific context of a concrete task.

Data preparation and analysis

The video data has been subtitled by the deaf assistant using the German words corresponding with the signs, preserving the linguistic structure of German Sign Language as best as possible. These subtitles served as basis to identify the students' use of the fraction terms to then reconstruct their iconic reference.

KINDS OF ICONICITY IN STUDENTS' SIGNS FOR FRACTION TERMS

The investigation of the iconic aspects reflected in the students' signs used in the first part of the interview showed diverse types of iconicitiy, that is, diverse kinds of iconic similarity as reflected in the sign. In the following, the different categories will be presented by means of illustrative examples.

Innerlinguistic iconicity

A large amount of 'mathematical signs' used by the students when talking about the fraction terms has been found to be based on signs used in everyday sign language. That is, the sign resembles another, possibly nonmathematical, sign in handshape and/or motion of the hand, and placing of performance of the sign. Assuming that the iconic reference fosters a stronger link to the idea referred to in the similar sign, the reference of the *innerlinguistic iconicity* and its 'fit' with the corresponding mathematical idea need to be considered for the development and appropriate use of 'mathematical signs'.

For example, the DGS-sign for 'zählen' has been used as 'mathematical sign' for the term 'Zähler' (*numerator*). As nominalization of 'zählen' (*counting*), hence 'the one that counts', the idea of 'Zähler' could be conceptually linked to 'counting' the given number of the parts the whole is divided in, embedded in an understanding of fractions as 'part of a whole' (e.g. Kieren, 1980; Lamon, 2012).

Fig.1: Sign used for "Zähler" *(numerator)* as innerlinguistically iconic to "zählen" *(counting)* in DGS (from two perspectives)

Another sign used for 'Zähler' reflected innerlinguistic iconicity to the DGS-sign for 'Zahl' (*number*). That is, the innerlinguistic iconicity to the sign for 'number' provides a link to a more general feature of the 'Zähler' – being a number – rather than providing a conceptual link to some idea of what the 'Zähler' could be understood as within the concept of fraction.

While potentially chosen due to the similarity of the written word 'Zahl' to the word 'Zähler' –-some kind of innerlinguistic iconicity in written language –-the sign for 'Zahl' furthermore also evokes innerlinguistic iconicity to the sign for "rechnen" (*calculating*). The shape of the hands matches for both signs, but the signs differ in movement insofar as the hands move down for 'Zahl' while they move up and down as opposed to each other for 'rechnen' (see Fig 2).

Fig.2: Sign used for 'Zähler' *(numerator*; left side*)* as innerlinguistically iconic to 'Zahl' *(number)* in DGS. On the right side, the sign for 'rechnen' (*calculating*).

That this actually seems to influence conceptualization is revealed in a student's choice for grouping the terms in the second part of the interview. Being asked "what fits together?", she explains her choice of grouping 'Zähler' and 'Bruchrechnung' together by pointing at the card 'Bruchrechnung', performing the sign for 'rechnen', then performing the similar sign for 'Zahl', placing the hand beneath the card for 'Zähler' and nodding before continuing with her explanation for the rest of her grouping.

The signs the students used for 'Nenner' (*denominator*) have been found to be similar to each other, all providing an innerlinguistic iconicity to the sign for 'Name' (*name*) or 'nennen' (*naming*). However, differences have been found in the features the sign used as 'Nenner' shared with the one of 'Name'/'nennen'. The signs can coincide

- by only sharing the same shape, the DGS-sign for the letter "n" in this case. Since this is a rather general match, the link provided through innerlinguistic iconicity is a rather weak one.
- by sharing the same shape and the same motion.
- by sharing the same shape and the same motion and by furthermore being performed at the same place, the cheek in this case. The link provided here between "Nenner" and the idea of "Name"/"nennen" is a stronger one.

Iconic-symbolic and iconic-physical reference

Iconic-symbolic reference in this context concerns a signs' reference "to a symbolic, written inscription, which in turn represents a specific mathematical entity or procedure" (Edwards, 2009, p. 138). *Iconic-physical* reference, on the other side, concerns the similarity to real objects or physical actions (Edwards, 2009). Although the students' referred in their explanations of 'fraction', 'numerator' or 'denominator' often to the symbolic representation of the fraction as one of the numbers being located above the fraction bar, the other one below, none of the signs referring to the fraction terms where purely iconic-symbolic or iconic-physical. Nevertheless, all of the students used a sign for ‘kürzen’ (*simplifying*) that combined both (see Fig. 3).

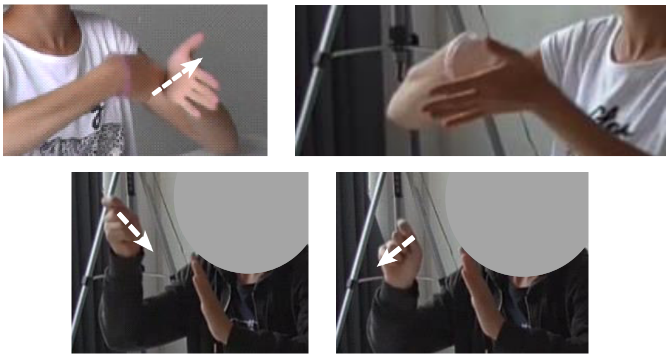

Fig. 3: Sign used for ‘kürzen’ *(simplifying)*, reflecting the action of striking off in the symbolic representation of the fraction while simplifying (above: from two perspectives; below: subsequent movements)

The signs reflect the idea of striking off when dividing the numerator and the denominator by the same number. It therefore refers iconically to a physical action that is performed within the symbolic notation of the fraction. With this, it recalls an aspects of the procedure performed when simplifying a fraction.

Iconic aspects of fraction talk in fraction comparison: an enacted iconic approach

8 out of 10 students approached the comparison of the fractions by activating area models of cake, chocolate or pizza pieces (Lamon, 1999). For this, they subsequently ‘placed’ respective imagined ‘wholes’ in the signing space in front of their body and ‘cut’ them into parts. This *enacted iconic approach* reveals an interpretation of the fraction as ‘part of a whole’, providing a *visual basis* to solve the task by means partitive division within the ‘quotient subconstruct’ (e.g. Marshall, 1993). However, all of these eight students mixed up the roles of the dividend and the divisor and identified the denominator as providing the number of wholes and the numerator as giving the parts of each whole. Since all the students visit the same class this might be explained by being prompted by some approach to fractions followed in the lessons, but not yet being fully elaborated.

CONCLUSIONS AND DISCUSSION

In this paper, I have presented diverse ways of how signed fraction talk might feature iconic aspects of mathematical ideas in the signs and gestures used and proposed how these aspects might influence the way these ideas become perceived and processed. For example, these iconic aspects might concern a certain similarity to other signs that are already used as conventionalized with another meaning and in this sense, bear an *innerlinguistic iconicity* within the specific sign language. The mathematical idea might then become linked to and interpreted against the background of some association the conventionalized meaning might evoke. Also, a sign can refer to a symbolic representation of a mathematical idea or to some sort of procedure carried out in its context. That way, it might foster a link to this representation or procedure by means of providing *iconic-symbolic* or *iconic-physical* reference to them. Furthermore, explanations carried out in sign language can provide a *visual basis* to the mathematical idea.

Grote points out that "assuming that epistemic processes are processes inherently mediated by signs, the similarity that forms the relationship between icon and referential object is constituted actively" (Grote, 2010, p. 312, translated by the author). That is, a sign does reflect iconic aspects of a referential object, or idea; it does so only for those who are aware of this iconic relationship. For signs referring to mathematical ideas, the reference has to develop hand in hand with the mathematical idea. Therefore, two intertwined processes of meaning making – of the mathematical idea and of the corresponding sign – have to be combined. In (Krause, *in press*) I describe this reconstruction of the 'process of iconization' to survey the gestures and signs used by a teacher while introducing the concepts of axial symmetry and point symmetry in an all-deaf classroom. The former becomes grounded in the activities of folding and mirroring, the latter in the activity of rotating around a point. The corresponding signs the teacher conventionalizes for "axial symmetry" and "point symmetry" respectively reflect these ideas iconically, showing aspects of innerlinguistic iconicity (mirroring) and iconic-physical references to folding and rotating. This raises the questions, are there general ways in which certain iconic relationships develop in processes of encountering mathematical ideas? Are these observable in the mathematics classroom?

As has also been seen in the description of the results, students do not necessarily use the same signs in their fraction talk. Still, there needs to be some degree of conventionalization if they want to communicate in the mathematics classroom. How does the use of multiple diverse signs for one mathematical idea influence the variety of conceptual links available for a student with respect to the signed idea?

The different types of iconicity presented in the examples are by no means thought of as exhaustive categories but rather as providing a first approach to describing the features of signed mathematical talk, based on a specific empirical basis. Further research needs to be done to widen the scope and uncover other categories so as to investigate the nature of mathematical signs and related visual-gestural representations as they develop and become established and used in the mathematics classroom.

Making claims about what makes a mathematical sign beneficial or hindering for learning mathematics is beyond the scope of this paper. The results presented here moreover raise awareness of how a 'mathematical sign' can be more than just a mere 'name' for a mathematical idea and how visual aspects of sign language can influence the shaping of mathematical thought. On the one hand, this provides an important baseline for attempts of developing dictionaries of 'mathematical signs', a current discussion in the DGS-community. On the other hand, knowledge about the influence of the shaping of mathematical signs provides a starting point for the elaboration of teaching methods in the mathematics classroom of deaf and hard-of-hearing students. In addition, research towards a more comprehensive knowledge about how those visual-gestural representations influence learning might also shed another perspective on how our body in general and gestures in particular might contribute to and shape the learning of mathematics.

References

Fernandes, S.H.A.A. & Healy, L. (2014). Algebraic expressions of deaf students: Connecting visuo-gestural and dynamic digital representations. In C. Nicol, P. Liljedahl, S. Oesterle, & D. Allan (eds.), *Proceedings of the joint meeting of PME 38 and PME-NA 36.* Vol. 3 (pp. 49-56). Vancouver, Canada: PME.

Grote, K. (2010). Denken Gehörlose anders? Auswirkungen der gestisch-visuellen Gebärdensprache auf die Begriffsbildung. [Do Signers think differently? Influence of the gestural-visual sign language on conceptualization.] *Das Zeichen - Zeitschrift für Sprache und Kultur Gehörloser 85,* 310-319.

Grote, K. (2013). 'Modality Relativity?' *The Influence of Sign Language and Spoken Language on Semantic Categorization*. Dissertation. Retrieved 01/15/2017 from: http://publications.rwth-aachen.de/record/211239/files/4546.pdf

Healy, L. (2015). Hands that see, hands that speak: Investigating relationships between sensory activity, forms of communicating and mathematical cognition. In Sung Je Cho (ed.), *Selected regular lectures from ICME 12* (pp. 298–316). New York: Springer.

Kieren, T. E. (1980). The rational number construct: Its elements and mechanisms. In T. E. Kieren (ed.), *Recent research on number learning* (pp. 125-149). Columbus, OH: ERIC Clearinghouse for Science, Mathematics and Environmental Education.

Krause, C. M. (2016). *The mathematics in our hands: How gestures contribute to constructing mathematical knowledge*. Wiesbaden: Springer Spektrum.

Krause, C. M. (*in press*). Embodied Geometry: Signs and gestures used in the deaf mathematics classroom – the case of symmetry. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, D. Wagner (eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners.* Rotterdam, Netherlands: Sense.

Lamon, S.J. (1999). *Teaching Fractions and Ratios for Understanding*. Lawrence Erlbaum Associates, New Jersey.

Marshall, S.P. (1993). 'Assessment of rational number understanding: A schema-based approach'. In T.P. Carpenter, E. Fennema and T.A. Romberg (eds.), *Rational Numbers: An Integration of Research* (pp. 261–288). Lawrence Erlbaum Associates, New Jersey.

Wilson, R. A. & Foglia, L. (2016). Embodied Cognition. In E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (Summer 2016 Edition), URL = http://plato.stanford.edu/archives/sum2016/entries/embodied-cognition/