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Math Moves:
A Lesson in Embodied Functions

By

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Abstract

This project takes a close look at a lesson in which the mathematical concept of “function” is introduced through the medium of dance. After investigating the prospects of using the fine arts to teach mathematics, and specifically, using an embodied medium such as dance/movement, a lesson is designed for implementation in a pre-algebra math classes an urban middle school. The lesson is revised for implementation in another pre-algebra class at the same school. Through data collection of video, student work, teacher reflections, and surveys, the lesson is analyzed for engagement, motivation, enjoyment, student perceptions of the connections between math and dance, and whether or not it addresses the difficult-to-learn aspects of “function”. In general, students enjoyed the lesson, were engaged and motivated to participate, made relevant connections between the art medium of dance and mathematics, and the lesson addressed nine things which are difficult to learn with respect to functions. The paper goes on to propose future changes and discuss the semiotic potentialities of “dance functions.”

Introduction

> *“I assumed that they would not learn a different way of thinking about what it means to know mathematics simply by being told what to do, anymore than one learns how to dance by being told what to do. I assumed that changing students' ideas about what it means to know and do mathematics was in part a matter of creating a social situation that worked according to rules different from those that ordinarily pertain in classrooms, and in part respectfully challenging their assumptions about what knowing mathematics entails. Like teaching someone to dance, it required some telling, some showing, and some doing it with them along with regular rehearsals” (Lampert,1990, p. 58).*

My Journey

One size does not fit all. At twelve years old, while I sat in my eighth grade algebra class glaring at the textbook (begging it to come to life and explain its contents to me), I became acutely aware of this fact: That which is effective for some people is not effective for everyone. My teacher’s philosophy was that the students would read the book together in groups and figure it out - with no instruction or demonstration from her. I would think to myself, “I like this stuff! I want to do well on this,” while my classmates, often frustrated and confused, said they hated math. I vowed right then and there, “When I grow up I will be a middle school math teacher, and my students will not give up on math.”

Also in middle school, I could not help but notice the difference in my classmates' attitudes, levels of participation, and desires to succeed, depending on the subject of the class. In math, students were generally frustrated and did not care and did not want to go above and beyond. But in art class, students were generally excited and eager and would spend hours outside of class working on their projects. In fact, I myself went above and beyond in nearly every art assignment, such as a papier mâché replica of a VolksWagen Beetle, calculated to scale for my American Girl doll.

Why are the students in art generally more engaged and motivated than in math? There is something which art class –as well as drama, music, and dance class –has to offer that is unique from other subjects: an interactive, low-pressure space for students to explore and showcase their creativity and talents within the bounds of the medium. Art seems accessible to most students, as if there is no "right" or "wrong" whereas mathematics seems to be only for a few students who can find the "right" answers, not for those who find the "wrong" answers. Noticing these differences, I wondered, "Is there a way to make math as engaging and motivating as the arts?"

Growing up and moving on to college, I chose to double major in Mathematics and Interdisciplinary Fine Arts, and my senior distinction research paper combined my interests of arts, math, and teaching young people. In "Using the Arts to Teach Middle School Mathematics" I discussed the development of the preteen mind, the social and physical changes of preteens, the theories of multiple intelligences and varied learning styles, and how the fine arts are an ideal medium for the teaching of other academic subjects, especially middle school mathematics. I closed the paper with four lesson plans, one for each of music, theatre, dance, and visual art. I was not content to leave it at that, though, so I came to the Masters And Credential for Science and Mathematics Education (MACSME) Program to become a math teacher and to conduct research exploring this concept of using the arts to teach math.

My Vision

"The arts and applied arts –music, drama, drawing and painting- provide the main opportunity for creativity and self-discipline in school. They are vital outlets and often provide learning success for the more than eighty percent of our students who do not lend themselves to the linear-logical approach to learning, upon which most schools exist" (Rubinstein, 1994, p. 57).

My ideal math instruction would incorporate outside influences from many other academic subjects, like history, science, or literature, and especially from the fine arts. In particular, information would be presented in the context of those other subjects (for instance, dance), and students would extract mathematical concepts from there (cf. Engle, Meyer, Clark, White, & Mendelson, 2010), as opposed to the more common use of subjects like art in a math class, where students learn a mathematical concept in the abstract, math-logic framework, then present the math they have learned in a project that uses sculpture, collage, a song, or a dramatic presentation (possibly on video) to demonstrate their knowledge.

Ideally, I envision the medium through which math is taught would be music, dance, theatre, and the visual arts. With educational budget crises and monetary constraints these days, I have noticed that some of the first programs and academic subjects to be cut from schools are the arts. I feel strongly that students benefit from participation in creative, artistic activities, and if these programs are cut, I would like to provide opportunities to explore the arts through math.

Going even further, my ideal math class would have the opportunity to share the story behind the artwork and the artists, because that would give students a concept of how the artist created the art (perhaps the artist has a degree in mathematics, and felt the best way to express mathematical ideas is through this work of art), and also to give students an idea of the types of occupations that exist with the arts and math to broaden their horizons when considering a career. Given all the time in the world, I would introduce the artist, show his/her work, and find math in it by doing some of our own. For example, students can learn the stained glass art of Frank Lloyd Wright, and through analyzing his geometric designs can learn properties of triangles of perimeter and area, then expand their knowledge as they create their own geometric stained glass projects and analyze their own work (Morris, 2004). Or to learn a different concept, along the lines of the work of Bamberger and diSessa (2003), students can listen to African drumming music, breaking down the rhythm and beats, discovering the underlying “groups” of beats (like measures) and the divisions of rhythm in the “melody” atop, demonstrating properties of fractions and ratios. Then students can come up with their own rhythms and analyze them for the “whole” and “parts” and describe each rhythm with fractions (Stevens, Sharp, & Nelson, 2001).

This idea of using art to teach math is not about teaching students to memorize the quadratic equation through a song (like, “negative b, plus or minus square root of b-squared, minus four a c, all over two a” to the tune of “pop goes the weasel”). While these artful settings of math concepts seem useful to many students in remembering the math facts, they do not lead a student to discover mathematics by dissecting the art. The idea of using art to teach math is also not about teaching students how to find the surface area of a rectangular prism by an algorithm, then having them make a sculpture and find the surface area of it. This is fine activity, but the math is not discovered by working with the art, it is learned first and then imposed on the art. It is not that these artful representations of math are “bad”, and I am not opposed to their use in a math classroom; these examples are wonderful tools for students to express their math knowledge. However, that is not my vision for math instruction, and it is not what this project attempts to explore. My vision is for students to engage in the art *first*, and from that recognize and find mathematical elements in the art form.

My interests also include those underprivileged students who are low-performing in mathematics. I believe that an artistic, embodied approach to mathematics may be beneficial to learning for these students. Having worked with youth of varied ethnicities and socio-economic backgrounds –from privileged suburban Minnesota, to inner-city immigrant families in Los Angeles, to secluded orphanages in Ghana –I am aware not everyone approaches life in the same way. That is why I endeavor to form curricula which approach math from many different angles, including real-world experiences, exploratory and discovery methods, as well as from the standpoint of the arts.

Finally, my vision is wrapped in the hope that others will learn to admire mathematics the way I and other math aficionados do. Nearly every time I tell someone I want to be a math teacher I hear, “Oh *math* –ugh! I hate math.” Why is this? Why do so many people dislike math? Perhaps by coming to the math via a more active, engaging medium, the math itself will be more accessible or engaging, and students’ math aversions will be averted.

Support for My Vision

In this section, I present progressively more specific forms of support for my vision. Given that there is little research specifically focused on using dance to teach mathematics, I begin by considering general perspectives and research in education that support my vision. I then move to considering literatures on using the arts first to teach academic subjects in general, and then mathematics in particular. I close by specifically examining embodied, kinesthetic approaches to mathematics instruction and considering how they relate to using dance to teach mathematics.

General support for my vision.

There are as many different ways to support learning as there are learners, because everyone is different. Educational researchers strive to dissect the different ways in which people learn in order to reach all learners by developing different ways of teaching (Bransford, Brown, & Cocking, 2003). These different ways of teaching differ from the "traditional" way of instruction which relied upon direct instruction, memorization of facts, and practicing a process correctly until it becomes automatized. We now see there are different instructional methods, which include lecture (oral and written), inquiry (open-ended projects and problems), individual contrasted with group based (self-study vs. cooperative learning), and skills oriented (drill and practice and modeling) (Bransford, Brown, & Cocking, 2003). Educational research has found that there is not one "universal best teaching practice", but that given a particular situation and specific learners, some methods of instruction can be more effective than others (Bransford, Brown, & Cocking, 2003, p. 22).

One major proponent of a theory that assumes different learning styles is Howard Gardner, who believes:

> …all human cognitive competence is better described in terms of a set of abilities, talents, or mental skills, which I call *intelligences*. All normal individuals possess each of these skills to some extent; individuals differ in the degree of skill and in the nature of their combination (2006, p. 6).

Gardner describes seven intelligences, but allows for the notion that there might be more which he has not yet considered: logical-mathematical, linguistic, musical, bodily-kinesthetic, spatial, interpersonal, and intrapersonal intelligence (2006). In the same area of thought, Ackerman and Heggestad (1997) conducted a review of many intelligence theories, and seeking an overlap between these intelligences and peoples' personality traits versus interests, Ackerman & Heggestad classified four "trait complexes": intellectual/cultural, clerical/conventional, social, and science/math (Ackerman & Heggestad, 1997).

Whether a theory of multiple intelligences can be proven or not, studies have shown that providing multiple external representations for a single concept, each attending to a different style of learning, is helpful for students' understanding (Ainsworth, 1999). In this practice, a concept will be demonstrated through several representations, giving learners different points of view and foundations in which to ground their understanding. The goal is not only that the representations be motivating, but also that at least one representation will make sense to that learner and promote comprehension (Ainsworth, 1999).

Recognizing that there are individual differences among people and thus among learners, psychologists developed the attribute-treatment interaction hypothesis to address the fact that a person's cognitive and metacognitive abilities, goals, and personality affects their learning (Gully & Chen, 2010). In a study regarding the verbal-visualizer aspect of the attribute-treatment interaction hypothesis, researchers found that students who self-identified with either a verbal learning style or a visual learning style seek those representations for gaining understanding of a process or concept (either a written explanation for a "verbal" learner or an image or model explanation for a "visual" learner) (Massa & Mayer, 2006). Learners might also look to other representations, in addition to the representation with which they are most comfortable, but they gain the bulk of their understanding from their most desired representation (Massa & Mayer, 2006).

We can see there is a need to vary instructional methods and styles to meet the needs of learners focused on different "intelligences." There are many ways to vary instruction, but for purposes of this paper and for aligning with my vision, I focus on research regarding the use of the arts to teach mathematics.

Research and other support for using the arts to teach academic subjects.

The arts have many places in the classroom: As the sole subject of the class, as informational displays for students to observe class concepts, as the medium through which students present projects and class work about areas other than the arts, and as a means to facilitate learning in an academic subject other than the arts, where the instructor leads the students through an artistic activity in order to teach them math, science, languages, and more.

In a classroom where art is present in informational displays for students to learn course concepts, the art is meant to demonstrate, describe, clarify, or explain another academic subject (Tarr, 2004). These visual aids are purchased or created by the teacher, and can be such things as posters around the room of graphs or patterns, a 3-dimensional geometric model, a video played in class about solving for variables. The students observe the art to understand or remember the other academic subject, but they do not participate in the production of any art, nor do they observe its production. While these things can be helpful for aiding in visualization of abstract mathematical ideas, the students are mere observers of the pre-constructed artistic aids (Tarr, 2004).

Students in any academic subject can use the arts to create their own "visual aids"; art can be the medium through which they present projects and class work in academic areas other than the arts. After learning a unit on spheres, students can use their knowledge to sculpt spheres out of clay, or choreograph a very spherical dance, or write a play about the "life of a sphere", embodying and portraying the characteristics of spheres which the student has learned. These projects require a thorough understanding of the mathematical concept at hand (be it spheres or something else), and that understanding is then used to create a work of art displaying the student's knowledge. But this is all *after* learning about spheres. In this case art is not as much the subject of the work as the math concept.

My vision is to facilitate the actual learning of a discipline other than the arts by having the instructor lead the students through an artistic activity in order to teach them the math, science, language, or other academic subject. This method of instruction enables different modes of learning, through physical activity in dance/movement, or visual representations in the more static visual arts, or an aural learning mode through music. Educational research has studied the benefits of providing multiple representations and varied means of instruction, finding that students have different styles of learning and should be given opportunities to receive information in various ways.

Children who participate in art activities, take courses in the arts, or who attend art-focused schools where much of the academic day is devoted to the practice and development of an artistic form, the students' performance in other areas is better than in schools where the arts are not offered or supported (Forseth, 1980; Welch, 1995; Oddleifson, 1997; Graziano, Peterson, & Shaw, 1999; Deasey, 2002; BouJaoude, Sowwan, & Abd-El-Khalick, 2005; Cokadar & Yılmaz, 2010). The United States Department of Education reported on a three-year study which used arts processes to teach other academic subjects that found that use of the arts not only improved "self-regulatory behavior", it improved understanding of other content (Oddleifson, 1997). In *Schools, Communities, and the Arts: A Research Compendium*, the U.S. Department of Education found that when art activities were used to teach other academic subjects, the students understood the material, and even grew in their skills of problem-solving, cooperating, and class preparedness (Welch, 1995).

Several studies using "creative drama" to teach science to middle grade students have shown that students can learn scientific concepts by acting out scenarios through the artistic representation of drama (BouJaoude, Sowwan, & Abd-El-Khalick, 2005;Cokadar & Yılmaz, 2010). In fact, the students who participated in the creative drama instructional units displayed higher achievement and better understanding of abstract scientific concepts than those students who were taught in a "traditional" fashion (BouJaoude, Sowwan, & Abd-El-Khalick, 2005; Cokadar & Yılmaz, 2010). It has been shown that art can successfully facilitate learning in other subject areas than the arts.

Arts in the math class.

There have not been many studies on using the arts to teach math , and those that have been conducted have not demonstrated increased achievement in mathematics due to artistic interventions (Eisner, 1999). Two studies (Forseth, 1980; Luftig, 1994) found that students who participated in art activities that related to and reinforced mathematical concepts performed slightly higher on mathematics achievement tests than the control groups which were taught in a traditional manner. Though the results in both studies were not statistically significant, the scores were not lower than the control group, and students in one study (Forseth, 1980) were later found to more easily and accurately transfer their mathematical knowledge to other situations and contexts.

The art of music, with its temporal, rhythmic, and harmonic nature, is another good medium for teaching logical-mathematical ideas. Neurological researchers Graziano, Peterson, and Shaw (1999) conducted a study on “Enhanced Learning and Proportional Math through Music Training and Spatial-Temporal Training,” which used piano keyboard training to enhance understanding of fractions and ratios. Their results showed that students who received the keyboard training scored significantly higher on a standardized test than students who did not receive this extra teaching (Deasey, 2002). It is apparent that connecting math to the arts can improve learning as well as performance on assessment tests, so we look closer at the research relating to the art of dance/movement in education with a focus on mathematics.

Not only do the arts appear a good medium through which to teach math, students express interest in it (Michelson, 2008). In a survey of upper-secondary school students, students wanted to see mathematics taught through various other disciplines, including the arts (Michelson, 2008). Though my vision includes using all art forms in mathematics instruction, this lesson will focus on the use of one: The art of dance/movement.

Embodied, kinesthetic math.

“The best way to learn an activity, is to perform it” (Freudenthal, 1971, p. 415).

Dance/movement, a performance art, is an action embedded in the body. Jean Piaget believed in a theory of action before concept: That children, when developing intelligence, first explore and elaborate in the realm of action, and by exploring through actions, learners construct ideas, or “conscious conceptualizations” (Piaget, 1974). Now there is a field of mathematical cognition research that assumes an embodied view of mathematics. Lakoff and Nunez (2000) provide a theory of embodied mathematics, that "Human mathematics is embodied; it is grounded in bodily experience in the world" (p. 366). Support of this view, according to Abrahamson and Howison (2008), has come from

- "theoretical analyses of human reasoning (Barsalou, 1999; Goldin, 1987; Lakoff & Núñez, 2000),
- "empirical studies of human's activity in general (Barsalou, 2008; Hatano, Miyake, & Binks, 1977),
- "interpretations specifically of mathematics student behaviors (Fuson & Abrahamson, 2005; Nemirovsky, Tierney, & Wright, 1998),
- "or, in particular, evidence of gestures accompanying speech acts in the solution of mathematical problems (Alibali, Bassok, Olseth, Syc, & Goldin-Meadow, 1999; Edwards, Radford, & Arzarello, 2009)" (p. 1).

Current educational researchers have put this idea into practice, conjecturing ways in which bodily activity relates to the learning and understanding of mathematics, such as the conjecture that "mathematical abstractions grow to a large extent out of bodily activities having the potential to refer to things and events as well as to be self-referential" (Nemirovsky, 2003). For example, the Oskapmin children in a remote village of Papua New Guinea ground their enumeration system in the human body, by associating a certain body part with a number, starting with the right-hand thumb as "1" and proceeding along the right arm, over the shoulders and head, coming back down the left arm, assigning "27" to the left pinky finger (Saxe, 1981). Not only can body actions be easily referenced, but these are inherently abstractions (such as, if one is counting how much change to return in a purchase, one might count on his/her fingers, which are not directly the money, and therefore are an abstraction). Studies have been conducted using kinesthetic means to introduce and support mathematical concepts such as graphs and functions (Nemirovsky & Rasmussen, 2005), finding that with the kinesthetic experience a person is able to construct symbolic, formal mathematical representations and expressions by transferring and generalizing the experience.

Mathematician, professor, and trained mime Tim Chartier attempts to "embody the invisible" in his art of mime performance (Chartier, 2010, p. 27). In his performances, Chartier first presents mime acts dealing with concepts such as the infinite number line or topology, and through post-performance discussions, gets audiences grappling with and reasoning about these abstract, advanced mathematical ideas (Chartier, 2010).

Many people, mathematicians in particular, consider mathematics to be an art in itself. In an effort to reveal how mathematics is created, and to get at the heart of the creativity of mathematics, Sririman (2004) studied five creative mathematicians and asked them about the imagery they use when they think of mathematical objects. One creative mathematician responded that he often "think[s] of functions as very kinesthetic, moving things from here to there" (Sriraman, 2004). This mathematics expert considers functions to be an active, physical thing, which could be a useful framework for learners when approaching functions.

The Lesson: Math Moves

In this section, I describe the process of this specific project, "Math Moves," and how a teacher-researcher-designer develops an idea, sets goals, creates a lesson, and implements it.

Design Process

The demographics of the particular study population were influential in the design of this lesson. Knowing my participants would be seventh grade pre-algebra students, the range of mathematical concepts to be taught was narrowed to those of that subject area (which according to the National Council of Teachers of Mathematics, 2000. includes fractions/decimals, integers, properties such as the commutative and associative properties, problem solving strategies, exponents, algebraic manipulations, functions, and much more). The particular class of seventh graders included a wide range of mathematical and academic skills, as well as learning disabilities and other difficulties; in the total of twenty students, there were nine designated special needs (including dyslexia, dyscalculia, Attention Deficit Hyperactive Disorder (ADHD), Attention Deficit Disorder (ADD), fine motoric disabilities, and delayed information processing disorders), five honors students (working on eighth grade algebra concepts in an afterschool program), and two English Language Learners.

Throughout the semester, many students voiced positive opinions about active lessons, wherein there was an element of moving around the room to do math in different “stations”. The usual class period was 95 minutes long, which many students lamented for being “too long to be doing math.” The student with fine motoric disabilities was often frustrated by activities with a lot of writing and note-taking, as were the students for whom English was not a comfortable language. The students with ADHD often had to stand up and walk around the room throughout the class period, which was in turn distracting for the students with ADD.

Another factor of designing this lesson was resources. There was no budget for this lesson, so any supplies or art materials (such as paint, paper, mosaic tiles, drums, instruments, etc.) would be purchased out-of-pocket by me. Having no spare money, this was quite a limiting factor in the design of the lesson.

Considering all of these constraints and affordances, I decided the best fine art for this lesson would be dance/movement. The students with attention and fine motoric disabilities would benefit from the active elements, the English Language Learners would not need to read or write a lot of English, the 95 minutes would be more active for all students, and it was free. In addition, several students in the class were involved with a hip hop afterschool program and often shared stories with their classmates of how they enjoyed dancing.

Once the art medium was decided, the lesson needed to focus the dance/movement around a mathematical concept. My vision is to have the dancing be the first apparent activity, from which follows mathematical understanding. The dance/movement inspires a discovery which can be seen in other contexts as well, especially in numbers and mathematics. In order to do this, I had to think more deeply about dance: What is dance? From where do dances come? What are the properties of dance that make it what it is? I hearkened back to my days in dance history, and my trip to Ghana, West Africa, to learn African Dance, and recalled that dance moves go through transformations over years and through different groups of people, and a dance style can start off one way in a certain context, and then being passed through dancers and generations, it can change and become a somewhat different dance, but still reminiscent of the original movement.

This is much like the mathematical concept of function: Functions can be considered as a set of objects which relate to another set of objects by a certain “rule”, and a function also can be thought of as an object itself, the properties of which can be transformed. Functions must also have the property that objects from the first set can only relate to one object in the second set, but multiple objects in the first set can relate to the same object in the second set (many-to-one, but not one-to-many). It was also convenient that students in seventh grade pre-algebra need to encounter the concept of functions during the course of the year.

With all of these dance/movement and function characteristics in mind, I had to find what was difficult for students about learning functions, and what are common misconceptions among students in regard to the idea of “function”, so I could design a lesson to hopefully address some of those issues, struggles and/or confusions.

Goals for the Lesson

1. To promote motivation and engagement by explicitly using dance/movement in the teaching of mathematics.
2. For students who generally express a dislike for math lessons to enjoy math lessons in the framework of dance, and for students who express a general like for math lessons to still enjoy them but in a new way.
3. For students to see connections between math and dance.
4. For students to interact with functions in the context of dance/movement, dealing with issues of functions which are considered difficult to understand.
5. To discover, as an instructor, the affordances and constraints of teaching an introduction to “functions” in a seventh grade class through the context of dance/movement.

Research: What Is Difficult About Learning Functions

Functions are an essential aspect of the world of mathematics (Martinez & Brizuela, 2006), appearing in mathematics as far back as ancient civilizations, such as the Babylonians who created tables of ordered pairs with functional correspondence (Jones, 2006). Through the history and development of functions, mathematicians grappled with the concept, how to describe it, and how to define it, coming 70 years ago to a definition which can be found in current mathematics textbooks:

"Let *E* and *F* be two sets, which may or may not be distinct. A relation between a variable element *x* of *E* and a variable element *y* of *F* is called a functional relation in *y* if for all *x* contained in *E* there exists a unique *y* contained in *F* which is in the given relation with *x*" (Kleiner, 1989).

As is seen in this definition, variables play a major role in the idea of function, and now in the Twenty-First Century many instructors have adopted an approach to teaching and learning algebra through the lens of functions (Kooij, 2002; Schleimann, Carraher, & Brizuela, 2001; Martinez & Brizuela, 2006). Being so important in understanding and developing abstract mathematical concepts (Piaget, Grize, Szeminska, & Bang, 1977), educational researchers have studied the difficulties of learning functions and ways in which students understand and misunderstand the idea of "function".

The tasks in approaching functions can be categorized by: (a) action -whether engaged in interpretation or construction of functions; (b) situation -the setting of the task, such as math class or social studies, and the context of the problem, or the "problem situation" (National Council of Teachers of Mathematics, 2000); (c) variable -either the static substitution of letters for numbers, or the dynamic representation of a graph; (d) and focus -attention on an aspect of function, such as the location of a coordinate pair or the shape of a graph (Leinhardt, Zasalvsky, & Stein, 1990). For the purposes of this lesson, as an introduction to the idea of function for early middle school students in pre-algebra, we will look at literature which refers to the usual ways in which "function" is introduced and taught, the idea of function as a relationship between values and as a "rule", and not address the issues involving graphing and transformations (Leinhardt, Zasalvsky, & Stein, 1990).

One main aspect of the concept of "function" is the manipulation of variables, thus alphabetic variables have become a mainstay in instruction of functions. Because of this, some instructors bring in the construct of substituting the letter variable *x* or the letter *y* for a numerical value sooner than the students are comfortable (Hitt & Morasse, 2009). Students can develop the need for variables once they see the situation and represent the unknown value with something representational, and it need not be the institutionalized letter *x* (Hitt & Morasse, 2009). When dealing with functions, there can be functions which deal with other variables, not necessarily *x* and *y*, and not even necessarily letter variables that stand for numbers.

Further, in a study of beginning algebra students working with variables to solve situated problems versus symbolic algebraic equation, students had difficulty manipulating and solving for the unknown value "*x*" in the conventional symbolic equation (only 42% successfully solved the problem), but when presented with a problem situation, 70% of students successfully solved for the unknown value (Koedinger & Nathan, 2009). The use of pure numerical values and alphabetic variables can cause trouble for students in solving problems whereas a kinesthetic and real-life problem situation can lead to correct understandings and development of generalization and variables (Carlson, 2002).

Understanding functions in their most basic form is an understanding of "action" or "a repeatable mental or physical manipulation of objects" (Dubinsky & Harel, 1992). In this beginning level of abstracting functions, students need to see the action of a function, and functions are viewed as an algorithm or "rule" that transforms an "input" into an "output" (Jones, 2006). To help students at this level of understanding, many instructors use the idea of a "function machine" which takes in a number and puts out a different number, following some rule, and some researchers believe this to be an effective introduction to the idea of functions (McGowen, DeMarois, & Tall, 2000). However, other researchers believe this method confuses students because they cannot see how the machine "works" (Seldon & Seldon, 1992).

In a study on constructing function concepts with middle school students, researchers found that using the function machine model proved more useful for students than using real-life, manipulatible situations such as turning two spools of different diameter pulling strings, or hanging weights on springs of different strengths (Meira, 1998). Students using real-life manipulatives felt more comfortable at first, as though the activity were too obvious, but as they tried to generalize and create and algebraic formula for the situation, they had many troubles. The students using a computerized "number machine" were troubled at first, and did not know how to begin thinking about the situation, but as they noticed a pattern they were more quickly able to generalize into an algebraic formula for the function (Meir, 1998).

Functions, as a concept involving variables and abstractions, are formally introduced in algebra classrooms, with students who have not had instruction in set theory, because they are likely in middle school and set theory appears in college courses. However, the working definition of function includes the concept that there are coordinate pairs comprised of members of sets, and each member of the first set can only be paired with one member of the second set (Kleiner, 1989). Research has shown that students in high school and below, who are unfamiliar with set theory, can be confused by these concepts and have no intuitive understanding with which to associate this idea of ordered pairs from specific sets (Jones, 2006).

Building on the idea of sets, these sets need not be comprised of numbers, by the arbitrariness of functions (Seldon & Seldon, 1992). Students rarely consider functions to consist of anything other than real numbers (Sand, 1996; Jones, 2006), when in fact, functions can relate people (relating a son or daughter to a mother or father), or a function can relate animals (a kitten to itself as a grown cat, a puppy to itself as a grown dog, etc.). The variables involved need not be letters, and the things they represent need not be numbers, but because instructional methods in schools generally only use these conventions, these are the elements students consider when they define functions (Sand, 1996).

Another difficult concept for students to grasp is the many-to-one concept (Sand, 1996; Jones, 2006), the concept that although the elements of the first set can only correspond to *one* member of the second set, there could be many elements of the first set which relate to the same member of the second set. Furthering the concept of many-to-one, there are also constant functions, where every member of the first set corresponds to one member of the second set. Students have trouble grasping this concept in the earlier stages of function work because school textbooks do not put much emphasis on constant functions (Sand, 1996).

Finally, some mathematics cognition researchers believe that for higher mathematical understanding, it is important to view function as an object (Sfard, 1992). This view, this “structural conception” of function, makes for more economic and faster thinking processes in higher mathematics. Though it is considered an ideal way to think of functions, this “object” perspective is embedded in more complex mathematical structures, and should only come after the process perspective, of "operational conception", and the idea of functions as object will emerge from that process (Sfard, 1992).

This lesson aims to address some of the troubles students encounter when first learning functions: Working with variables (especially variables which are not necessarily letters), working within a problem situation instead of a pure algebraic equation, seeing functions as an action, adapting the idea of the "function machine," addressing the idea of sets, demonstrating the arbitrariness of functions by using non-numerical examples, explaining the one-to-one and many-to-one concepts, all building from an operational conception.

Math Moves Lesson -As Planned

For the two class periods prior to the Math Moves lesson, students take a survey (see Appendix, part i), the same survey each day, to gauge engagement and enjoyment of each day of math class.

The first day of the Math Moves lesson (see Appendix, part ii, for complete lesson plan), the class enters the classroom to find the desks all pushed to the perimeter and a large open floor, on which the students are asked to sit in a circle. This is to facilitate movement and creativity of dancing. The first activity is to answer, silently and independently, the following question on a note card: "What does dance have to do with math?" Without discussing the answers, we go into the lesson.

The teacher begins the lesson with an introduction about dance. I, being the teacher, told a quick story of my experience in Ghana, West Africa, and how dancing traditions in Africa influenced the styles of dance we see in America today. For instance, the class is sitting in a circle because many social dance forms, especially in Africa, are in a circle. Everyone is a community, nobody is on a stage. Also, hip hop dance in the United States has "breaking" where a dancer will show off his/her moves during the instrumental break in a song, and this "showing off" usually happens in the middle of a circle of people dancing. That idea comes from Africa. Furthermore, individuals often adopt each other's dance moves, but alter them slightly to make them their own, or to make them "better" or more crowd-pleasing. A person might change a dance move by performing it faster or slower, or doing it in reverse, or changing directions, or adding other elements, but the original dance move is apparent within the new move.

That idea is the focus of the day's lesson: To take a dance movement, call it an "input", and change it into a new movement, call that an "output." While explaining this, the teacher draws a T-table on the whiteboard, with "input" for the left column and "output" for the right column.

Standing in the left side of the T-table, the teacher demonstrates a dance movement (see Appendix part viii for some suggested movements), calling it the "input". Then the teacher moves to the right side of the T-table and demonstrates the corresponding "output" dance movement. Students will be asked to guess what changed from the input to the output movements, and the teacher is to take student suggestions, entertaining as many different ideas as possible. Teacher notes that all of the guesses could be right, according to that *one* pair of movements. But what if there was another pair of movements, with the same relationship as the first pair of input and output?

The teacher must demonstrate another pair of movements for students to narrow down the possibilities of what changed from the input to the output. Students will again be asked to guess what changed from the input to the output, and the variety of suggestions will likely be less than the first time. The teacher impresses the need to see just one more pair of movements to be certain of the relationship between input movements and output movements.

Once a relationship has been defined which all students believe describes this situation (these pairs of inputs and outputs), the teacher introduces the terminology of "function" -that the relationship between the input movement and the output movement is called a function- and writes the word "function" on the board. The teacher also explains that the function is "of" the dance move, and gives the function notation "*f* ()" where  represents the dance movement. The teacher models the verbal phrasing that this means "a *function of* the dance movement."

To get more practice in the finding of a relationship between movements, the teacher demonstrates three more pairs of movements, this time bringing up two student volunteers to help demonstrate the output movements. When the class has reached consensus about what the function is for this second example, the activity shifts to allow the students to try their hands (and feet, and arms and legs) at creating input movements and applying a function to create the related output movements.

Students are broken into groups of four or five and each group is given one page with instructions (see Appendix, part iii), a poster paper, and a marker. Each group is to create one movement per person in the group (so each person can create his/her own movement, but there is opportunity for assistance or suggestions from group members), and record these movements as images in a T-table on the poster paper, in the "input" side. The group is then to apply the function given on the instruction page to each input movement, and all agree on the output which would follow the relation. Once all group members agree an output follows the function relationship properly, the output movement is recorded as an image in the right side of the T-table, across from the corresponding input movement.

After a group is finished with the prescribed function, they are to create their own function. The group is to draw another T-table for this new function, and using the same input movements from the previous function, they are to agree on the output movements for each input given this new function. As they decide on the output movements, they record the movements in the T-table.

Reaching the end of the class period, the students fill out a survey (the same as the previous two days). For homework, students are to teach their dance function, with inputs and outputs, to a friend, neighbor, or family member, and report back to the class about the person's reactions.

The second day of the Math Moves lesson (see Appendix, part iv), the classroom has again an open floor (no desks). The teacher asks students about their homework -to teach their function to a friend or relative- and those who would like to can share with the whole class. Next, students answer, silently and independently, this question on a note card: "What does dance have to do with math?" Just as before, without discussing the answers we go into the lesson.

Students break into their groups from the previous day to quickly refresh their memories and limbs of the functions, inputs, and outputs which they created the previous class day, using the posters as reminders. The groups then present to the class their pairs of movements, each input with its corresponding output, and the class is to guess the function. When a group has finished showing all of their movements, and the function is correctly discovered (or is revealed by the group presenting), the function is written on the board in function notation (for instance, *f* () = 2  could be a representation of two dancers performing the input).

After all groups have presented, the students will get their math notebooks and brainstorm as a whole class, "What *is* a function?" After some discussion and student suggestions, the teacher gives the class a formal definition of *function*: "A relation that uniquely associates members of one set with members of another set." To reinforce the formal definition, the class is asked to explain each element of the definition, such as "What is a relation?" and "What does it mean to 'associate'?" and tying the definition to the dance/movement activity, "What do you think the 'members of one set' are?" Students share their ideas with the class to more fully understand the definition.

After that, the teacher elaborates on the idea of uniqueness by demonstrating that there can only be one output movement for each input movement if it is correctly following the relationship of the function. However, there can be a function which relates every input movement to the same output movement, such as *f* () = clap. This says that no matter what movement you have as the input, your output will be a clap, because the dance movement represented by  is not present in the output, there is just a clap.

To begin moving the class toward functions with numbers and variables, the teacher gives examples of functions which are no longer dance/movement, but which contain real life, real world situations. First, the teacher gives the function of making a telephone call for a certain cost per minute, drawing a T-table on the board which the students also draw in their notebooks. The teacher next gives a realizable example which demonstrates the many-to-one property of certain functions that multiple inputs can lead to the same output: Mail goes to mailboxes via the mail carrier (Sand, 1996). If a person wishes to mail a letter, s/he wants that letter to go to a specific destination, and to do that the letter must have an address on it. That specific address talks about only one destination, so there is only one place that letter can go: To the desired mailbox (if it doesn't get lost somewhere!). However, that mailbox can receive multiple letters; it receives all letters with that address. All of this is recorded in a T-table on the board, and the students write their own T-tables in their notebooks.

After two real-life examples, the teacher moves into explicitly mathematical, linear algebraic functions. Drawing a T-table on the board, the teacher labels the left column "x" and the right column "y", and gives the function $f(x) = 2x + 1$. The students are invited to give some values for the input column, the x values, and together as a class figure out the corresponding y values, the output column. Then the teacher constructs a new T-table, filling in some coordinate pairs (some x and y values) and some single values (either the x or the y but not both), and the class must guess the function, and come up with the correct output or input values to fill in the missing x and y spots in the T-table.

To close out the lesson, the students will answer -for the last time- this question on a note card: "What does dance have to do with math?" Students will fill out another survey (the same survey as the previous three days), and the day is done.

If there is time later in the semester, the teacher can revisit the lesson with the class, bringing in videos of dance from Africa and from hip hop today, and students can look for correlations between the African traditions and the United States' alteration of those styles and movements. Also, the teacher can show examples of other dance/movement works which relate to mathematics, such as choreographer Merce Cunningham and his use of probabilities to put dance phrases together for a performance. Furthermore, the Dr. Schaffer and Mr. Stern Dance Ensemble, which has a repertoire of Math Dance choreographies, publishes a book of activities for Math Dance in the classroom to explore concepts through body movement, such as combinations, symmetry, and polyhedra (Schaffer, Stern, & Kim, 2001).

Methodology

Guiding Research Questions

For purposes of this study, to deepen my exploration of teaching math through the medium of dance, in hopes of adjusting and validating my lesson plan choices, I chose to focus on my first three goals to create these guiding research questions:

1. Might the explicit use of movement/dance in the teaching of "function" promote motivation and engagement?
2. Do students enjoy math in the framework of dance, and further, do students who express a general dislike of math lessons enjoy math lessons in the framework of dance?
3. Can a lesson in which functions are taught through dance help students see connections between math and dance?
4. Does a lesson in which students interact with functions in the context of dance/movement deal with issues of functions which are considered difficult to understand?

Data Sources

To address my specific guiding questions, I collected the following data for analysis.

Video

To capture the activities of the students, the style of instruction, and the very things that happened during the lesson, there were three video cameras placed around the room. One stationary camera in the back left corner to capture the lesson, my instruction, and the whole classroom, another stationary camera in the front right corner to capture the students' actions and reactions throughout the lesson, and a final moving camera to focus on students working in groups, and catch specific moments up close.

Survey

To gauge students' engagement, motivation, and enjoyment of math class, I gave a survey at the end of class on a total of five days: two surveys at the ends of the movement math lessons, and three at the end of non-movement math lessons. The survey, an adaptation of a survey designed by Koller, Baumert, and Schnabel (2001) had five statements, each with a Likert or summative scale from 1 to 4, where 1 = "strongly disagree" and 4 = "strongly agree". Two questions also had an open answer part, for students to explain their choice of number 1 to 4. The scale did not allow for a middle number, intentional in order for students to decide one way or the other (did they agree or not?). Students of one class marked each survey such that each student's responses can be tracked across the lessons. Students of the other class took all surveys anonymously, and thus their responses cannot be tracked by student, only the class as a whole. (See Appendix part i for the survey.)

Note cards

To gain insight into students' thinking about the connections between math and dance/movement, students answered the following question on a note card:
"What does dance have to do with math?"
Class A answered this question three times: before the movement math lesson, the morning after the first day of the lesson, and the morning after the second (and final) day of the lesson.
Class B answered this question once: the morning after the second (and final) day of the lesson.

Student work on dance functions

To support students' transitions from exclusive movement functions to exclusive math functions, part of the lesson involved students writing or drawing representations of the dances being done by the teacher, their classmates, and themselves. The sheet on which students recorded the movements was structured into "T-charts" of "inputs", "outputs" and the functions which related to two columns. (See Appendix part vi for worksheet). These charts show students' representations of dance movement in a math context.

Teacher reflections

At the end of each day of each lesson, I recorded my immediate reactions and responses to the happenings of the day, including thoughts on whether activities went as planned or not, and how that affects the lesson for the next day. These reflections were used to inform decisions on how to change the lesson for use in the second class.

Informal whole-class conversation

Recorded on video, at the end of the second day with Class A, I led a brief, open-ended, discussion on how they felt about the movement math lesson. I felt that day, in the interest of getting out of work, the students might not have expressed themselves honestly in the written surveys, thus I decided in the moment to ask a couple of questions to get at whether they thought it was weird, or cool, or if they liked it, or if they would ever want to do it again, etc.

Implementation of the Lesson

This lesson was implemented with two separate classes of seventh grade pre-algebra, at Martin Luther King Middle School in Berkeley, California. As reported by the California Department of Education, Educational Demographics Office (CBEDS, 7/7/09), the student population of the school is comprised of many ethnicities: approximately 35% White, 23% African American, 16% Hispanic, 8% Asian, 1% Filipino, and 17% Multiple Ethnicities or decline to state. The two classes in which this lesson was conducted reflected the diversity of the school. I will call the first class with which I worked “Class A” and the second class “Class B”. Class A had 19 students, 10 female and 9 male, whereas Class B had 18 students, 9 female and 9 male. The movement math lesson took place over two days per class, with one week between Class A and Class B to revise and re-plan. Class A was very familiar with me and my usual instruction; we had an established relationship as I was their primary teacher. Class B did not know me before the first day of the movement lesson, thus they were not familiar with my instructional style and did not expect me to teach in any certain way.

Class A

On the first day of the lesson the students entered the room to find, unexpectedly, that there were no desks, just a large empty room -a significant change from the normal classroom experience. Many reacted with great excitement and activity, curious to know what was happening, not quite sure what to do with themselves. Upon asking the students to sit in a circle on the floor, many responded adversely, saying they were not in kindergarten. Other students embraced the idea of being in kindergarten again, and started running around the room, not sitting in a circle.

When the class settled down, and sat in a circle, they were attentive and eager to see what the day would bring. The first activity, to answer the question “What does dance have to do with math?” on a note card, did not go as planned. Before the instruction of “do not say anything aloud, just think to yourself” could be spoken, students began calling out their ideas of how dance and math are related, which then influenced the responses of the whole class.

During the teacher demonstration of ways people change and stylize dance movements, some students felt compelled to mutter jokes, and cause the whole class to laugh. For the most part, though, students were receptive of this new and different activity, genuinely following along and asking relevant questions. As the notation and terminology of “function” was introduced, students expressed confusion and a hint of frustration, explaining that they understood what happened with the dancing, but then writing it down and calling it “a function of the dance move” no longer made sense.

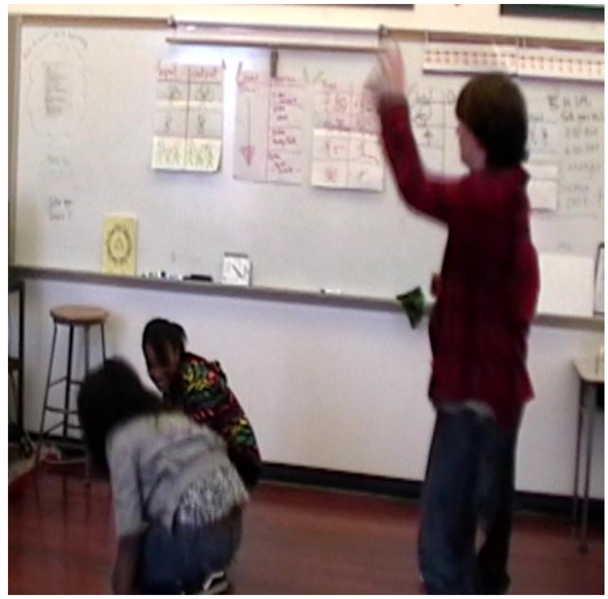
With a little further explanation and referencing the previously demonstrated functions, many of the confused students vocalized their realizations with “ah-ha” and “oh, now I get it” expressions, allowing the class to move to group work. The usual expectation for group work is for each group to receive a page of instructions, read the instructions together, and complete the task while the teacher rotates around the room helping answer questions. On this day, however, many groups neglected to read the instruction page, or at least did not do so very carefully or fully, and thus the intended activity was not done by most groups.

An unforeseen challenge to the group activity of creating dance movements was the level of noise and disruption the class would create for the room directly below, which had a class of sixth grade math students taking a standardized test. The teacher from the classroom below made several telephone calls requesting that the students stop moving around, which put a damper on the creativity and energy of the dancing students. The class was frequently being told to be more quiet and less jumping or walking –though the prescribed activity, to create input dance moves and corresponding output dance moves, called for a degree of noise and movement.

Although the group work time was supposed to last until the end of the class period (a good thirty minutes total, to create posters for two different functions: The given function and their own invented function), several of the students with specified learning disabilities were having trouble with the activity. One student with ADD was overwhelmed by all of the movement and the various groups doing different things, so he stopped participating altogether and sat on a couch along the wall alone. Another student with social disorders was unsure of how to conduct himself in this free setting, so he hid his head in a desk and did not participate, and he responded to any person who tried to communicate with him by repeating back to them everything they had said to him, but using a robot voice. One other student was frustrated with the disorganization, and she pulled her hood over her face, tied the strings as tight as they would go (so no one could see her), and sat on the floor below her group.

One group included three students who are enrolled in the Honors math program (a voluntary group that does extra math for the sake of math, not for any extra school credit). These are students who enjoy math on a regular basis and who are dedicated to achieving and understanding the math content. During the group work time of the dance/movement lesson, these students were exceedingly discouraged by trying to do the activity described on the information sheet while seeing other groups doing entirely different things. They did understand the activity, and were following the instructions, but they were troubled by the way their dances and movements were nothing like their classmates'.

Seeing that things were chaotic, the groups were not following the directions, and that the activity was too disruptive for the class below, the lesson was altered to have groups share their functions at the end of the first day instead of at the start of the second. It was revealed in this time of sharing that most groups did not understand the activity, and there were not distinct input movements which related to output movements by a function. The teacher inquired into each group's presentation, to have the presenters break the movements into coordinate pairs related by a function, but many groups were unable to do so. Some groups had created whole dances which followed the function "rule" (See Images 1 and 2). The intention was to perform the input facing the left, but the group created a dance which faced the left –or rather, their right- without distinct inputs facing front.

Images 1 and 2. Group 4, Class A, Day 1- dancing function, but misinterpreted the assignment; the function given was *f* (🯅) = 🯅 facing left *; the entire function was danced to the left.*

Other groups simply created entire dance sequences, with no resemblance to the given function (see Image 3). One group presented coordinate pairs which were distinct input and output movements, but each pair followed its own function as opposed to the whole group demonstrating the same function through many pairs of inputs and outputs.

Image 3.

Group 1, Class A, Day 1- dancing function, but misinterpreted the assignment; the function given was *f* (🯅) = 5🯅 *; they are all dancing a different movement.*

The final activity of the day, the survey, was met with many complaints from the students because they had already filled out two surveys, one for each of the previous days of class, and they did not see a need to fill out yet another. Realizing the students did not understand the concepts nor the activity (and realizing that there would be significant modifications to the second day of the lesson to ensure the concept is taught and understood), the homework assignment to "teach your dance function, with inputs and outputs, to a friend or neighbor or family member over the weekend" was omitted.

The second day of the lesson required some adjustments to clear up confusions from the first day. The desks were not moved away from the center of the room, so the setting was the usual math classroom setting and students entered as they usually did, calmly and comfortably knowing where they belonged and what to do. It began with the note card activity as planned, and it was made very clear that students were not to say their thoughts aloud but they should answer on the note card silently, "What does dance have to do with math?"

The next activity was very similar to the beginning of the previous day, with the teacher dancing an input and a corresponding output, but this time, the students and the teacher were recording the movements in T-tables of their own (the teacher on a giant poster T-table, and the students on a worksheet with many T-tables –see Appendix, part vi). All students were to think to themselves about what the function relationship could be, without saying it aloud, to give everyone a chance to come up with their own ideas. Once three pairs of input and output moves were given, then students were invited to share their ideas for the function. The function was then written in proper notation under the T-table with the respective coordinate pairs.

This was done a couple of times, taking student volunteers to perform the output movements or to invent input movements, and all students guessing for him or herself what the function relationship should be. For the last T-table on the worksheet, the students broke into groups with the peers sitting around them, and the group drew a function from a hat. Given only five minutes, the groups were to come up with three input movements and apply the function (drawn from the hat) to find the corresponding outputs. Each group then demonstrated their movements and the class guessed at the function relationship.

Returning to the originally planned lesson, the students wrote a formal definition for function in their composition notebooks, which built from the understanding of the first day, that the input and output are related by a function. The teacher then guided a discussion on how the definition describes the dance functions (that the members of "one set" are the input moves, and "another set" is the output moves). Continuing in the composition notebooks, the class followed the real-world examples of functions (telephone calls and sending mail), but as soon as the explicitly mathematical, algebraic function was introduced, $f(x) = 2x + 1$, some students became confused, distraught, and discouraged because it no longer made sense to them.

Spending time to try to clear up the confusion, there was not time to do an ending worksheet to practice algebraic functions with numbers. Before the students left, they answered on a note card for the final time, "What does dance have to do with math?" Many students grumbled, feeling that they had already answered this question twice and they had nothing more to say. However, they all did write their answers on a note card, and followed that with another survey, which they again did not appreciate.

In hopes of getting a true response from students about their feelings on the dancing in math lesson, there was a brief, informal discussion prompted by one question: "Hey guys, I just want to know: What did you think of dancing in math class?" Students gave individual responses, many said they liked it more than anything else, and a couple said they disliked it and did not want to do it anymore. About 90% of the students raised their hands in response that they liked it enough to want to try something like this again. Unfortunately, there was not time near the end of the semester to do a revisit of dance in math.

Class B

Having done the lesson once already, it was clear that many revisions were necessary (see Appendix, part v for the revised lesson as implemented with Class B). The first revision was to not move the desks away and to not sit in a circle on the floor. The desks were in their usual places, the students were in their usual seats, and the lesson began after thirty minutes of "regular" math class had already happened, so the students were very calm and comfortable.

Also, the class of sixth graders was out at lunch, so there was no worry of disturbing them if students were jumping or moving around or making noise.

Another slight change was to remove the three rounds of answering the question "What does dance have to do with math?" on a note card. The students only answered the question once, the day after the second day of the lesson.

Class B heard the introduction about African dance influencing dance around the world, particularly hip hop in the United States, and (possibly because they were unfamiliar with the teacher) they were not inclined to act out or mutter jokes during this, which went faster with no interruptions. The lesson then moved to using T-tables on the worksheet used for Class A on the revised second day (found in the Appendix, part vi).

The first function was entirely demonstrated by the teacher, and was accompanied by the teacher writing the function notation on the board, as well as modeling how to represent each dance move in the T-table. The second function included student volunteers, and the whole class guessed at the function relationship. The third function was given, and student volunteers came up to demonstrate a movement that followed the function relationship, and the class decided whether it fulfilled the relationship or not.

The fourth function was done in groups (formed by sets of four desks near each other). Each group drew a function from a bag, then created three input movements and figured out the corresponding output movements according to the function relationship. Each group member recorded the movements in the fourth T-table on their own worksheet. After five minutes, the groups shared their pairs of inputs and outputs with the class, and everyone guessed what the function was which related the pairs (see Images 4 and 5). The last activity of the day was to complete a survey, as they had done the previous day.

Images 4 and 5. Class B, Day 1- group sharing the input and the output for the function *f* (🯅) = mirror 🯅 *, demonstrating within the T-table on the board.*

The second day was again the latter half of a "regular" math class (see Appendix, part vii for the revised lesson for the second day). The class began by getting their composition notebooks, thinking back to the previous day with the dancing activity and brainstorming some ideas for the meaning of a "function" according to what they had done. The class then wrote a formal definition of "function", and related the terminology in the formal definition to the aspects of the dance functions.

Following the realization that there can only be one output movement for each input movement, the class went into real-life examples of functions (telephone calls and mail). Having understood those quite well, Class B moved easily to algebraic functions, and noted that this function concept was much like a game they played previously in the year with a horizontal T-chart, filling in missing parts of the coordinate pairs and guessing the rule.

As class ended, the students filled out another survey. Though the plan had been for the students to answer the "What does dance have to do with math?" question on a note card at the end of this day, there was not time, and thus the students answered the question the following class day, which was a few days later.

Analysis

Each data source serves each of my three guiding research questions in different ways. In order to address the guiding questions as fully as possible, and to get at the purposes of this study, I analyze data sources differently per guiding question according to the following criteria:

Addressing Guiding Question 1, *“Might the explicit use of movement/dance in the teaching of “function” promote motivation and engagement?”*
The relevant data sources to address this question are the daily surveys and videos.

Surveys

Item 1 of the survey relates to a student's motivation for that particular day, as it asks about their attempt to engage in the lesson of the day. Item 3 of the survey is aimed at students’ interest and/or investment in the lesson, which is a leading factor in motivation. Item 5 of the survey references a student’s sense of “losing track of time”, intended to indicate engagement, as a student who is engaged in the activity will likely not be distracted by thinking about how much time has gone by and how much time remains.

To analyze items 1, 3, and 5 of the surveys for motivation and engagement, I tabulated students’ responses in a spreadsheet, correlating the responses of each student in Class A, whereas Class B surveys were reported anonymously and could not be correlated, but were independent. I then averaged all responses of non-dance/movement lessons, and averaged all responses of dance/movement math lessons, for each of the classes separately. With the averages and standard deviations for each student for each style of lesson, I ran t-tests on each item of the survey for each class. For Class A, I used correlated samples t-tests as I was able to keep track of who had filled out each survey to be able to match them. For Class B, I did not have this information so therefore needed to use the less powerful independent samples t-test. I then compared the responses for dance/movement lessons versus “regular” math lessons, for each class, looking for trends of motivation and engagement among the students, specifically to see if the dance/movement lessons produced at least as much motivation and engagement as a regular math lesson.

Video

The three video cameras located around the classroom captured the activities of the students, tracking closely their engagement throughout the dance/movement lesson. Video also captured verbal expressions of motives to participate, as well as verbal indicators of the particular focus of the students at a given time.

To analyze the engagement of the students as seen in the videos, I apply Engle and Conant's (2002) "productive disciplinary engagement" framework, where productive disciplinary engagement is taken to be "students' deep involvement in and progress on key concepts and/or practices characteristic of the discipline they were learning" (Engle, 2009, p.6). There are six facets to measure students' engagement:

a) More students in the group sought to make, and made, substantive contributions to the topic under discussion;
b) Students' contributions were more often made in coordination with each other, rather than independently of each other (Barron, 2000; Chi, Siler, Jeong, Yamauchi, & Hausmann, 2002);
c) Few students were involved in unrelated 'off-task' activities;
d) Students were attending to each other assessed by alignment of eye gaze and body positioning (McDermott, Gospodinoff, & Aron, 1978; Shultz, Florio, & Erickson, 1982);
e) Students often expressed passionate involvement by making emotional displays (Tannen, 1989); and
f) Students spontaneously got reengaged in the topic and continued being engaged in it over a long period of time." (Engle & Conant, 2002, p. 402)

I used these six facets collectively to identify engagement in my data. Then, watching the videos, I kept a log of the time students spent engaged in particular disciplines. The engagement was broken into four disciplines: "dance/movement", "mathematical reasoning", "real-world", and "other." Two interdisciplinary engagement categories were also identified: "both math and dance/movement" and "both math and real-world." The whole class is considered focused on "dance/movement" when they are inventing initial movements, "math" when they are manipulating numbers and mathematical symbols, “both math and dance/movement” when they are applying functions or ‘rules’ to an input move to get an output move and when they are trying to relate math and dance (i.e., note cards), “real-world” when they are discussing the workings of phone bills and mail carriers, “both math and real-world” when they are trying to relate the idea of ‘function’ to the real-world situations, and “other” when they are doing administrative or other things (getting settled, cleaning up, etc.) (cf. Bergmann, 2008).

Addressing Guiding Question 2, *“Do students enjoy math in the framework of dance, and further, do students who express a general dislike of math lessons enjoy math lessons in the framework of dance?”*

The relevant data sources to address this question are the daily surveys and videos. There was also an informal, whole class, teacher-led discussion with Class A, which was captured on video. The second portion of this question can only be addressed in Class A, for reasons explained below.

Surveys

Item 2 on the survey directly refers to students’ enjoyment of the lesson of that day, in comparison with ‘usual’ math lessons. Item 4 on the survey asks if the student would like to spend more time doing lessons like the lesson of that day, with the intent that if a student enjoyed a lesson, s/he would want to do more lessons like that one, and thus Item 4 relates to students’ enjoyment of the lesson.

To analyze items 2 and 4 of the surveys for enjoyment, I tabulated students’ responses in a spreadsheet, correlating the responses of each student in Class A, whereas Class B surveys were reported anonymously and could not be correlated, but were independent. Just as was done for items 2, 3, and 5, I averaged all responses of non-dance/movement lessons, and averaged all responses of dance/movement math lessons, for each of the classes separately. With the averages and standard deviations for each student for each style of lesson, I ran t-tests on each item of the survey for each class. I then compared the responses for dance/movement lessons versus "regular" math lessons, for each class, looking for trends of enjoyment among the students, specifically to see if the dance/movement lessons produced at least as much enjoyment as a regular math lesson.

As the surveys in Class A were correlated, I was able to analyze responses for changes between regular math lessons and dance/movement math lessons using a paired t-test, to see if the average enjoyment for that particular student increased on a dance/movement day from a regular day. Students who responded with a 1 or 2 on items 2 and 4 of the survey on regular days were considered to express a general dislike of math lessons, and a response of a 3 or a 4 on those same items would express enjoyment. The surveys in Class B were not correlated, so a comparable analysis cannot be made.

Video

The three video cameras located around the classroom captured the visible expressions and the verbalizations of the students' attitudes during the lesson.

To analyze the videos for students' enjoyment of the lesson, I noted behavioral and physical expressions of enjoyment (laughter, smiles, etc.) in comparison with expressions of dislike (sighs of frustration, groans, etc.). I also logged verbal expressions of enjoyment, such as "math is fun!" versus verbal expressions of dislike, such as "do we *have* to do this?"

In Class A, because I was the teacher and was familiar with students' usual mathematics classroom behavior, I observed videos for instances of students who usually do not participate or who often complain about math activities to see if they show any different behaviors. Specifically, do they express enjoyment or more distaste toward the activity than a regular math lesson? Furthermore, I looked for behaviors of students who usually enjoy math class to see if the dance/movement framework affected their enjoyment.

Whole-class Discussion

At the end of the second day with Class A, I led a brief, open-ended, discussion on how students felt about the movement math lesson, after observing students' attitudes toward the survey that day (they were not pleased about filling out another survey, and thus might not have expressed themselves honestly in them).

To analyze student enjoyment from the whole-class discussion, I listened to the video records for student responses which indicated enjoyment of dance/movement lesson, and eagerness -or at least willingness- to do another dance/movement lesson in the future.

Addressing Guiding Question 3, *"Can a lesson in which functions are taught through dance help students see connections between math and dance?"*
The relevant data sources to address this question are note cards and video.

Note Cards

The note card activity directly asked students to make connections between math and dance.

To analyze note card responses, I entered all responses in a spreadsheet, and categorized responses into six groupings according to the ways in which they relate dance/movement to mathematics: (a) counting steps and/or beats; (b) timing; (c) geometry; (d) functions; (e) nothing to do with each other; (f) other (such as, "they are both hard" or "they are both fun"). Some students wrote several connections, and each connection was considered separately, thus, there are more connections than students.

In Class A, which answered the question on a note card three separate times, I followed each student's progression from their responses before any dance/movement math lessons, between the dance/movement math lessons, and after the dance/movement math lessons. I looked for ways which their connections changed, expanded, or decreased.

Video

The three video cameras captured students' verbal expressions of connections between the dance/movement and mathematical concepts.

In logging the times when students were engaged in one of four disciplines ("dance/movement", "mathematical reasoning", "real world", and "other"; or a combination, "math and dance/movement" and "math and real-world"), I recorded verbal expressions of connections between math and dance/movement, such as usage of numerical operations and alphabetic variables in describing the dance moves.

Addressing Guiding Question 4, *"Does a lesson in which students interact with functions in the context of dance/movement deal with issues of functions which are considered difficult to understand?"*

The relevant data sources to address this question are lesson designs, student work, note cards, and video.

Lesson Plans

To address this question, I analyzed the lesson plans for examples of pedagogical choices in the teaching of functions which are meant to address issues of learning functions (refer to Section II, Part iii: What is Difficult About Learning Functions):

a) working with variables
b) working within a problem situation, not just algebraic equations
c) seeing a function as an action
d) adapting the idea of the "function machine"
e) addressing the idea of sets
f) demonstrating the arbitrariness of functions by using non-numerical examples
g) explaining the uniqueness and many-to-one concepts
h) building from an operational conception.

Student Work, Note Cards, and Video

I also looked for these nine pedagogical choices in student work and note cards, and in watching videos of the lesson implementation, I tracked students' reactions and interactions with each of these elements of the lesson.

Findings & Discussion

Using the analysis methods described above, I combined my various sources of data to address my three guiding questions. Each class was treated separately during analysis, due to the differences in the implementation of lesson which led to differences in data collection. Also, comparisons are made between the two classes, and possible explanations for similarities and dissimilarities.

Motivation and Engagement

Findings

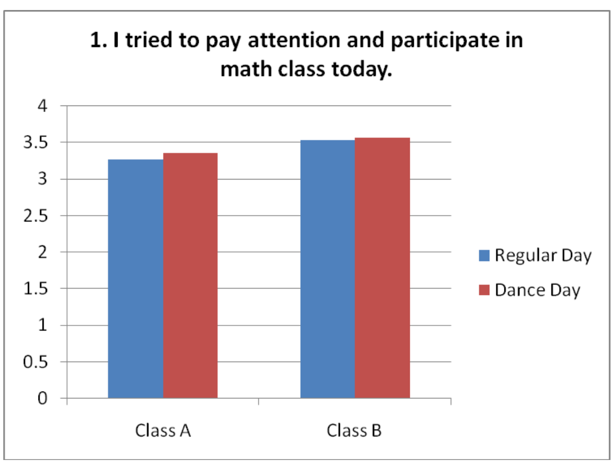
Analysis of the daily surveys showed that there was no significant effect in either class of the dance/movement framework on students' motivation to participate in math lessons. (For numerical survey results, see Table 1.)

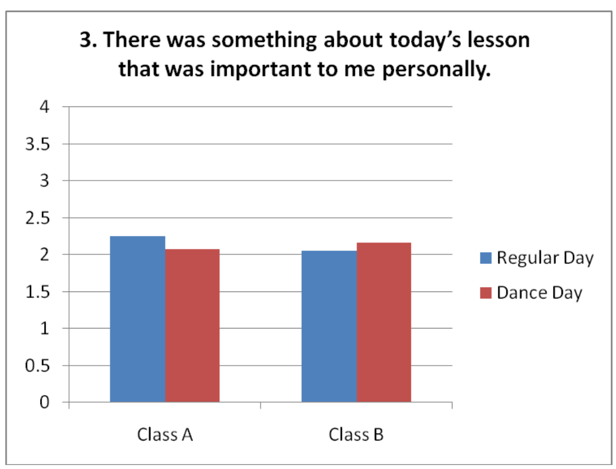
Table 1 – Average student responses per class

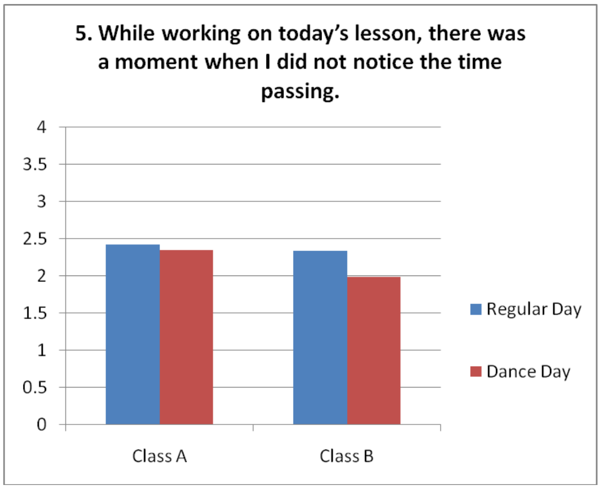
	Class A			Class B		
Question	non-dance	dance	Difference?	non-dance	dance	Difference?
1. I tried to pay attention and participate in math class today.	3.27	3.36	no effect	3.53	3.56	no effect
2. I enjoyed working on the activities today more than I usually enjoy class.	2.38	2.78	effect: dance>math $p = 0.019$	2.79	3.18	effect: dance>math $p = 0.032$
3. There was something about today's lesson that was important to me personally.	2.25	2.07	no effect	2.05	2.16	no effect
4. I would like to spend more time doing lessons like today.	2.12	2.79	trend: more dance $p = 0.067$	2.89	3.16	no effect
5. While working on today's lesson, there was a moment when I did not notice the time passing.	2.42	2.34	no effect	2.33	1.98	no effect

On a day of a “regular” math lesson, students self-report in Item 1 of the survey that they are relatively motivated to participate, Class B a little more so than Class A (Class A average, 3.27; Class B average, 3.53). Both classes showed slightly higher motivation to participate on dance/movement lesson days, more so in Class A, but the t-tests show the change is not significant (Class A dance lesson average motivation, 3.36; Class B dance lesson average motivation, 3.56). In response to Item 3, students in both classes felt fairly neutral about the importance of any math lesson -regular or dance/movement- to their lives personally. In fact, Class A reported slightly lower personal importance of the dance/movement lesson as compared to a regular math lesson (dance lesson average, 2.07; regular lesson average, 2.25), but the t-test shows this difference is insignificant. Class B showed slightly higher personal importance of a dance/movement lesson as compared to a regular math lesson (dance lesson average, 2.16; regular lesson average, 2.05), but again, t-tests show this difference to be insignificant. For Item 5 of the survey, students in both classes reported relative neutrality about noticing the passing of time on any given day, regular or dance/movement. Although t-tests show the difference is insignificant, both classes show slightly more notice of the passing of time during the dance/movement lesson as compared to a regular lesson (Class A dance lesson average, 2.34, vs. regular lesson average, 2.42; Class B dance lesson average, 1.97, vs. regular lesson average, 2.33).

Histograms of Survey Data Averages –Measuring Motivation and Engagement







Video analysis of both classes reveals many students are motivated to participate because of their interest in the activity of dance/movement, as well as showing that several other students were un-motivated because of their lack of interest in the activity of dance/movement. There were a few instances of motivated, engaged students encouraging their disengaged, unmotivated peers to participate, and the unmotivated students became engaged, presumably motivated by their peers' motivation and not necessarily the activity of dance/movement. Furthermore, in Class A three students who generally do not participate or engage in math lessons, group work, or individual work were seen to be very engaged in the dance/movement lesson, two even going so far as to volunteer to dance with the teacher within the first five minutes of the start of the lesson.

In both classes, students verbally expressed interest in the activity of dance/movement, saying things such as "Dance is life!", "Ooh, ooh, I'm a dancer," and "I do karate, and that is like this." Their interest motivated them to create dance movements in groups, and to help their peers create movements as well. In Class A, there were a few students who verbally expressed a disinterest in the dance/movement activity, saying "Do we *have* to do this?" and "I'm going to fail at this [imitating sobbing]…" However, all students who showed no motivation to participate were encouraged by their classmates, and in turn, did participate.

Table 2 –Percentages of Time Spent Engaged in Different Disciplines

	CLASS A					CLASS B			
	Day 1		Day 2		Lesson Over all	Day 1		Day 2	Lesson Over All
Activity	Intro	GroupWork	T-tables	NoteBks		Intro	T-tables	NoteBks	
TotalTime	0:20:03	0:38:43	0:45:46	0:34:14	2:18:46	0:26:44	0:24:16	0:31:00	1:22:00
MATH & DANCE	40%	23%	58%	19%	36%	78%	49%	32%	52%
MATH	0%	0%	1%	43%	11%	0%	0%	46%	18%
DANCE	21%	6%	6%	0%	7%	7%	0%	0%	3%
OTHER	39%	18%	18%	22%	22%	15%	12%	4%	10%
REAL WORLD	0%	0%	0%	4%	1%	0%	0%	6%	2%
MATH & REAL WORLD	0%	0%	0%	12%	3%	0%	0%	11%	4%
VARIES ACROSS GROUPS	n/a	53%	17%	n/a	20%	n/a	38%	n/a	11%

Video analysis also shows disciplinary -and interdisciplinary- engagement. (See Table 2 for percentages of time spent focused on each discipline, per class, per activity.) Students in both classes were found to be engaged in one of the three desired disciplines (“math”, “dance”, or “real-world”) or a combination of the desired disciplines (“math and dance” or “math and real-world”) most of the total lesson time. A new category (“varies across groups”) emerged to describe the engagement when students were working in groups because not all groups were engaged in the same discipline at the same time. During the time categorized “various”, groups were found to be either focused on “dance” or on “math and dance” as they applied the function to their dance inputs. It must be noted that Class A did a group activity (as in the original lesson plan) that Class B did not do because of the restructure and redesign of the lesson, thus classes will be compared by activity and not day of lesson. Table 2 includes all activities.

On the first day of the lesson, Class A participated in a group activity, during which two groups were engaged in “dance” the whole time, and the other two groups were engaged in “math and dance” as they were attempting to make sense of the functions and how to apply them to dance movements. A couple students, were engaged in “other” for much of this group work time, being overwhelmed by the activity and finding other things to do (such as sit alone on the couch or the floor).

During the introduction to the lesson, which was delivered similarly for both classes, Class A spent more time engaged in “other” things (such as asking about video cameras and getting settled on the floor) than Class B; proportionally, Class A spent over twice as much time by percent than Class B engaged in “other.” The T-table activity was comparable between both classes in terms of student engagement in particular disciplines. Class A spent more time discussing ‘math and dance” than Class B because the teacher felt the need to clarify things which had happened during the previous day, whereas Class B was starting fresh. Students in Class B spent proportionally more time working in groups to create input and output movements as Class A, due to allowance by the teacher because Class A already spent time the previous class day creating dance movements. To apply the engagement framework (Engle & Conant, 2002, p. 402), there is evidence from both classes that students were collaborating to create dance movements, suggesting things their group mates could do; group mates were coordinating ideas with each other; a total of only three students were off-task from two of the groups (and only in Class A); at times, students would attend to one student in the group to hear their suggestions (see Image 6); most students were demonstrating “passionate involvement” through their full investment into dance movements; students who were momentarily distracted by something like a hat (everyone wanted to wear one students’ hat), they would spontaneously abandon the hat to rejoin the dance activity.

Image 6. Class B, Day 1- group engaged in both math and dance, at the moment in this image, recording their dance moves in T-tables.

When writing in composition notebooks, Class A spent a significant amount of time engaged in “other” because of distractions such as the vice principal entering the room, and the enrollment of a new student to the class. Less time was spent engaging in “math and dance” in Class A because of the time lost engaged in “other”, and the teacher was moving the activity along to ensure time to address real-world and algebraic functions.

Overall, Class A spent nearly one hour more than Class B on the Math Moves lesson, due to design changes and differences in activities. As an effect of that difference, Class A spent significantly more actual class time engaged in “dance” (0:09:18) than Class B (0:01:57), and Class A spent over three times as long engaged in “other” (0:30:31) than Class B (0:08:25).

Discussion

Students in both classes self-reported that they are relatively motivated to participate and try in math class on any given day, regular or dance/movement. This could indicative of students' true motivation in math class; students might be motivated to participate and try in math class most days. However, students might respond in this manner because of the setting in which they were asked to self-report; students might have reported higher levels of motivation to please the teacher, who gave them the survey and who would be reading and analyzing them.

Responses from students about the relevance of the lesson to their personal lives are quite reasonable. A regular math lesson might not pertain to anything a middle school student encounters in his/her personal life. By choice of wording on Item 3, a response of disagreement does not imply that the student did not feel the lesson was important, it just refers to the *personal* importance of the lesson. Many students, whether they enjoy dance/movement or not, are likely not involved in dance in their personal lives, and thus a dance lesson would not personally relate to their lives.

Item 5 of the survey was meant to measure students' engagement: If a student does not notice time passing, they are engaged in the activity of the lesson. However, it is possible that many students did not understand the statement about time passing, and either responded out of confusion with a guess, or interpreted the statement to mean that they felt time had slowed down or stopped, which might indicate boredom. Item 5 might not be a valid measure of students' engagement in math lessons, but video evidence shows student engagement.

The video analysis showed great motivation among some students and a lack of motivation among other students. The students demonstrating motivation could have been motivated by their personal interests in dance, either as a hobby or extra-curricular activity in which they participate, or from popular culture (music and entertainment media) which involve dance in many forms (music videos, reality competition shows, stage performances, etc.). As students were creating input and output movements, many danced movements similar to those of the popular artists Michael Jackson, who had recently passed away. Other students seemed motivated by the prospect of impressing their classmates, either with their Michael Jackson imitations, their karate movements, walking on their hands, or with their strengths in doing push-ups while clapping. Many students were motivated by their group members and said they did not want to dance alone, but were pleased to dance with other students.

As was mentioned in Findings, three students of Class A who generally lack motivation and engagement in math lessons were actively participating. Two of these students have information processing disabilities, and usually need to work at a slower pace than their classmates. During this activity, however, the students were able to stay involved at the same level as their group members, and contribute to the task of creating dance movements. The third student is leery of school in general, and is frequently absent. His absences cause the days he is present to be confusing, because he does not have the background knowledge to keep up with the lesson, therefore he sits idly during class. On the days of the dance/movement lessons, the activities required no prior knowledge of functions or dancing, and this student was able to participate fully in all aspects of the lesson.

Among students demonstrating a lack of motivation, which was only seen in Class A, several seemed to be seeking attention from their classmates and/or the teacher, and were easily re-directed and demonstrated motivation thereafter. Some were feeling embarrassed or shy about movement, but all students had enough motivation to give it a try and work with their group.

Nearly all students in Class A and all students in Class B are engaged in the intended activity as directed by the teacher at almost any given moment. There were instances in Class A during the group activity (which was not done by Class B) when the students were not engaged in the teacher's intended activity, however, the students were engaged in the activity as they perceived the teacher intended, and they were engaged in one of the intended disciplines, dance. It must be noted that when students are engaged in "other" they are not necessarily off-task. Often, they are on-task, doing things like gathering materials, making a transition from one group presentation to the next, or other teacher directed activities.

In the teacher reflections, the first impression of Class A after the first day was that they were off-task and distracted by their friends for much of the lesson. However, after analyzing the video data, it is seen that they were engaged in the activity as they perceived it: making up a dance that has changes in it. They were not deliberately “goofing off” they were trying to create their best dance moves to show the change throughout.

The disparities between percentages of time spent in each activity per class can be attributed to the design and re-design of the lesson. The redesign allowed for students in Class B to spend less time overall, and more of that overall time engaged in “math and dance”, which relates to the concept of function. Engagement in *just* dance is the means for creating the functions, but does not imply the concept of function, so students in Class A needed to re-visit the function creation activity, because it was seen that they were not engaged in applying the functions to their dance movements.

Differences in engagement must also be considered in terms of the differences in students; the two classes are made up of different students with different personalities and different learning styles. Each class might not need to be engaged in the same activity for the same length of time in order to achieve the same content understanding. Students in Class A verbalized their confusions to the teacher throughout the lesson, and thus spent more time on explanations of concepts than Class B, which did not ask clarifying questions.

In the end, both classes were engaged in instructionally-intended disciplines nearly the whole time, and participated in the intended activities to the best of their knowledge.

Enjoyment

Findings

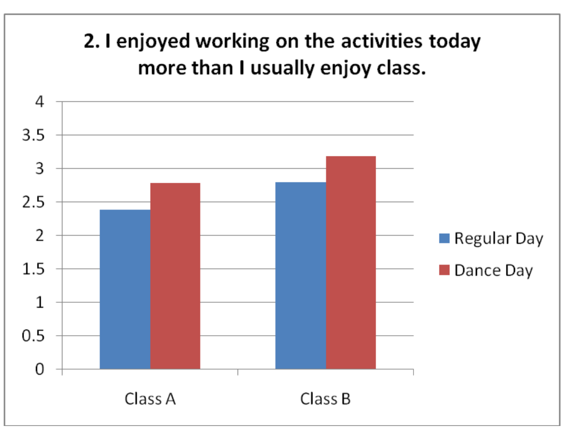
Analysis of the daily surveys showed that there was an effect of the dance/movement framework on students’ enjoyment of math lessons. (For numerical survey results, see Table 1, on page 39.) On Item 2 of the survey, students in Class A self-report that on a day of a “regular” math lesson their enjoyment is relatively neutral (average, 2.38), whereas on a day of a dance/movement lesson their enjoyment is more positive (average, 2.78), which is shown by a t-test to be a statistically significant effect of the lesson ($t(19) = 2.54$, $p < .05$). On the same item of the survey, Class B reports their enjoyment of a regular math lesson is more than neutral (average, 2.79), and that they have more enjoyment on a dance/movement day (average, 3.18), which is shown by a t-test to be a statistically significant effect of the lesson ($t(30.59) = 2.26$, $p < .05$). Students from both classes showed desire to do more lessons like the dance/movement lesson than a regular lesson. A t-test shows that the desire of Class A to do more dance/movement lessons is a trend of the lesson ($t(17) = 1.95$, $p < .07$, regular lesson average, 2.12; dance lesson average, 2.79), whereas a t-test for the responses of Class B shows no effect (regular lesson average, 2.89; dance lesson average, 3.16).

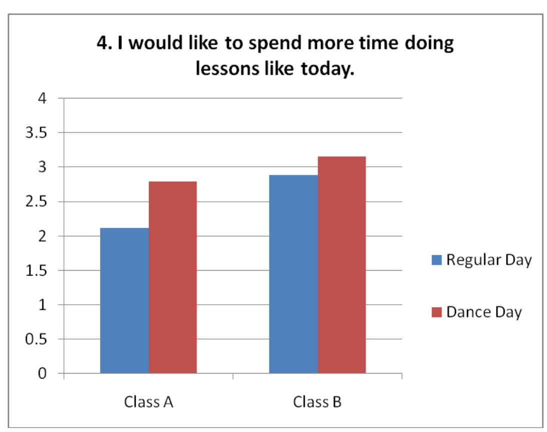
In regard to specific students who show a general dislike of math and whether they enjoy math in the framework of dance, results can only be seen in Class A, where the surveys were correlated and a given student's responses can be tracked across the course of regular lessons and dance/movement lessons. Among the seven students who expressed a general dislike of regular math lessons, five students reported more enjoyment of a dance/movement lesson, and two students' responses did not change. No responses among those students who express a general dislike of math lessons decreased in enjoyment of a dance/movement lesson.

Responses to whether a student desires to spend more time on a lesson such as that day's lesson show that students who generally do not want to spend more time on a regular lesson want to spend more time on a dance/movement lesson. Of the ten students who do not want to spend more time on a regular lesson, eight students want to spend more time on a dance/movement lesson and two students' responses did not change from the regular lesson to the dance lesson.

To see if and how the dance/movement lesson affected students who express a general enjoyment of math lessons, surveys showed that among the thirteen students who express general enjoyment of math lessons, three students' responded with a general dislike of the dance/movement lesson, seven students enjoyed the dance/movement lesson even more than the regular math lesson, and three students' responses stayed the same. Of the six students who express a general desire to spend more time doing lessons like the regular lesson, three express a desire to spend more time doing dance/movement lessons, two express a desire to spend less time doing lessons like the dance/movement lesson, and one student expressed the same amount of desire to do a regular lesson as a dance/movement lesson.

Histograms of Survey Data Averages –Measuring Enjoyment





Video analysis shows that most students enjoyed the dance/movement lesson. Some students in Class A expressed dislike both verbally and behaviorally, whereas there were no found instances of verbal or behavioral demonstration of dislike in Class B. Students in both classes eagerly volunteered to dance the output movements in the introductory activity, letting out verbal “Ooh-ooh!” sounds to draw the teacher’s attention in order that they might be chosen. During the introduction to the lesson in Class A, one student said “Math is fun!” to which his peers responded with noises of happy agreement and laughter. Students in Class B muttered “This is cool…” in reference to the introduction of the lesson.

While working in groups, many students were smiling and laughing, and showing signs of satisfaction with themselves and their dance movements. Some students called their friends over to where they were dancing to show off the movements they created. Several times in each class students told the teacher “this is fun!” and some asked “can we do this every day?”

Though there is evidence that every student enjoyed the activity at least at one moment of the lesson, some students’ enjoyment fluctuated during the course of the lesson, expressing dislike at some point during the activity. All verbalizations of dislike came from students in Class A, saying things like, “Noooo!” in response to getting in a group, and “I’ll just be the supervisor, I’m not going to dance,” in response to presenting inputs and outputs to the class. Other vocalizations of sighs and exasperation were also heard at some point during group work, mostly from students who express a general like of math lessons and who are involved in the honors program.

Among students in Class A who usually display a dislike of math class (most of whom have specified learning disabilities) all were seen enjoying themselves. They were eager to participate, and were smiling and laughing while creating dance movements in groups. Some of them said they were shy to perform in front of the class, but rose to the occasion with giggles and excitement. This same analysis cannot be done in Class B because no observations were made to see the students’ behavior and expressions of enjoyment in a regular math lesson.

Although this was not captured on video, a parent from a student in Class B sought me out a couple of weeks after the dance/movement lesson to let me know how much her child enjoyed the lesson. She said her child came home that day talking all about math class, and explaining how dance functions work, and showing the different dance functions to her family. By this mother's account, this student does not usually want to talk about school, much less about math class.

The whole-class discussion with Class A showed that many students responded positively to the dance/movement lessons, and generally enjoyed dancing in math class. One student, who generally does not participate in math class, said that he enjoyed it, and though he would not want to do it every day, he would like to do it again. Another student, a fairly reserved honors program student, felt it was fun to watch the dancing, but "it can be awkward" to do the dancing herself. One very talkative student said "I think it's way better than just doing lessons on the board, because I get really bored just sitting, that's why I talk a lot, 'cause I get bored, so it's better to get up…" One high-achieving English Language Learning student "thought it was OK", and a student with a specified learning disability said "it was alright." Overall, eleven students reported that they thought it was "fun", three students thought it was "OK once-in-a-while", two students said it was "so-so", and two students did not like it at all and would not like to do it again.

Discussion

Students self-report that they enjoy dance/movement lessons, and on average, they enjoy dance/movement lessons more than regular math lessons. There are many factors which could contribute to students' enjoyment of the dance/movement lesson: group interactions, active movement, freedom to create movements that had no "right" or "wrong", and the prospect of "dance" as such.

Many students self-report that they are "social" and want to talk with their friends all the time, so a lesson in which they have free time to collaborate with friends will likely be enjoyable. Other students express needs to move around during the long block class period, so a lesson in which students can move around the room and be active would likely appeal to them. Still other students feel distraught about their successes and failures in math class, some eve crying when they do not arrive at the correct answer for a math problem. For these students, the open-ended activity to create input dance movements has no correct answer –it can be anything the student can enact with his/her body. Finally, a few students felt a strong connection to dance and dancing, and enjoyed participating in the act of dancing because dancing is something they enjoy.

A few students, all in Class A, expressed dislike for the dance/movement lesson. These students explained that they felt awkward dancing, they did not want to perform, or they were overwhelmed by all of the hustle and bustle of the classroom. A couple of students who expressed dislike toward the dance lesson but a general like of a regular math lesson were clearly frustrated by the group activity and the confusion of the instruction sheet directions as compared to the activities of their classmates. Even those students who expressed a dislike for the dance/movement lesson were seen smiling and laughing and enjoying themselves at some point, though this might not be a product of the lesson but a product of the peer interaction which could take place in another setting than dance.

Connections Between Math and Dance

Findings

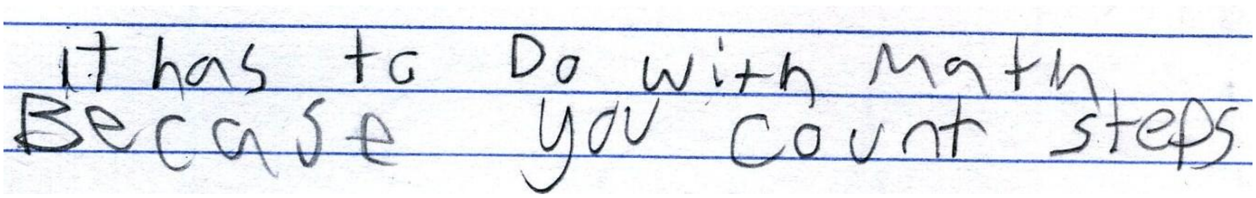
Students gave a wide variety of responses to the note card question "What does dance have to do with math?", making connections to counting, time, space/geometry, functions, and other things, as well as responding that there are no connections at all. (See Table 3 for number of responses per connection between math and dance.)

Table 3- Number of Responses per Category of Connection between Math and Dance

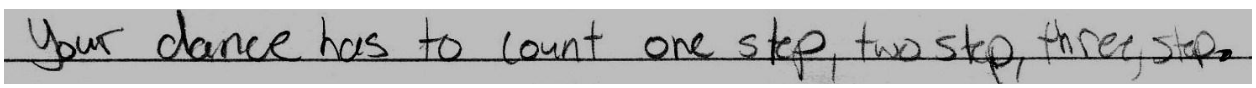
	Class A				Class B
	Before Day 1	Before Day 2	After Day 2		After Day 2
counting steps and/or beats	14	7	6		6
timing	2	2	3		1
geometry/ spatial	0	0	0		2
functions, input/output	0	5	9		8
no connection	1	1	1		2
other	3	3	2		6
total connections	20	18	21		25
total students	17	15	17		19

Before participating in the dance/movement lesson, fourteen of the seventeen total students in Class A answered that both math and dance involve counting: there is counting of one's steps in dance and counting is math (see Images 7-11 for some examples). Two students made a connection between math and dance through time –that there is timing in dance and time is a mathematical thing. One student recognized that both math and dance have "notes", and one more student believed math and dance have nothing to do with each other, that dance is "an expression of energy".

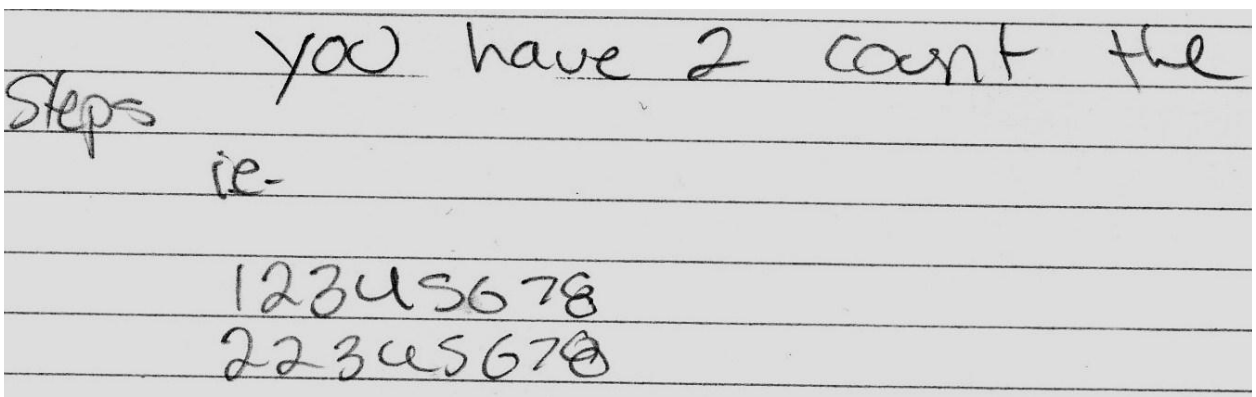
Images 7-11. Some Note Cards from Class A, Before Day 1



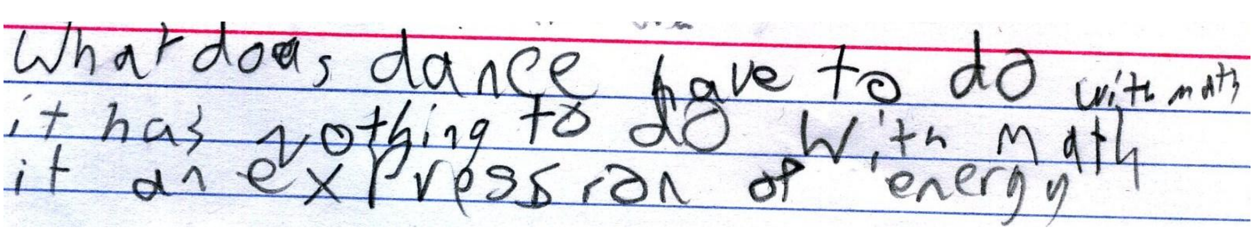
"it has to Do with Math Because you count steps" –Class A, Before Day 1



"Your dance has to count one step, two step, three step." –Class A, Before Day 1



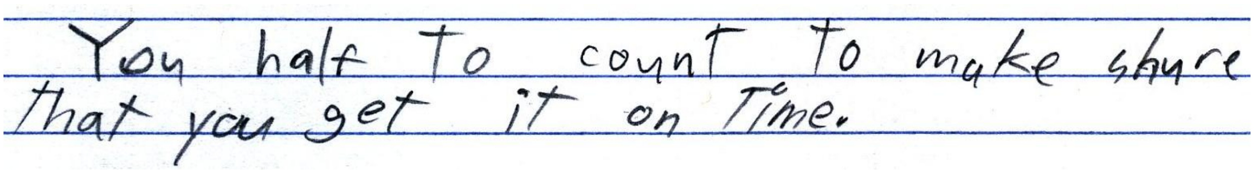
"you have 2 count the steps i.e. 12345678 22345678" –Class A, Before Day 1



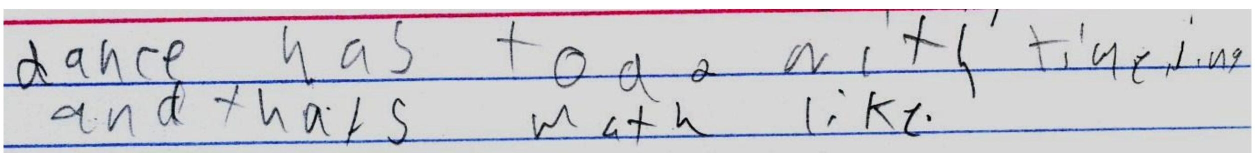
"What does dance have to do with math it has Nothing to do With Math it an expression of energy" –Class A, Before Day 1

The second set of responses from Class A, written the morning between the two dance lesson days, were much more varied than the first set (see Images 12-## for some examples). Seven responses mentioned the counting of steps, two of which included other connections as well. Four students claimed functions, and inputs and outputs, are a part of both dance and math, and one student elaborated on the function idea, saying that “In writing, that’s like cause and effect.” One student said that both dance and math are difficult, another said he did not know how they were related, and the student that believed math and dance have nothing to do with each other still believed they have nothing to do with each other.

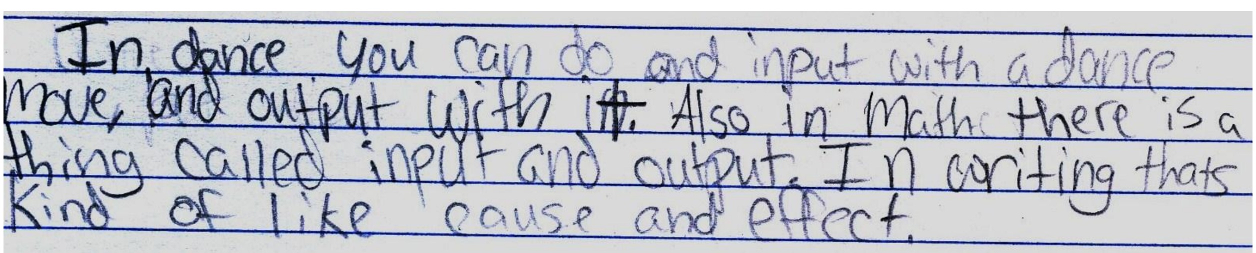
Images 12-14. Some Note Cards from Class A, Before Day 2



“You half to count to make shure that you get it on time.” –Class A, Before Day 2



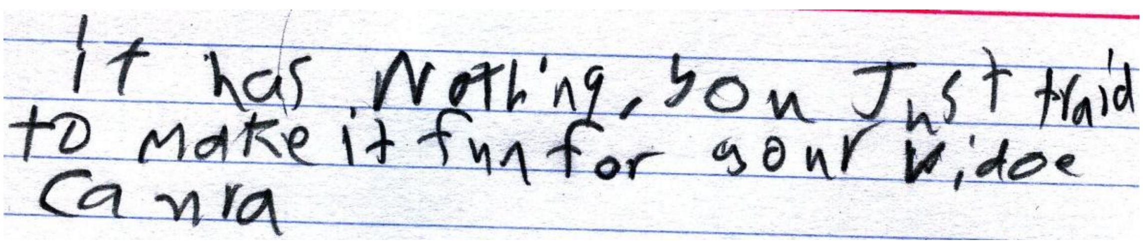
“dance has to do with timeing and that’s math like.” –Class A, Before Day 2



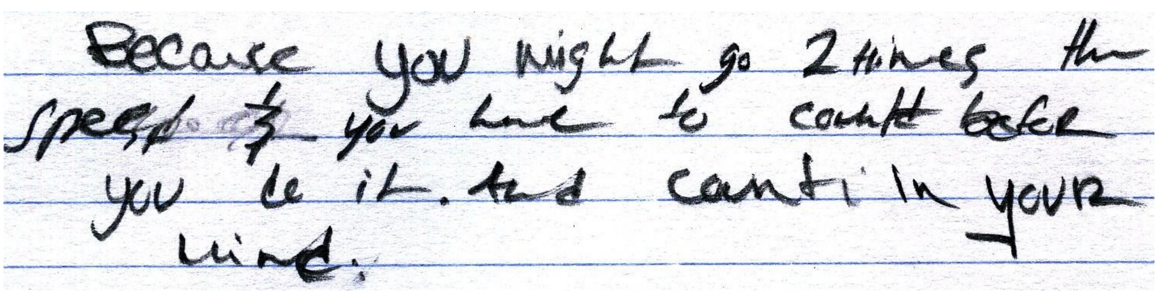
“In dance you can do input with a dance move, and output with it. Also in math there is a thing called input and output. In writing that’s kind of like cause and effect.” –Class A, Before Day 2

The final set of note cards from Class A were composed at the end of the second day of the dance/movement lesson contained the most widespread responses (see Images #-# for some examples). The number of students who noted counting as a connection decreased to six (down from fourteen on the first day), and three students included timing/time in their responses. Nine students referred to functions, either by describing inputs and outputs, describing functions, or giving specific examples of functions which were danced in class. Students giving "other" responses included the procedural relationship, that math and dance "both have steps", and the student who did not know how they were related the day before still stated that he did not know. Finally, the student who said that math and dance were unrelated in his previous two responses still believed that they are unrelated (see Image 15 below).

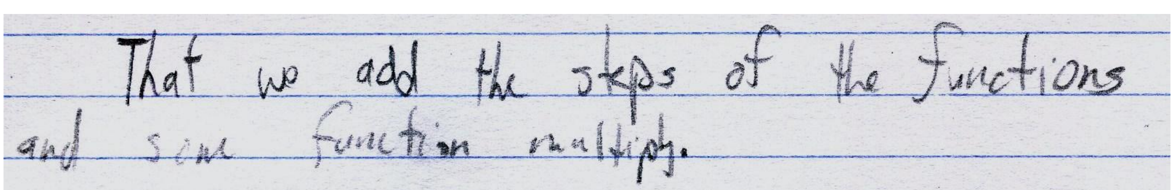
Images 15-18. Some Note Cards from Class A, After Day 2



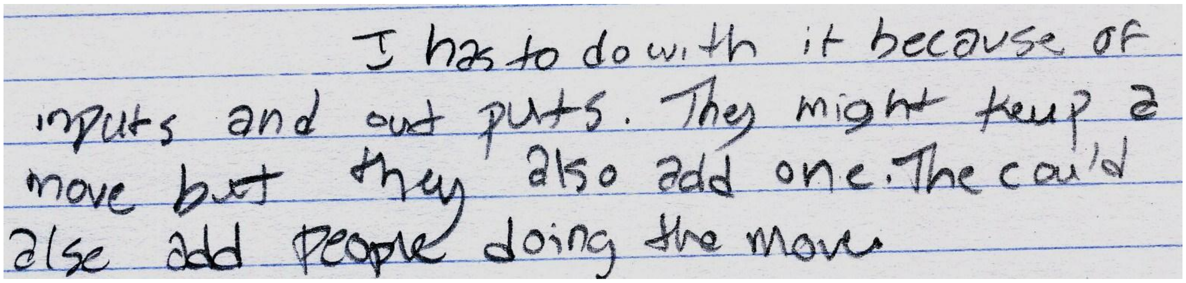
"it has Nothing, you Just traid to make it fun for your video camra" –Class A, After Day 2



"Because you might go 2 times the speed & you have to count befor you do it. And count in your mind." –Class A, After Day 2



"That we add the steps of the functions and som function multiply." –Class A, After Day 2

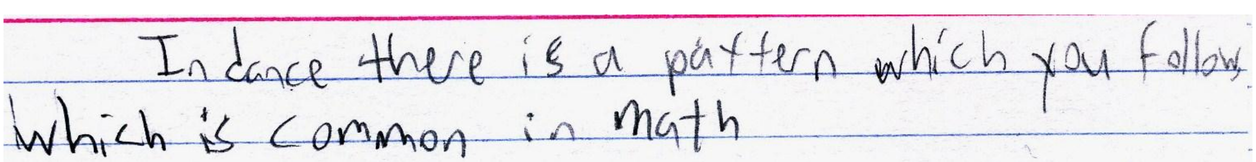

I has to do with it because of inputs and out puts. They might keep a move but they also add one. The could also add people doing the move.

"I has to do with it because of inputs and out puts. They might keep a move but they also add one. The could also add people doing the move." –Class A, After Day 2

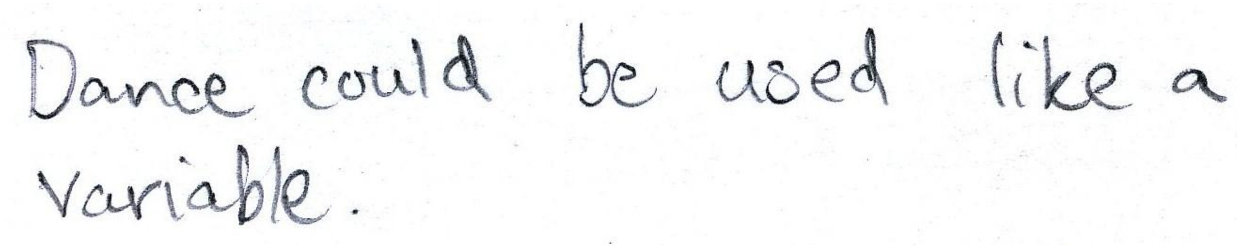
As can be seen in Table 3, twelve students changed their responses at some point between the first note card and the last, and four students gave essentially the same response each time. Of students who changed their responses, three kept their original idea and added to it, and ten different students mentioned functions.

Class B only responded to the question once, the next class meeting after all of the dance/movement lessons were completed. Overall, responses were more varied than those of Class A. Five students mentioned counting and/or steps as a connection between math and dance, and no students mentioned timing or time. Eight of the eighteen responses referred to functions, or inputs and outputs. Two students in Class B did not believe there were any connections at all between math and dance. "Other" responses include: One student noted the procedural relationship that "one step leads to another" in both math and dance; two students believe there are patterns in both math and dance; two students addressed ideas of geometry, that you can measure angles of dance moves and there are formations in both math and dance; one last student claimed that dance can make it easier to understand math.

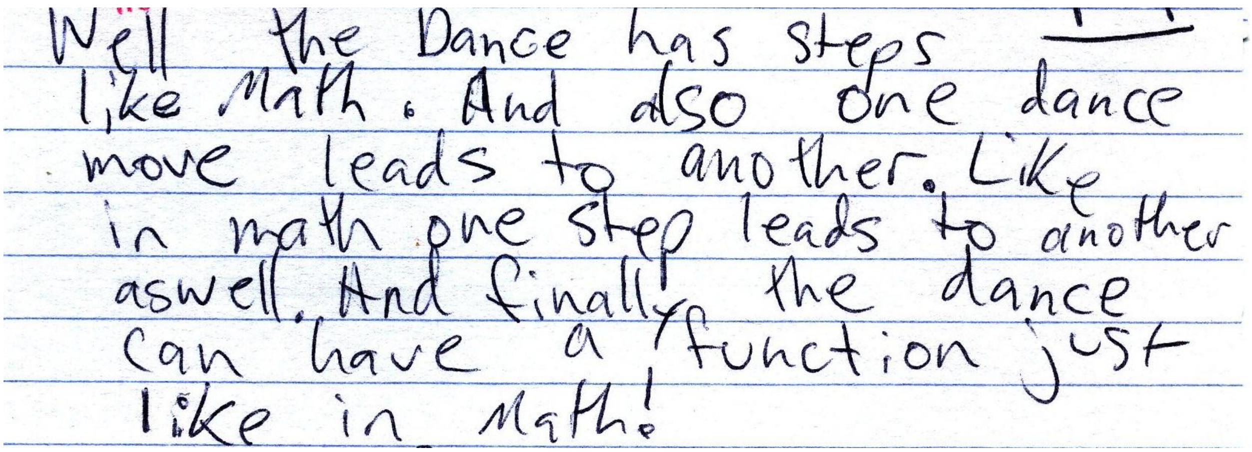
Images 19-23, Some Note Cards from Class B, After Day 2


In dance there is a pattern which you follow which is common in math

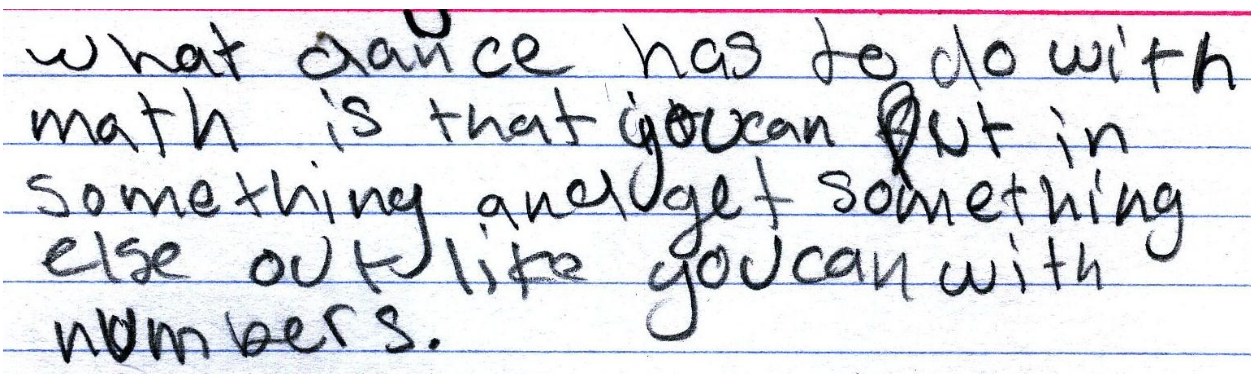
"In dance there is a pattern which you follow which is common in math" –Class B, After Day 2


Dance could be used like a variable.

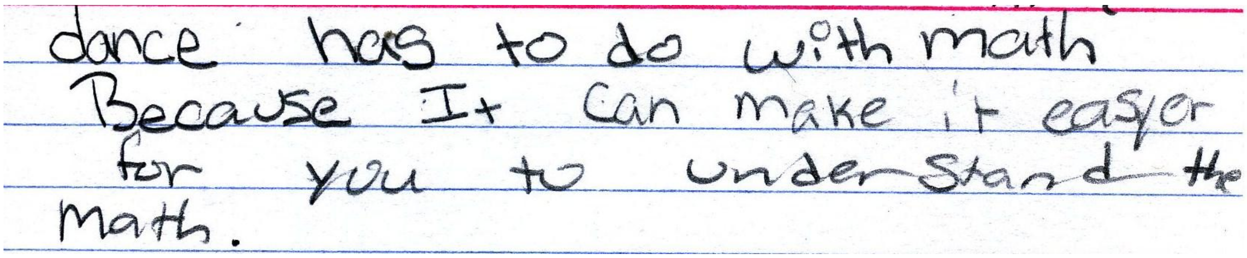
“Dance could be used like a variable.” –Class B, After Day 2


Well the Dance has steps like Math. And also one dance move leads to another. Like in math one step leads to another aswell. And finally the dance can have a function just like in Math!

“Well the Dance has steps like Math. And also one dance move leads to another. Like in math one step leads to another as well. And finally the dance can have a function just like in Math!” –Class B, After Day 2


what dance has to do with math is that youcan put in something and get something else out like youcan with numbers.

“what dance has to do with math is that you can put in something and get something else out like you can with numbers.” –Class B, After Day 2



"dance has to do with math Because it can make it easyer for you to understand the math." –Class B, After Day 2

Analysis of the videos gives more insight into student thoughts about possible relationships between math and dance, especially in Class A. Before the first note card activity, at the beginning of the first day of the lesson, students responded aloud to the question as it was written on the board (whereas students in Class B remained silent). As students read the question, they called out ideas which they did not later write on their note card, such as "physics!" or "it has nothing to do with…" One self-proclaimed dancer loudly responded, "No! You have to count!" after which a group of students began chanting, "One, two, three, four, five, six seven, eight! One, two…" This one students' verbal response of counting appeared on 14 of the 17 students' note cards.

During the introduction of the T-table of "input" and "output" movements, students in both classes mentioned a resemblance between this T-table and a numerical T-table, saying in Class A "Oh, don't you do this with the number thing?" and classmates responding "oh, yeah" and "ahh, yes…" Class B also made this connection, in reference to a game they play (which Class A did not play) that is a horizontal T-chart with pairs of numbers, and as a whole class, the students must recognize the pattern to fill in the blanks.

At another moment during the introduction, when the students gave suggestions as to what changed from the input to the output, student responses often involved mathematical terminology. Students in Class A suggested "times three" (referencing multiplication) or that "you start with a third, and you get one" for an input which was performed by one person and an output performed by three people. In Class A, students suggested the function (which was to perform the move while raising the left arm) was to "add something". For the same function, students in Class B suggested "plus *x*" was the rule, bringing in algebraic terms by using a variable. For a function which could be described as “dance the opposite” or “dance the mirror image”, one student in Class A suggested the function was “the inverse” of the input. This appeared in a different form in Class B, when one student suggested the output was “negative” the input –very much like the inverse or opposite of addition is subtraction –both of these students saw the relationship of oppositeness.

Discussion

Written on note cards and expressed verbally during the lesson, students made a wide range of connections between math and dance: counting, time, spatial/geometric, functions, and other things. Not knowing the dance background of the students, it was expected that answers would vary, because a student with no dance experience might think of a media portrayal of dance and make a connection from there, whereas a student who is involved with dance as an extra-curricular activity might make a connection based on first-hand experience.

Students in Class A responded to the note card question three times, yet their answers may have been influenced by one very vocal student in the class. When the question was first posed, students responded aloud instead of keeping their thoughts to themselves and simply writing their ideas on the note cards. When answers were called out, students heard a self-proclaimed dancer student say “You have to count!” and proceed to rhythmically count from one to eight, gaining more students in the chant as it went on. Because of the calling out of answers, students who had verbally said math and dance had to do with “physics!” or “they have nothing to do…” did not write their ideas on their note cards, but instead wrote about counting. Had the answers not been said aloud, there might have been a wider variety of connections between math and dance, and more students might have replied that the two were unrelated.

The next two times that Class A responded to the question, it was made clear that they should write their responses quietly and not call out answers so they would not influence their peers. In these next two sets of note cards, the answers are more varied, and many students changed their answer from the previous card. Perhaps students had more time to think about the question over the weekend, or had talked with classmates to hear what they had written the previous class period. It is also possible that by participating in the Math Moves lesson many students had their first encounter with dancing, and had a larger view of dance when answering the note card question the second and third times.

Class B did not answer the question on a note card three times, but only once, the day after the entire Math Moves lesson was completed. The question was answered in silence, so student responses were not influenced by other students' ideas. Students in Class B were all answering the question having at least one common dance experience: The Math Moves lesson. However, the responses still reflected a large range of understandings of dance and dance backgrounds.

Aside from the note card activity, students in both classes vocalized connections between math and dance when describing functions as they were being demonstrated. Students recognized the dance/movement T-table as a mathematical tool, directly tying it to an activity like a "function machine." This was not used in this particular class prior to the Math Moves lesson, but students must have encountered T-tables in previous years of math instruction.

Students also connected the dance functions to math by describing the functions in terms of mathematical operations like addition and multiplication. Students in Class B used the variable *x*, and described a relationship as "the negative" which would be the mathematical concept of inverse. When the student in Class A suggested that the function "dance the opposite" was "do the inverse", this could become a great discussion about what it means to "do the inverse" of something, and whether or not that is the "opposite" of it. These students have had mathematical encounters and explorations of the idea of inverses in addition and multiplication, so in theory they know that operating with the inverse leads to the additive or multiplicative "identity."Thus, this proposal to "do the inverse" would imply that one dances an output move which, when combined with the input move, creates the "dance identity" –and what would a dance identity be?

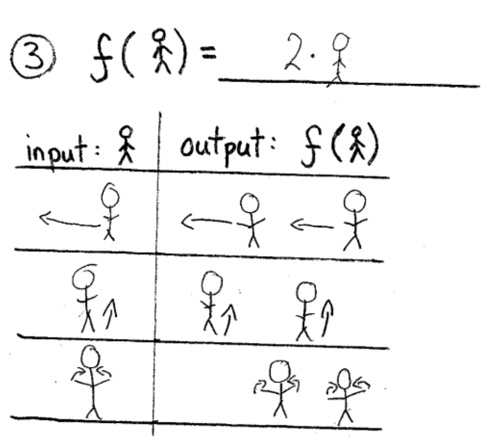
These descriptions of dance movements may have been mathematically-inclined due to the setting in which they were presented –math class. Perhaps if these same students had been asked in physical education class to "find a relationship between one dance move and another" their answers would not have included as much math terminology.

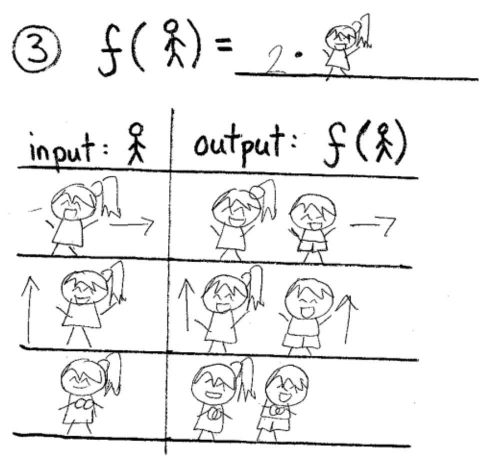
Elements of Math Moves Lesson

Findings

Instances of each pedagogical choice were found in the plan of the lesson, in student work, the note card activity, and observed as they were implemented in the video data.

a) working with variables: the teacher introduces a variable when the lesson begins, drawing function notation on the board using a ‘stick figure’ to represent the dance movement, as in “*f* (🯅)”. The lesson uses this notation whenever a dance function is defined. The students write with variables when constructing dance T-tables (see images 1 and 2):





Image##- Students’ representations of the function “twice the dance movement” -Image ##

On the second day of the lesson, when talking about real-world functions, the teacher and the students draw telephones as variables to represent time spent on the phone, and envelopes to represent letters being mailed. Finally, the lesson closes with the use of x as a variable to represent the number input of an algebraic function, $f(x) = 2x + 1$.

The students in both classes did not express any confusion at the use of stick figures to represent the dance movements. All students drew their own representations of the dance movement functions, using their own version of the stick figure (as seen above). No students questioned the use of telephones or envelopes as variables in the real-world examples. Students were already familiar with variables as letters which represent numbers, so the transition to using x instead of an image was not an issue.

b) working within a problem situation, not just algebraic equations: the lesson is based on a sort of problem situation (what is the relationship between these pairs of movements?), and being in the context of dance/movement, it is not just algebraic equations. This is not the "problem situation" which the literature supports, though. The two real-world examples are more like the problem situations referred to in Section II, Part iii (how much does this phone call cost? and where will all of these letters go?). The lesson does progress to working within algebraic equations, but it does not start there and does not focus on that context.

c) seeing a function as an action: dance/movement functions embodied the idea of an output being and action of the input. Students acted out their functions.

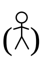
d) adapting the idea of the "function machine": the activity to "guess the relationship" or "guess the rule" between the input and the output dance/movements is essentially the function machine in action; students see one item, then see another item, and must guess how to get from the first to the second. The function "machine" turns the input into the output, and in Math Moves the dancer turns the input into the output.

Students in both classes who had worked with "function machines" prior to this lesson (in previous years of math classes) made connections between the dance functions and the numerical activity, saying things such as "Don't you do this with the number thing?"

e) addressing the idea of sets: when a formal definition of "function" is presented, it included the phrase "members of one set to members of another set". The teacher asked the students, "What do you think 'members of one set' are? A student in Class B offered an explanation, that "I think members might be part, part of that group, or the set." The teacher elaborates, "Thinking about the dance functions, what would these members of one set be?" Students offer suggestions, saying “inputs are the members of the ‘one set’” and “the outputs are members of the ‘another set’.”

From the note card data, it was found that the idea of dance movements as sets stuck with one student from Class B, who wrote that “sets … are involved in both math and dance.”

f) demonstrating the arbitrariness of functions by using non-numerical examples: the main example of function as a dance/movement relationship does not use numbers per se, though there are instances of “twice” and “five dancers” and “half speed”, which are numerical quantities. The two real-world examples to not depend on numbers (mail goes to mailboxes because the address tells the postal worker where to take it), though numbers are useful in thinking about the telephone call example (to quantify the minutes of the call and the cost of each minute).

g) explaining the uniqueness and many-to-one concepts: the second day of the Math Moves lesson begins by demonstrating uniqueness of the dance functions; that if a person starts with a certain input dance move (for example, a jumping jack), and applies the function (for example, *f* () = turn ), there can only be one output (in this case, a turning jumping jack) and not something else (for example, a sideways slide, a kick, a forward lunge, anything but a turning jumping jack). Students quickly understand that there can only be one output for every input, and react as if this is an obvious fact (with eager shouts of “only one thing!” in reference to how many outputs can relate to a certain input, and statements of “well, yeah,” and “duh”). The lesson addresses the many-to-one concept through the letters and post service example. Some students in Class A were reluctant to think about “snail mail” because they said the “only do email”, but an email example would not demonstrate a function because emails can be sent to multiple recipients, which would be one-to-many, and not a function. Students in both classes did claim to understand the concept that one piece of mail can only go to one mailbox, but one mailbox can receive several pieces of mail.

h) building from an operational conception: the idea of starting with an input dance move, “applying a function” to that input, and that gives an output dance move is very operational, and the understanding of function in the Math Moves lesson is built from this concept. In the videos, students are seen performing "operations" on their dance inputs, such as "add a clap", to get the intended output dance move. From this operational concept beginning, the lesson progresses to describing the function as a relationship between an input and an output, and then expands that thinking to consider a whole set of inputs and a whole set of outputs.

Discussion

Because the lesson was designed to address the difficult issues surrounding an introductory understanding of "function", it incorporated the nine intended elements in the plan. But planning something is not the same as implementing it. Though each of the nine elements appeared in both classes, Class B received further instruction in certain parts than Class A (such as a more extensive discussion on 'sets' and an exploration of uniqueness instead of just being told about uniqueness and students agreeing that it must be true), probably due to the redesign of the lesson, which made the activities more compact and allowed for more exploration of the concepts. There was no assessment between the two classes to measure if these further discussions and explorations led to deeper understanding of functions, but if the research reviewed in Section II, Part iii, is an indication, further discussion of these topics would probably be beneficial, not detrimental.

Reflecting on the Lesson

The Math Moves lesson was a design experiment in using dance/movement to teach functions, and a lot has been learned from these first two iterations of the lesson. Using the findings from the data analysis and the daily teacher reflections, I look for benefits and detriments of the lesson for these two classes of seventh grade pre-algebra students. While reflecting on each element of the lesson, I suggest ways in which the lesson can be altered for future implementation.

Affordance of a Dance/Movement Lesson to Introduce Functions

From the literature on embodied mathematics (see Section I, Part iii, Topic 4: Embodied, Kinesthetic Math), some would argue that an embodied, kinesthetic approach provides certain learning and understanding that a lecture-style approach would not attain. But this does not mean *any* body activity leads to learning a certain math concept. There are certain aspects of the body activity, certain affordances of the embodied approach, which might make parts of the math concept more salient than if the embodied approach were not taken. What are the affordances of the Math Moves lesson? What do these dance functions reveal that might not have been seen through a worksheet of a "function machine"?

One affordance of the dance function design is the way it makes the process of finding a relationship between the input and the output very salient. In a purely numerical "function machine" or in a T-table of values, a learner just sees the two quantities represented as symbols – say, 2 and 5. But the learner can't see how the machine turned a 2 into a 5, they just see "2" and "5." Then the task is, "What is the relationship between 2 and 5?" or "What can you do to a 2 to get a 5?" There are infinitely many ways to relate a 2 and a 5: add three, times two plus one, times five-halves, etc. It is necessary to see several examples to notice a pattern, and once a pattern evolves, the creation of function "rule" to describe it is difficult to visualize (Seldon & Seldon, 1992).

In the case of a dance function, however, the input is visible in the output: If the function is *f* (🯅) = 2 🯅, one can see the same dance movement being performed simultaneously by two dancers, and the input was the movement performed by one dancer. This is different from its numerical equivalent, $f(x) = 2x$, where putting in a 9 gives out an 18, and one cannot still see the 9 in the output; the symbol "18" is its own entity, and could be described as 2*9, but it could also be 10+8, or 12(3/2), or many other combinations of things. In the dance function, however, the input is seen in the output, making the identification of a relationship between the two much more transparent.

The dance movement functions also have the semiotic potential of objectifying functions as a process; by going through the actions of the "input" movement, then "applying the relationship" in your mind and attempting to act out the resulting "output" movement, the learner has taken the symbols on the page (for example, *f* (🯅) = 🯅 half speed), and had to create an "object", the dance itself, which embodies that relationship. Thinking through "how do I perform this movement at half the speed?" a learner is engaged in the idea of function as a process, and semiotically objectifies the process as the dance function.

Constraints of a Dance/Movement Lesson to Introduce Functions

Though we see some affordances of introducing functions by way of dance/movement functions, the design is not perfect, and a correlation between dance functions and more purely mathematical functions is hard to extend beyond the scope of this "process-oriented" introduction –that a function is a process of getting an output from an input. This lesson only went so far as to demonstrate that "function" is a relationship between members of one set and members of another set, the uniqueness of this relationship, and the many-to-one possibility, but the concept of "function" in mathematics includes the process, the relationship between sets, the object (for example, a graph taken as a whole), and the problem situation which it all describes (see Section II, Part iii). Can dance functions be used to teach the object or problem situation views of 'function"?

Dance has the property of being "temporal," it is stuck in time; it is ephemeral. This means that the dance movement is happening in the instant it is being danced, and there is no residual object to which one can refer and say, "This is the dance," because the dance is comprised of all moments of it, which happen in time, then are gone. (This was lucidly experienced when I, the teacher, was dancing example movements, and students wanted to see them repeated several times because "I didn't get it all" –the students were trying to keep track of everything that happened in the dance, as there was nothing to grasp after it had been danced, it was gone.) Numbers, as symbols on a page, do not have the issue of disappearing once they "happen." The symbol will be there on the page, where you left it, when you want to refer to it later. So a dance function has the constraint that it happens in time and is not observable as a static "object."

Does this mean the idea of a function as an object cannot be taught through dance functions? Not necessarily. The function as an object means all of the related members of each set are compiled and present in one representation, such as a graph in the 2-dimensional coordinate plane, each point of which describes both the members of each set and the relationship between the two. Perhaps dance functions can present a temporal version of this idea; one could create an entire "dance" wherein each movement is a member of either set A or set B, and each movement in set A is related to a member of set B by some function. Then the two sets could be danced simultaneously, as one large dance, where the viewer can see both the movement of set A and its corresponding member of set B. The choreography as a whole would be considered the "object", though it will still be evanescent.

And what about the problem situation view of functions, that the function describes a situation, such as the temperature outside at a particular time of day? Though the idea for this embodied function approach came from my own realization that historically, dance movements have evolved over time but still retain certain elements that relate them, this does not present a "problem situation." We cannot predict what will happen as dance movements continue to evolve over time, what elements will be retained and which will be dropped; therefore there is no function that describes this historical situation.

Furthermore, these dance movements are enacted by the human body. Thus, the possibilities of functions described by this context are constrained to those movements which are humanly possible. Can there be a flying dance function? Maybe if it was danced by birds –but that would remove the embodiment for the students learning functions.

Classroom Configurations

The original idea to have no desks in the classroom and have the students sit on the floor was unexpectedly met with adverse reactions from Class A. Students are able to compose themselves in a physical education class (which lacks desks and students sit on the floor) because it is expected that there are no desks, and one is wearing gym clothes which are intended to get dirty. In math class, however, students are used to the structure of having a desk where each student will work and have his/her belongings. In math class, students are also wearing their everyday clothing, which in some cases does not lend itself to sitting on the floor. Since the lack of desks was such a great difference from the norms of this math classroom, many students did not know how to conduct their behavior and were running wildly around the room and shouting. Along with the "outdoor" behavior, students did not know where to place their belongings, and when the activity called for sitting in a circle, some students did not want to sit on the floor for fear of getting dirty. Due to these reactions from Class A, the lesson was altered to occur in an unchanged classroom for Class B, complete with desks in usual row formation. All dancing in Class B took place around the perimeter of the room and in the aisles, which students successfully maneuvered and did not request more space.

For future implementation, an instructor could make several changes to the space for the Math Moves lesson. An instructor could create a classroom culture wherein the absence of desks is not completely out-of-the-ordinary, perhaps by having a large space in the room where students sit on the floor for various activities (playing math games, making posters, etc.). If students interacted with this open space at least once per week, there would be classroom expectations about how to behave productively, and a lesson wherein students sit in open space and dance would fit in the students' concept of what happens in that classroom.

Another issue which arose during Math Moves in Class A was the noise and disruption factor. The teacher in the classroom below made several telephone calls requesting that Class A be still and quiet because their movement was distracting and disturbing the students below. When implementing the lesson with Class B, timing was arranged such that the class below was at lunch while Class B was moving and dancing.

Some schools do not have classrooms above or below other classrooms, thus the noise and disturbance would not be an issue. However, an instructor wishing to implement the Math Moves lesson could take the class to a different location than the usual math room. This new location can be a place where the students are familiar with being in an open space and sitting on the floor, such as the school gymnasium. In the gymnasium, students could be noisy and active and would not interrupt another class. Furthermore, in the gym students would be able to adopt the norms and expectations of a physical education class, and would not be taken off-guard by a lack of desks and "order."

Structure of Activities

The students in Class A had difficulty maneuvering the first group work activity, where each group was given a sheet of instructions and were sent off to create input and output movements according to a function. Two of the four groups did not fully read the instructions, and three of the four groups did not do the activity. Many students with specified learning disabilities were overwhelmed by the freedom of the group activity, and these students stopped participating altogether.

When the lesson was revisited the next day in Class A, all students did well participating in a more structured activity. Each student possessed his/her own sheet of T-tables and together as a class completed the first few dance functions, the teacher demonstrating on the board. Students were each accountable for knowing the activity, and were more certain of what to do because they had already tried it. Class B did not express confusion or frustration about the instructions for any of the activities, probably due to the structure and teacher-guided examples in the same format as the group work.

In a future iteration of this lesson, the structure of the T-table work could be established as in Class B, then extended to include a less teacher-directed group activity, more like the intended activity of Class A on the first day. Because the structure will be established in the first activity, the future class would have a framework in which to create their own functions with less confusion or frustration and more actual functions work.

Mathematizing Dance/Movement Functions

Due to time constraints, the lesson did not allow for students to *discover* the concept of “function” through dance movement, as would be closely in line with my vision. Hans Freudenthal, the “father” of Realistic Mathematics Education (RME), believes students should discover the secrets of mathematics themselves, for they will develop a deeper understanding and learn it better than simply being told: “Telling a kid a secret he can find out himself is not only bad teaching, it is a crime” (Freudenthal, 1971, p. 424). Thus, without time pressures of standardized education, this lesson could give students the opportunity to create the spatial and conceptual organization of math functions from dance/movement themselves, rather than the instructor telling it all to them up front.

Perhaps in an afterschool program where time is not such a minimizing factor, students can analyze the history and progression of dance movement. Implementation could be re-structured to guide students through discovery of the mathematical idea of “function” by looking at examples of the progression of dance movements from different places of the world and different times in history. Students can watch three to four short clips of dances from different times and places which show a common movement or dance element, and the class can look for the common thread. This common thread is the relationship between the dances, and is thus the function.

From this discovery, that dances have common elements and are thus related historically and culturally, students can bring in examples of popular dances they have seen (maybe in music videos, or at concerts, or in their own dance classes, or in physical education class), and the class can analyze the different dances to find relationships and define functions relating them.

Of course, students should get up and do some dancing themselves, so once dance functions have been identified in the media, students can create their own dances following one of the found function relationships. These dances can be performed for the class, to guess which function is being applied, and/or there could be a dance functions concert, wherein other students and families and friends come watch the dance function creations, and the students in the performance can elaborate on the mathematics behind the movements.

Uncovering Students' Connections between Math and Dance

The note card activity most closely revealed students' connections between math and dance, but did not track changes in their understanding very accurately. Responses from students in Class A were influenced by their peers from the first time the note cards were written, and the students in Class B only wrote the note cards one time, after the Math Moves lesson. To track students' connections between math and dance in a future implementation of this lesson, the note card activity could be presented as a silent activity given at the very beginning of the lesson, and given again at the end of the entire lesson (not between days of the lesson, as happened in Class A). Further, the prompt could be expanded to include "Why?" which would more fully expose students' thinking about the relationship between dance and math.

Measuring Understanding

As it stands, this lesson does not include any assessments of student understanding, or better yet, learning. For accountability of both the students and the instructor –and effectiveness of this lesson – assessments before and after the lesson will reveal what students have come to understand about functions. An assessment before the lesson will show what students know about the idea of function from previous math instruction. If conducted early enough before the lesson, this pre-assessment could allow the instructor to customize the lesson, taking into account what the students already know and expanding on that, or addressing things the students specifically did not understand.

An assessment after the lesson would help the instructor determine if the learning objectives (the nine elements of the lesson that address what is difficult about functions) were addressed; did the students come away with better understandings of those elements than they had prior to the lesson? And of course, school administrators want to know that instructors are teaching the students, not just letting them run amok, and a good way to demonstrate that is through a post-lesson assessment.

Expanding and Extending the Math Moves

In going through two iterations of this lesson, and thinking more about the possibilities of dance/movement functions, a couple of expansions came to mind that could introduce more properties of functions, and provide new levels of challenge.

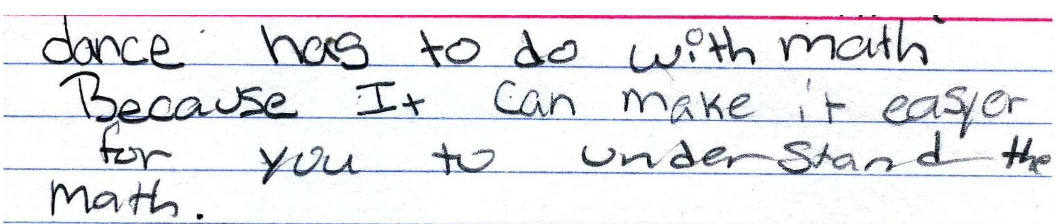
The dance framework of functions lends itself to the creation of “constant” functions: given any input movement, the output is just one movement (for example, input a turn, the output is a clap. Input a back bend, output is a clap).

Also, dance functions are enacted movements by a human body, which by principle, have to be continuous, because a human can’t get from one pose to another without passing through all movements between the first and the second. Therefore, dance functions can open a discussion on continuity, and how each movement is connected to those before and after it. Expanding this idea of continuity, and movements leading from one into another, the dance functions could be acting on an entire piece of choreography, not just disjoint movements separated into little phrases of “inputs”. In this way, the function will truly demonstrate continuity; every possible pose or instance of the “domain” (the original piece of choreography) is changed by the function to create the “range” (the new piece of choreography). The two works could be performed simultaneously, to create the function as one object of inputs and outputs.

Furthermore, to explore concepts of transformations of functions, one could take a work of choreography and call *it* the function –this is the function as an object. It is a relationship between the dancer and the movements in time, such that the movement is brought to life by the dancer according to a specific time of delivery, which is described by the choreography. Then, this dance “object” is transformed by changing it in some way (maybe performing it in a triple beat instead of a duple beat, or perform the whole dance facing to the left, or more complexly, perform the entire thing in reverse…). This shows the object being transformed, and students can explore what types of transformations lead to different dance movements.

Conclusion

Mathematics and dance are both multidimensional and relational, just like students grappling with the idea of “function.” The vehicle of dance provides many embodied approaches to teaching the different aspects of functions; from the procedural aspects as shown in the Math Moves lesson, to relationships in dance history, to transformations of whole choreography, students can use their own bodies to mathematize dance functions. In this lesson, I set out to share my zeal for seeing math through the lens of another of my interests –dance- and to discover how receptive students are to such instruction. I found students –some who generally dislike math class- enjoying themselves, eager to participate. Students were using mathematical terminology to describe movement patterns, and I witnessed several “ah-ha!” moments. Students were able to *experience* functions, not just witness them. In the words of one student,


dance has to do with math Because It can make it easyer for you to understand the Math.

In my future instruction, I plan on pursuing some of the possibilities revealed through this project. I hope to give students a chance to experience math with their whole being, because one cannot separate the mind from the body and the spirit.

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Appendix

i. Survey

SURVEY

Rate each of the statements below on a scale of 1 to 4, where 1 is you strongly disagree, and 4 is you strongly agree. Please base your answers for each question on today's class. Your answers here will not affect your grade in any way, so please be as truthful as you can.

	strongly DISAGREE			strongly AGREE
1. I tried to pay attention and participate in math class today.	1	2	3	4
2. I enjoyed working on the activities today more than I usually enjoy class.	1	2	3	4
3. There was something about today's lesson that was important to me personally.	1	2	3	4
4. I would like to spend more time doing lessons like today.	1	2	3	4
5. While working on today's lesson, there was a moment when I did not notice the time passing.	1	2	3	4

Item 3 — If so, what was it?____

Item 5 — If so, what moment? ______________________________________

Why?____

ii. Lesson Plan, First Day

MATH MOVES

Prior two days: Survey

DAY 1 (half of a "regular" Thursday class period, approx. from 10:30 to 11:15 AM)
- class is set up as a big open space, students gather in a circle, sit on the floor if possible

- Focus activity and data collection: Students answer on a note card, "what does dance have to do with math?"

- "We're going to dance a little bit today. A couple years ago I went to Ghana, West Africa, to teach math and to learn dance. Much of the world's dancing roots come from Africa, and it was very enlightening to learn their dance culture and see how that relates to dancing in America today.

For instance, we are sitting in a circle because many social dance forms, especially in Africa, are in a circle. Everyone is a community, nobody is on a stage.

You know how in hip hop, with break dancing, people gather in a circle and one person dances in the middle? [dance "cool" pushing-arms move clockwise] Then another person will get in the center and copy the moves but change them a little, to make the moves their own, with their own style? Maybe like this: [dance pushing arms move clockwise and counter clockwise, alternating]. Then **another** person will come in and change that move to their own style? [dance it and add running man feet]

We're going to do that in math class. We're going to take a movement, let's call it our "input" (write "input" on the board in the T-table), and we're going to change it to a new movement, let's call that an "output" (write "output" in the T-table).

Let's try one:

-"here is my input. [teacher dance left arm movement]
-"this is my output. [teacher dance left and right arm movement simultaneously]
-"Can we guess what changed?
-"here is another input. [teacher dance twist turn]
-"and the output. [dance twist turn with right arm movement]
-"what is the change? Both times the input went through the same change."
-take student suggestions. Come to the conclusion that I am "adding" the right arm move.

"The rule that changed the one move into another move is a *relationship* between the inputs and outputs. We call that relationship, that rule, a "function" (write "function" on the board).
-"if we want to write that rule, we say it is a function of the variable, or the dancer.

- Teacher dances a new set; shows an input and asks the students to perform their guess at the output (do three of these).

- Break class into 4 groups of 5-ish

- Give each group a sheet of directions, with a dance "function", each group is assigned a different function.
  - Groups must create their own set of "inputs", movements which will serve as the domain of the movement functions (create one movement per person in the group, about five).
  - Create images/directions (not words; don't say "draw"), somehow record them on a poster paper, represent the moves however makes sense to you all as a group!
  - Groups apply the teacher-given function to their set of inputs and all agree on the outputs.
  - Groups invent one of their own functions (write it down however makes sense to you all as a group!), apply it their set of inputs, and agree on the outputs.
  - Posters STAY in the classroom, in a file at the back of the room (data!).

- At 11:20 am clean up, put desks back, start the end-of-the-quarter activity (catch-up)

- At 11:50 am, Survey!!

- Homework: "teach your dance function, with inputs and outputs, to a friend or neighbor or family member over the weekend."

iv. Group Work Information Sheet

Dance Functions!

1) With your group, create a set of "input" dance moves, at least one move per person.
Record inputs in a chart on the poster paper.

2) Apply the following function to each input, and as a group decide what each "output" would be:

Every group had a different function, so here are two examples:

f **(🯅) = mirror 🯅**

f **(🯅) = 🯅 + clap**

Record outputs in the chart on the poster paper.

3) Invent your own dance "function"
Get a new poster paper from Miss Blessing. Re-record your inputs in a chart on this new poster.

4) Decide as a group on a new, different function. (think about: how will it change your inputs?)
Write the new function here:

5) Record your new outputs in the new chart using the function that you all invented together.

v. Lesson Plan, Second Day

<u>DAY 2</u>: (first half of a "regular" Monday class period, approx. 10:30 to 11:15 AM)
- as students wander in, "to who did you teach your dance function?"
- Warm-up/data collection: "Write your answer on a note card: What does dance have to do with math?"

- Break into groups again for a *quick* reminder of the functions, inputs, and outputs which were created Thursday (still keep all written work in the classroom –do not take it home!)

- Groups perform inputs and outputs for the class, we guess the rule "function".

- Bring it around to math functions: Students write in notebooks a formal definition of "function": a relation that uniquely associates members of one set with members of another set.

- Thinking about the dance moves: If you have a certain input, then apply the function, how many outputs will you get? [demonstrate a move with the function "twice in a row"; show that the only output can be that move twice in a row, not any new, additional moves, not that move in reverse, but only that one move, twice in a row.]
  - that is a special feature of "functions", that for every input there is only one output. [write that in your composition book]

- Tie this to other functions: We have seen that "functions" relate dance moves to each other, and there is only one output for each input, but there are other things which functions can relate, too.
- For instance, how much money it costs to talk on the phone is related to how many minutes a person talks. If you talk for 7 minutes (your input) and it costs 5 cents per minute (that's the

function), the call will cost 35 cents (the output). Then, when you see on your phone bill that a call cost $1.00, how many minutes long was that call? Remember the cost of a call is 5 cents per minute. The function, relationship, between minutes and total cost is the rate 5 cents per minute.

- Or if you want to send a letter to someone, what are you going to do to send that letter? [put their address on it] And to how many possible mailboxes can it go with that specific address on it? [only one -just like a function, only one output for each input] So, the input is the letter, the function is the address, and the output is the letter in that person's mailbox. A lot of people could send letters to that mailbox, right? All of the letters are different inputs, the function is the address, and the outputs are those letters in the correct mailbox.

- Are you still wondering what does this have to do with math? Well we can use functions on numbers, too! It is a lot like the dance functions we did.

Let's try this: if we say our input is "x" and our output is "y" we can get from x to y using a function, say "$2x + 1$". [draw a T-chart with all of this, and fill in some number examples]

And what if all you had were the numbers, and you had to guess the function? [draw a T-chart with coordinate pairs following this function: $f(x) = x/2$]

- Final wrap up/data collection: Answer on a note card: "what does dance have to do with math?"

- Spend rest of class (from 11:15 to 11:50 AM) on actual algebraic math functions
    - play one round of Silent Board Game

- 11:50 am, Survey!

RANDOM DAY (End of semester/near Christmas Break)
- recall: that functions dance thing we did?
- what about in hip hop and break dancing, when people steal and morph others' moves?
- professional dancers & choreographers do weird dance/math things, too!
- Merce Cunningham video

v. Revised Lesson, Day 1

<u>MATH MOVEMENTS: Revised</u>

Prior two days: Survey

<u>DAY 1</u> (second half of a "regular" Tuesday class period, approx. from 11:00 to 11:55 AM)

- "A couple years ago I went to Ghana, West Africa, to teach math and to learn dance. Much of the world's dancing roots come from Africa, and it was very enlightening to learn their dance culture and see how that relates to dancing in America today.

For instance, you know how in hip hop, with break dancing, people gather in a circle and one person dances in the middle? [dance "cool" pushing-arms move clockwise]

Then another person will get in the center and copy the moves but change them a little, to make the moves their own, with their own style? Maybe like this: [dance pushing arms move clockwise and counter clockwise, alternating].

Then **another** person will come in and change that move to their own style? [dance it and add running man feet]

- "We're going to do that in math class. We're going to take a movement, let's call it our "input" (write "input" on the board in the T-table), and we're going to change it to a new movement, let's call that an "output" (write "output" in the T-table).

Let's try one:

-"here is my input. [teacher dance left arm movement]
-"this is my output. [teacher dance left and right arm movement simultaneously]
-"Can we guess what changed?
-"here is another input. [teacher dance twist turn]
-"and the output. [dance twist turn with right arm movement]
-"what is the change? Both times the input went through the same change."
-take student suggestions. Come to the conclusion that I am "adding" the right arm move.

"The rule that changed the one move into another move is a *relationship* between the inputs and outputs.

****We call that relationship, that rule, a "function" (write "function" on the board).**

-"if we want to write that rule, we say it is a function of the variable, or the dancer.

-For that first set of moves, what did we decide was the rule? what was the function?
(add the right arm)

[HAND OUT WORKSHEETS]

- We're going to analyze some dance moves today to figure what is the "function", the relationship, which connects the input moves with their output moves.

- "I'm going to give you an input, and you are going to put that -either draw it or write it or however you want- under "input" in the first T-chart on your worksheet (show on a giant worksheet).
- Say the input was this: [dance right arm] I would draw this: [show on T-chart]
Then I will dance an output, which you put in the first T-chart, under "output".
- Say the output was this: [dance right arm and left arm] I would draw this: [show on T-chart]

- Next, I will give another pair of input and output, which follow the same rule or function that the first pair had.
- Say it was this: [dance turn] for the input, so I draw this [show T-chart],
and this: [dance turn with left arm] for the output, so I draw this [show T-chart].
- This new pair goes in the same T-chart because they follow the same rule as the first.

-I will give one more example of that function, which is one more pair of “input” and “output”, which you should put in your T-chart on the next line.

- Perhaps this: [squat] for input [show T-chart] and this [squat with arm] for output [show T-chart].

- After you have three pairs, take a guess (silently!) at the function or rule which relates the inputs with the outputs.

- Draw that function to the right of the T-chart, where it has: *f* (o-|-<”dancer”) =________.

Here is the second example, draw this on your own in your second T-chart.
- dance input: [right arm] pause... dance output: [reverse right arm]
- “In the same chart, here is the next pair: [squat] input, pause... [reverse squat] output
- and one more pair: [turn] input, pause... [reverse turn] output
- to the right of the second chart, write your guess at the function or rule which relates each input to its output.

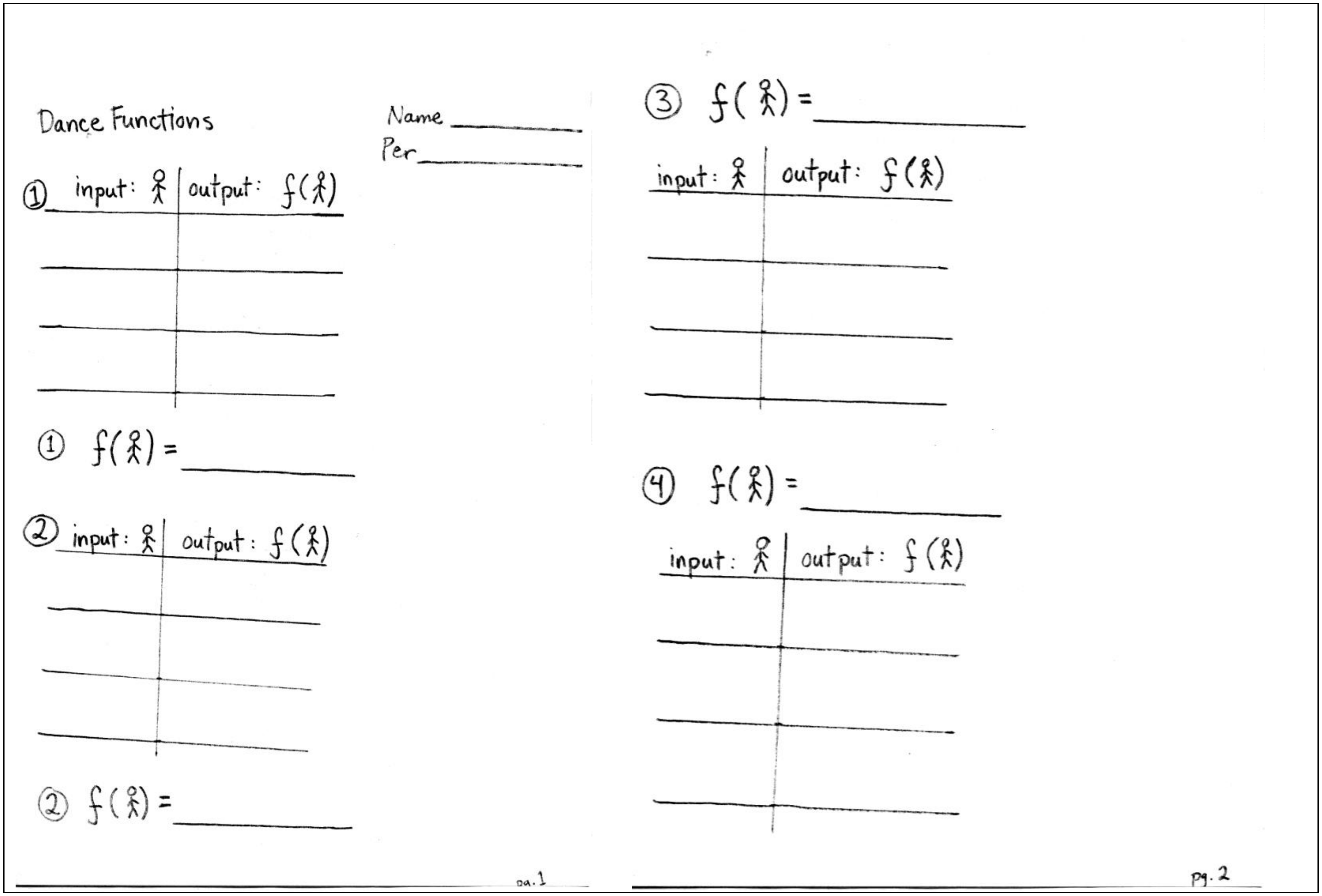
In the fourth example, I'm going to give you an input and the function, and you are going to guess the output. I will call on people who are calmly raising their hands to come up and show us the output.
- draw this on your own in your third T-chart.
- here is the function: *f* (o-|-<”dancer”) = twice as fast
- dance input: [slow right arm] pause... “who would like to give us the output?”
- here is the next output: [slow squat] pause... “Who wants to show the output?”
- and one more input: [slow turn] pause... “Does someone want to show the output?”
- “Does that make sense?”

For the last T-chart, you are going to come up with three pairs of inputs and outputs.
- using a function from this hat, you will make up your own inputs, then apply the rule to figure out the outputs. You have 4 mins and 37 seconds to work with your group.

- At 11:50 am, Survey!!

- Homework: “teach your dance function, with inputs and outputs, to a friend or neighbor or family member over the weekend.”

vi. Worksheet for Revised Lesson


Dance Functions

Name ________

Per ________

①

input: 🯅	output: f(🯅)

① f(🯅) = ________

②

input: 🯅	output: f(🯅)

② f(🯅) = ________

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③ f(🯅) = ________

input: 🯅	output: f(🯅)

④ f(🯅) = ________

input: 🯅	output: f(🯅)

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vii. Revised Lesson, Day 2

<u>DAY 2</u>: (second half of a “block” Wednesday class period, 11:59 to 12:40 PM)
- begin by getting your composition book.

>> recall from yesterday: draw T-chart with “input” and “output”. “Remember this chart?”
-What was a “function”? Can someone give me a definition?
- “function”: a relation that *uniquely* associates members of one set with members of another set.

- Thinking about the dance moves: if you have a certain input, then apply the function, how many outputs will you get?

[demonstrate a move with the function “twice in a row”; show that the only output can be that move twice in a row, not any new, additional moves, not that move in reverse, but only that one move, twice in a row.]

- that is a special feature of “functions”: for every input there is only one output. [write that in your composition book]

- We have seen that “functions” relate dance moves to each other, and there is only one output for each input, but there are other things which functions can relate, too.

- For instance, how much money it costs to talk on the phone is related to how many minutes a person talks. If you talk for 7 minutes (your input) and it costs 5 cents per minute (that's the function), the call will cost 35 cents (the output). Then, when you see on your phone bill that a call cost $1.00, how many minutes long was that call? Remember the cost of a call is 5 cents per minute. The function, relationship, between minutes and total cost is the rate 5 cents per minute.
- Or if you want to send a letter to someone, what are you going to do to send that letter? [put their address on it] And to how many possible mailboxes can it go with that specific address on it? [only one -just like a function, only one output for each input] So, the input is the letter, the function is the address, and the output is the letter in that person's mailbox. A lot of people could send letters to that mailbox, right? All of the letters are different inputs, the function is the address, and the outputs are those letters in the correct mailbox.

- Are you still wondering what does this have to do with math? Well we can use functions on numbers, too! It is a lot like the dance functions we did.

Let's try this: if we say our input is “x” and our output is “y” we can get from x to y using a function, say “$2x + 1$”. [draw a T-chart with all of this, and fill in some number examples]

And what if all you had were the numbers, and you had to guess the function? [draw a T-chart with coordinate pairs following this function: $f(x) = x/2$]

- 12:35 pm Final wrap up/data collection: Answer on a note card: “what does dance have to do with math?”
- Survey!

viii. Examples of dance/movement functions

Define:
"resting" is dancer standing upright with arms at sides.
"squat" start resting, both knees together, bend bringing body down then extend returning to "resting"
"lunge" start resting, reach leg out to side and lean weight onto reaching leg, knee bending on reaching leg, standing leg remain straight; return to resting

Function 1: "addition" > "add" right arm swing from "resting" to above head in angled arch

Input > left arm swing from resting to above head in angled arch
Output > both arms swing simultaneously from resting to above head in angled arch.

Input > squat
Output > squat and right arm swing to above head in angled arch, simultaneously

Input > cross feet and "unwind", turning 360^{o}
Output > cross feet and turn while right arm swings up to above head in angled arch

Input > left lunge
Output > left lunge while right arm swings up to above head in angled arch

Function 2: "multiplication" > do each move three times OR with three people?!

Input > left arm swing from resting to above head in angled arch
Output > repeat 3 times

Input > squat
Output > repeat 3 times

Input > cross feet and "unwind", turning 360^{o}
Output > repeat 3 times

Input > left lunge
Output > repeat 3 times

Function 3: "squared" > perform movement twice as fast

Input > left arm swing from resting to above head in angled arch
Output > repeat twice as fast

Input > squat
Output > repeat twice as fast

Input > cross feet and "unwind", turning 360^o
Output > repeat twice as fast

Input > left lunge
Output > repeat twice as fast

Function 4: "order of operations: parentheses" > perform movement then do a hop

Input > left arm swing from resting to above head in angled arch
Output > left arm swing from resting to above head in angled arch then hop

Input > squat
Output > squat then hop

Input > cross feet and "unwind", turning 360^o
Output > turn then hop

Input > left lunge
Output > lunge then hop

Function 5: "square root" > perform movement half as fast

Input > left arm swing from resting to above head in angled arch
Output > repeat half as fast

Input > squat
Output > repeat half as fast

Input > cross feet and "unwind", turning 360^o
Output > repeat half as fast

Input > left lunge
Output > repeat half as fast

Function 6: "reverse" > perform movement backward

Input > left arm swing from resting to above head in angled arch
Output > left arm start above head in angled arch, swing down to resting

Input > squat
Output > start in bottom of squat, rise to resting, then lower again to squat

Input > cross feet and "unwind", turning
Output > "wind up": from resting, cross foot behind and twist body opposite 360^{o}

Input > left lunge
Output > starting in bottom of left lunge, rise to standing, then return to lunge position

<u>Function 7: "composite"</u> > combines functions 1-4: perform each move adding the right arm swing, twice as fast, three times, then end with a hop

Input > left arm swing from resting to above head in angled arch
Output > twice as fast, both arms swing simultaneously from resting to above head in angled arch, repeat three times, then hop.

Input > squat
Output > twice as fast, squat and right arm swing to above head in angled arch, simultaneously, repeat three times, then hop.

Input > cross feet and "unwind", turning 360^{o}
Output > twice as fast, cross feet and turn while right arm swings up to above head in angled arch, repeat three times, then hop.

Input > left lunge
Output > twice as fast, left lunge while right arm swings up to above head in angled arch, repeat three times, then hop.

<u>Function 8: "rotation"</u> > perform movement in horizontal plane (if possible…)

Input > left arm swing from resting to above head in angled arch
Output > left arm extended behind body, swing to front of body in angled reach

Input > squat
Output > lying on back with legs straight, bring legs in to body, then extend

Input > cross feet and "unwind", turning 360^{o}
Output > lying on floor, roll like a log in place

Input > left lunge
Output > forward lunge on left leg