OPEN MATHEMATICAL PROBLEMS IN THE CLASSROOM:

A TEACHER'S GUIDE

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INTRODUCTION

ABSTRACT

Open mathematical problems are increasingly considered to be important instructional resources. Although practitioners and researchers are still debating over optimal ways of implementing these problems, they agree that preparation is critical. Just how teachers might best prepare, though, is under-documented and under-theorized. To understand teacher preparation of open problem implementation, I sought the expertise of mathematics teachers familiar with open problems. Three experienced mathematics teachers participated voluntarily in a semi-structured group interview, in which they were asked to evaluate a set of open problems with respect to perceived characteristics and potential implementation in the classroom. Through this process the following themes were identified: the spectrum of *open versus closed* problems, *content learning goals*, and *task features*. This thesis contributes to the literature on bridging mathematics-education theory to practice by delineating a set of pragmatic emphases for practitioners preparing to implement open-ended problems in their classrooms. In so doing, this thesis also dimensionalized this practice toward further research.

OVERVIEW

Mathematics education is increasingly focused on fostering mathematical practices as a means for improving instructional outcomes in the United States. The Common Core State Standards in mathematics describe mathematical practices that "educators at all levels should seek to develop in their students" (CCSS Mathematical Practices, 2010). These practices include making sense of problems and persevering in solving them, reasoning abstractly, and modeling. To foster these practices, educators need to be strategic in their use of instructional resources and methodology, such as carefully selecting mathematical activities.

A reading of relevant literature from mathematics-education scholarship indicates a variety of terms relevant to conducting research on open problems, such as "activity," "problem," "open-ended problem," "task," or "situation;" these terms may reflect slight differences in the authors' orientation toward these key practices. This thesis will be guided by the problems' unifying qualities of having the "potential to provide intellectual challenges for enhancing students' mathematical understanding and development" (NCTM Research Committee, 2010, pg. 1). This definition may be read as offering a response to the 'standard task' or 'textbook problem,' which are perceived as under-challenging of students. An important definition rooted in the student's orientation towards the problem articulates that a problem produces desirable outcomes when the individual encounters information that is subjectively novel: "If the individual can immediately recognize the procedures needed, the situation is a standard task (or a routine task or exercise)" (Pehkonen et al., 2013, p. 9).

I will be referring to open problems and problem solving as the focus of this paper. For now, I will define an open problem as a mathematical activity that centrally includes the task of solving a problematic situation, where the procedures for solving are not readily obvious and there are multiple logical paths towards the solution(s). Open problems can be but are not necessarily open-ended problems, which are characterized as having more than one correct solution. I will discuss key features of open problems in greater detail later on.

There is a history of use of open problems along with a wealth of research describing the successes of using them to develop students' content knowledge and problem-solving skills. Competence in problem solving is associated with greater long-term success in mathematics, greater success in application of mathematics outside the classroom, and views that mathematics is an ever-growing and dynamic field of study (Boaler, 1998; NCTM Research Committee, 2010; Schoenfeld, 1992). Much research has been done to describe these learning outcomes as well as the classroom environments in which excellent problem solving takes place (Boaler, 1998; Lampert, 1990; Schoenfeld, 1992).

As a new teacher entering the mathematics education field, I have become acquainted with the literature describing how the use of problem solving in the classroom "not only impacts the development of students' higher-order thinking skills but also reinforces positive attitudes" (NCTM Research Committee, 2010, p. 5). I have these same goals for my classroom and am excited to incorporate such problems into my lessons. However, I found myself asking, how do teachers plan for open-ended problem implementation? Since these problems are often more complex that standard exercises, how do teachers set learning goals for these problems? How, therefore, would I go about incorporating these problems into my classroom? The literature through the 1990s describes cognitive processes, learning outcomes, and classroom environments, without focus on how teachers have planned for classroom use (Lester, 1994). Even in 2010, NCTM acknowledged “knowing how to incorporate problem solving meaningfully into the mathematics curriculum is not necessarily obvious to mathematics teachers” (NCTM Research Committee, 2010). The literature about open problems does not often focus on the details of implementation, such as the diversity of characteristics of problems that lend themselves to problem solving, nor focus on the details of open problems.

This thesis project serves to fill this perceived hole. To answer my research questions, I sought out the expertise of experienced teachers who have themselves implemented lessons involving open-ended problems. I conducted one semi-structured group interview along with introductory and follow up surveys with three expert middle-school teachers affiliated with the Graduate School of Education at UC Berkeley. The initial survey asked participants about their background with problem solving implementation. The interview served to answer questions raised for me from the research. I also hoped to determine the similarities and differences in problem implementation as described by research in contrast with these teachers. A semi-structured interview was chosen as the key research instrument because it combine rigor and flexibility by way of embracing possible immediate follow-up questions; doing so was especially auspicious given that I could not foresee my participants’ reactions to the specific protocol items. I planned the group interview with the explicit objective of creating space for airing conflicting ideas and encouraging debate. Tradeoffs to this approach include participants influencing each other's ideas, however I know these teachers to be outspoken and articulate so I believed any such influence would only be conducive to furthering my research objectives. I created the follow-up survey for participants to reflect on specific instances of their own implementation of open problems.

THEORETICAL ORIENTATION

HISTORY

Research and a national educational agenda on problem solving began in the US during the 1970s as an answer to a perceived crisis in mathematics education. This growing interest in problem solving in mathematics education was in direct response to a "decade of curricula that focused on rote mechanical skills" with minimal mathematical success for students (Schoenfeld, 1992, p. 336). In 1980 the National Council of Teachers of Mathematics (NCTM) published the *Agenda for Action* and in 1989 the *Curriculum and Evaluation Standards for School Mathematics*. Each document had an impact on curricular development emphases and political action in education, respectively. In fact, the 1980s have been called 'the decade of problem solving' (Lester, 1994; Schoenfeld, 1992).

Research on problem solving then unfolded in waves: In the early 80s research was motivated by comparisons of varying degrees of success for problem solvers; in the mid-80s to early 90s research described metacognition and student beliefs during problem solving; and in the early 90s research focused on social and situated influences on students' work (Lester, 1994). In summary, research focused on the student and the individual approach, not on the problems themselves or the teacher moves. In an article describing the state of problem solving in 1994, Frank Lester stated three contemporary issues as directions for future research:

Issue 1: The Role of the Teacher in a Problem-Centered Classroom

Issue 2: What Actually Takes Place in Problem-Centered Classrooms?

Issue 3: Research Should Focus on Groups and Whole Classes Rather Than Individuals

I will outline two key studies, which I believe have addressed these issues since Lester's publication. The first is a three year case study of whole classes in two contrasting schools by Jo Boaler; the second is an action research study conducted by Magdalene Lampert of her own attempt to shift students' views of mathematics through problem solving. I believe these two studies inform my own by clarifying certain aspects of problem-based classrooms while still revealing gaps in research of the specific features of teacher planning and implementation.

In 1998, Jo Boaler published a case study of two schools that helps to answer Lester's Issue 2 and Issue 3 by taking a very detailed and extensive look at a school that used solely open-ended activities. The study compared learning outcomes and daily attitudes of students to those of students at a traditional school using a textbook approach. The study took place over three years and included observations, questionnaires, interviews, and quantitative assessments of both schools. In terms of classroom observations, the classrooms did not appear structured; the students could work in various spaces and determine their own working pace. Students had choice and freedom in their mathematical learning. As Bolaer (1998, p. 42) writes, “Supporters of process-based work argue that if students are given open-ended, practical, and investigative work that requires them to make their own decisions, plan their own routes through tasks, choose methods, and apply their mathematical knowledge the students will benefit in a number of ways.” These alleged benefits were found to include increased enjoyment, understanding, gender equality of opportunity, and enhanced transfer (ibid.).

Boaler also determined that problem-based classrooms are not dependent on rare, highly skilled teachers. That said, her study did not focus on the teachers’ actions. Mathematics education researcher Magdalene Lampert conducted an action research study of her own elementary mathematics classroom in which her goal was to change the way students perceived what it means to do mathematics. Her study focused on her own actions as teacher in a problem-centered classroom. As teacher and researcher, Lampert (1990) integrated knowledge from both educational scholarship and practical problem solving in teaching.

> *In the lesson I designed and enacted, I portrayed what I wanted students to learn about mathematical knowing both in how I constructed my role and in what I expected of the class. I gave them problems to do, but I did not explain how to get the answers, and the questions I expected them to answer went beyond simply determining whether they could get the solutions. I also expected them to answer questions about mathematical assumptions and the legitimacy of their strategies. Answers to problems were given by students, but I did not interpret them to be the primary indication of whether they knew mathematics. (p. 30)*

Lampert describes the many ways she shifted the mathematical authority and challenge onto students in order for them to experience the “zig-zag of discovery” (Lampert, 1990, p. 32). The problems she used determined predictable boundaries for classroom activity, enabling focused student work. She also set up patterns for student interaction that reflect mathematical discourse; students were taught to question each other’s hypotheses, students’ hypotheses were labeled by Lampert with the student’s name, and the class as a whole evaluated collections of responses. Teacher actions were described within three categories: a straightforward description of appropriate (or not) activities; modeling roles; or participating in the mathematics with students.

These two studies surveyed above clarify both student attitudes and actions in problem based classrooms as well as teacher motivations and in-class actions that promote problem-based learning. In what follows, I will first describe key characteristics of problems used in such settings, and then I will draw on the above two studies to describe what we currently know about teacher goals and desired outcomes for students.

PROBLEM CHARACTERISTICS AND GOALS

Literature describes characteristics of open problems in terms of both the inherent qualities of the problem and the skills that will allow students to relate to and be challenged by the problem. Open problems should solicit multiple approaches through different solution strategies, a characteristics independent of a particular classroom (Boaler, 1998; NCTM Research Committee, 2010). Many characteristics follow from this quality of multiple solution strategies. The different levels of sophistication of solving make these problems accessible to students of mixed mathematical abilities. Open problems also encourage student engagement and discourse, with the different solution strategies or positions to be generated, adopted, and defended. A problem solving research brief produced by NCTM also describes content based characteristics as linked with the ability to connect to “important mathematical ideas and promote the skillful use of mathematics” (NCTM Research Committee, 2010, p. 1).

Of students these problems require higher-level thinking and problem solving. A key aspect that makes these problems challenging to students is authenticity: “some aspect of the task is unspecified and requires that the solver re-formulate the problem statements in order to develop solution activity” (Cifarelli & Cai, 2005, p. 302). Therefore, when students first see this task they do not automatically know the solving procedure; it is authentic, new to them. Students may engage in informal experiment that can lead to whole class formalizations (ibid.) Through this decision-making and exploration, the work contributes to the conceptual development of students as well as to their reasoning and communication skills (NCTM Research Committee, 2010). This mathematical and communicative development occurs not through obtaining an answer per se, but through developing and enacting the strategies used en route to figuring out the answer (Lampert, 1990).

An important theme woven throughout the discussions of characteristics of open problem is how the strategies that students use reveal their assumptions regarding how mathematics works (Lampert, 1990; Schoenfeld, 1992, Stein, Grover & Henningsen, 1996). This focus on how mathematics works is reflective of the goals many researchers and teachers have for students in implementing problem solving: to bring students' ideas about what doing mathematics means closer to what it is that actual mathematicians do (Schoenfeld, 1992; Lampert, 1990; Freudenthal, 1968). The problem-based approach has been found to increase students' "beliefs about the adaptable nature of mathematics and the need for reasoned thought," which in turn increased students' examination performance, transfer, understanding, enjoyment, and equality of opportunity (Boaler, 1996, p. 58). Open problems create opportunities for students to experience the dynamic and evolving nature of mathematics, which potentially fosters a belief that "learning mathematics is empowering" (Schoenfeld, 1992, p. 335). In summary, the characteristics of open problems that promote a strong mathematical point of view are the presence of multiple strategies, engagement in exploration, giving reasons for solutions, and making generalizations.

To illuminate the stages of task implementation at the focus of this study, I will employ the mathematical task framework of Stein, Grover, and Henningsen (1996). This framework illustrates "the relationship among various task-related variables and student learning" (Stein, Grover, & Henningsen, 1996, p. 459). This framework describes the path of implementation of a mathematical task along with the factors that influence various phases. Though the authors delineate these separate phases, their work studies the last three, without giving great attention to the first three. Within this framework, the two studies previously outlined have studied both the *mathematical task as implemented by students in the classroom* (Boaler, 1996) and the *factors influencing implementation* (Lampert, 1990). My goals of describing open problems and the details of implementation can be clarified with this framework and fit within the first two rectangles: *Mathematical Task as represented in curricular/ instructional materials*, and

Mathematical Task as set up by teacher in the classroom. Although the Stein, Grover and Henningsen do not explicitly focus on open problems, they maintain a view of the mathematical task that is in line with the definition of an open problem as presented

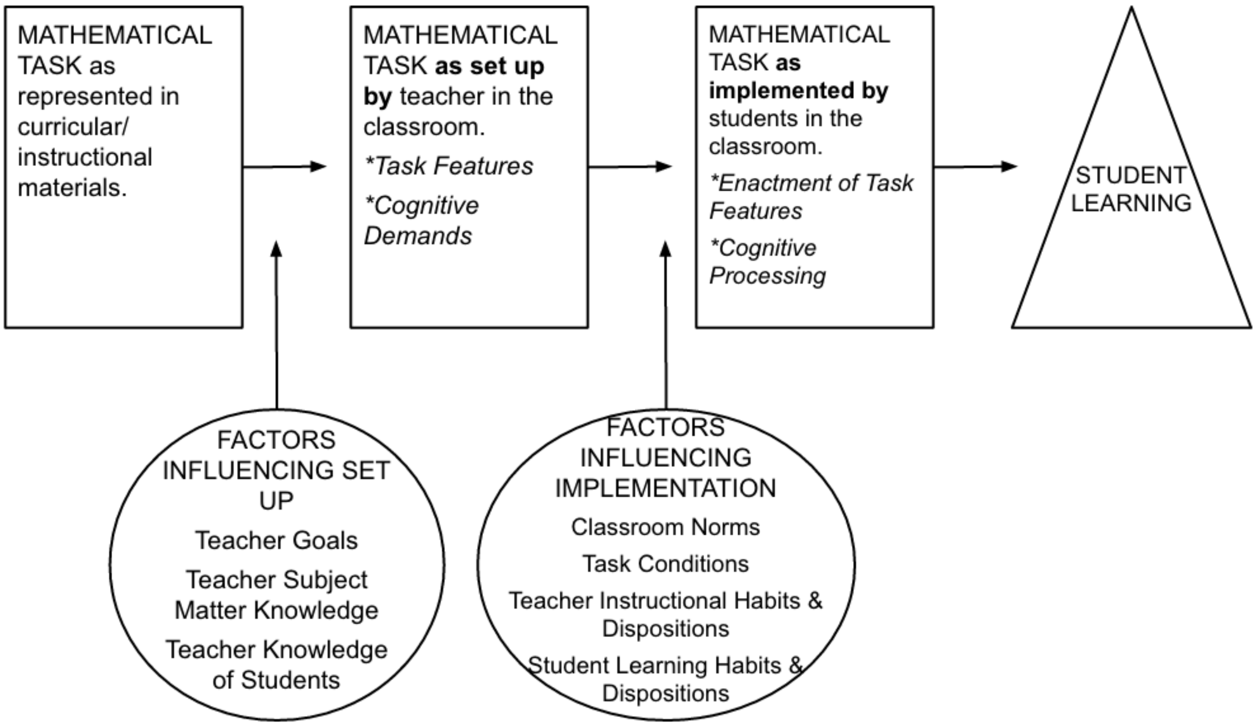

FIGURE 1: STEIN, GROVER, & HENNINGSEN, 1996, P. 459

earlier in this paper. *Task features* are characteristics that educators have determined are important for "student thinking, reasoning, and sense-making" which the authors typify as "existence of multiple-solution strategies, the extent to which the task lends itself to multiple representations, and the extent to which the task demands explanations and/or justifications from the student" (ibid, pg. 461). *Cognitive demands* refer to the kind of thinking processes involved in solving the task *as announced* by the teacher. This task framework gives a lens with which to separate variables in the classroom. With this framework in mind, I will reiterate the focus of this study:

How do teachers analyze mathematical problems as represented in instructional materials? How do teachers plan for the setup of open mathematical problems in the classroom, especially in terms of determining learning goals, task features, and cognitive demands?

METHODS

PARTICIPANTS

I wanted the participants in this study to be classroom teachers who had over 5 years of teaching experience along with frequent implementation of problem solving. Recruitment involved identifying two eligible participants along with one of their colleagues, for three participants total. The three expert teachers are associated with the Graduate School of Education at The University of California, Berkeley. I will use pseudonyms to refer to the three participants: Jason, Amy, and Hannah. Jason taught for 18 years at a sub/urban public middle school, Amy taught for 7 years in an urban private school, and Hannah taught for 12 years at a sub/urban public middle school.

MATERIALS

Eight problems were chosen to provide grounds for a conversation of open problems, with a focus on their task features and cognitive demand. Four were chosen from Philip Heaford's *Great Book of Math Puzzles,* one from John Mason's *Thinking Mathematically,* and three from Dor Abrahamson's course, co-developed with Betina

Zolkower, *Paradigmatic Didactical Mathematical Problematic Situations*. The problems were chosen for their perceived differences with respect to prerequisite skills they would demand and their range of possible solution paths. They were all word problems and each had only one correct solution. I thought of half of the problems as being open, half as being closed (see Figure 2). For a full description of each problem, please see the appendix. Going into the dialogue, I had my own assessment of the problems' task features and cognitive demands; however, to the best of my ability, throughout the conversation I did not affect my participants' opinions or preferences.

Open Problems	Closed Problems
Three Boxes *Births* *Staircase* *Two Trains*	*The Safe* *Three Curves* *Water Lily* *Warehouse*

FIGURE 2: PROBLEM BREAKDOWN

PROCEDURE

I conducted an initial survey, a semi-structured group interview, and a follow-up survey. The diagram below describes these three phases (see Figure 3). The initial survey contained four questions to elicit participants' past experience with open-ended problems. I was particularly interested in better understanding how these experienced teachers selected and used open-ended problems, and the bases for their decisions about integrating them into the classroom. Because there are many ways to conceive of open-ended problems, I clarified by stating the following to the participants at the beginning of the initial survey:

> *Because 'open ended' can take many forms and mean different things to different people, here are some examples of the problems I have in mind. They might be slightly different from those you've worked with, so I'll be*

curious if you've used problems that feel just like these do, or if they have slightly different qualities.

I hoped to provide a narrowing of characteristics of problems by providing three examples that my colleagues were familiar with for the purpose of ostensive definition. I did not provide a detailed definition of open-ended problems for the participants because I did not wish to influence them with my own opinion.

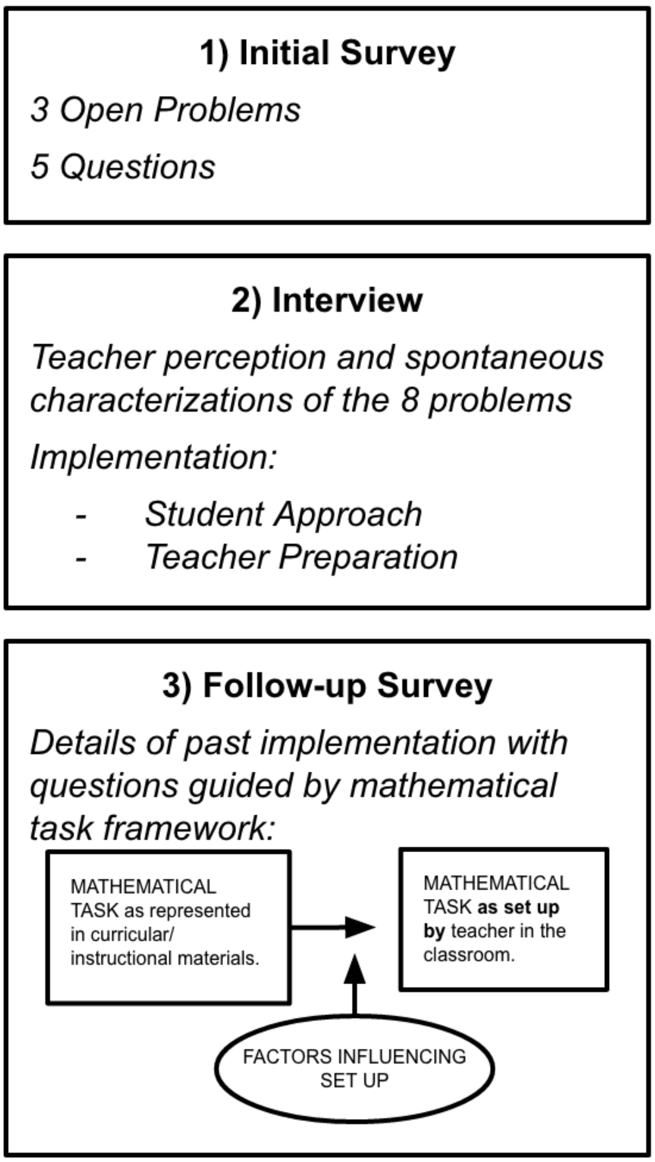

FIGURE 3: TIMELINE

The core interview took place over an hour and a half at the campus of UC Berkeley. It was a single group interview taking the format of a semi-structured interview. The interview had two pieces to it: discussing characteristics of open problems and discussing the implementation process. To begin the interview, I presented participants with the eight mathematical word problems and asked them to describe and distinguish them. The questions I asked attempted to reveal how the teachers viewed the characteristics of open problems as contrasted to the closed problems also included in the set. Next, I shifted the conversation to considerations for implementation, including: what preparation they would engage in to implement a given problem, how students would approach each problem, and how they would determine the learning goal for the problem as implemented. The interview then shifted towards the practical considerations teachers have for problem-based lessons.

Following the interview, participants responded to a survey asking for specific problems they had implemented in their classrooms outlining their thinking around implementation. The survey asked participants to discuss one problem at a time, with the opportunity to respond multiple times. I used the task framework of Stein, Grover & Henningsen (1996) to have participants describe the original problem, "factors influencing setup," and "the task as set up by teachers in the classroom" (p. 457).

DATA GATHERED

I collected field notes and video footage of the interview, as well as the survey data from the three participants. I also generated a transcription of the interview.

DATA ANALYSIS

I engaged in a reflective ethnographic qualitative analysis in an attempt to interpret the teachers' concepts of open problems. In particular, I sought out remarks that were either reflective of or in line with current research to see which ideas of current research were most important to teachers, or those that were outside the realm of current research for contrast. I then categorized their remarks by *characteristics* of problems, thoughts around *implementation*, and themes that emerged.

RESULTS AND DISCUSSION

From the initial survey, I found that each teacher had previous experience integrating open problems into a middle-school mathematics classroom in different capacities. These problems were implemented through problems of the week, formative assessment lessons, or on a uniquely scheduled day with odd length periods. There was not one particular time during a unit that problems were used; instead they were used as an anchor “before, during, or after any new skill, concept or method.” Participants also had experience discussing pedagogical implications through their work at the graduate school. The participants’ combined experiences of mathematics education as researchers and teachers helped them connect “features as discussed in mathematics education research communities as associated with the development of students' capacity to think and reason mathematically” to actual classroom practices (Stein, Grover, & Henningsen, 1996, p. 463).

The interview revealed several key themes in teachers’ orientations towards open problems. As we discussed the eight problems in front of us, I felt many conflicts as I pushed the teachers to stay grounded in the problem text, conflicts that created friction with the views I had of open problems before entering this dialogue. These conflicts are illustrated by the following episodes.

SELECTION

The initial part of the interview was centered on differentiating the eight problems to reveal their essential characteristics, identified along the spectrum from open to closed. My study participant teachers discussed this spectrum in terms of a proposed “difference in entry points,” an access issue originated in multiple factors. For example, Jason characterized the Safe Problem as a problem students “couldn't do anything with without formal knowledge of diameter or circumference.” Hannah characterized the Two Trains problem as “more of an intimidation problem” where students might think “'there’s no way I'm going to get anywhere, it sounds scary'” and thus creating an access problem. Hannah could imagine students feeling they have no skills for solving this problem. This characterization of the Two Trains problem as *closed* by Hannah is the first conflict in perception of open versus closed that occurred. Amy noted that this problem has a “trick solution.” I had originally characterized it as open because there are multiple ways to solve it, including an elegant solution as Amy referenced.

The second conflicting perception of open versus closed concerned the Water Lily problem. I originally categorized it as closed because for me, there was no work to be done to figure it out, there was just recognition of exponential growth and a quick response. However, instead of determining this problem to be closed, Hannah had the perspective that the answer was “more of a trick.” Her reasoning was her “aversion to problems like that, because if it's something I did in first period the trick might be shared with later periods. I don't want to set myself up for failure.” When the goal of the problem is one answer that can be solved with an elegant solution, learning can be jeopardized by sharing of that answer. However, by changing the initial setup of the problem to instead focus on the process, the learning goals and access point for the problem also shift towards ‘open.’ This shift also makes teaching the problem across multiple periods easier. Instead of focusing on the ‘“trick” answer, the problem can be presented with a focus on mathematical argument, which is how Jason thought of the problem: “Water

Lily is about coming up with a justification, and how would you model, how would you explain, create a mathematical argument, you could even give away the answer. That would be an interesting way to do this problem to remove the threat of giving away the answer." By revealing the answer from the beginning, the challenge presented to students shifts to figuring out *why*, and shifts what I had perceived to be a classic textbook problem to a multi-day open exploration.

Finally, multiple approaches to implementation of the Three Curves problem were discussed. The problem asks students to state the three types of curves made when slicing a right circular cone in directions other than parallel to the base. Jason observed that the problem is very challenging without formal knowledge of conic sections. Wondering how the problem could possibly be made more accessible, the teachers suggested to change the task goal to asking, "How you would know when you had the answer?" This version of the problem, the teachers believed, would make it into an open 'cases problem', one where you need to explore different pathways to multiple answers (here, distinguishing between curves to identify three distinct types). As such, rather than attempting to recall a set of memorized facts, students would explore a right cone and justify their groupings. The study participant teachers' change of the problem falls under the "setup of the material" variable.

In short, the discussion of the problems' access points revealed the differing selection criteria each teacher had in mind, an indicator of *whether* the teacher would use the given problem for problem solving.

When the conversation turned to learning goals, I asked, *how do you set a learning goal for an open problem?* I believed this to be a relatively negotiable question, and expected content learning goals to be part of the conversation. Throughout the interview, I had to clarify my desire to talk about these content goals, each time with pushback. Stein, Grover and Henningsen (1996) describe mathematical tasks within the initial textual phase and with the purpose of focusing “students' attention on a particular mathematical idea” (ibid). Instead of focusing on the ‘particular mathematical idea’, the conversation turned to the challenges of integrating such content, particularly to do with the issue of timing. Interestingly, each teacher had a different perspective to offer about the challenge of content integration. These perspectives reflected the settings they had each taught in, the expectations held of them, and the resources available to them.

The concerns Hannah brought to the interview were characterized much more by practicalities of teaching than the two other teachers. She taught in a public school with a specific textbook and described the challenge of integrating problem solving into their already packed curriculum. Her open problem integration happened on days with odd schedules or when the designated content happened to be better fit to open problems. Therefore, her integration of open problems happened less frequently and in a more contained manner.

Amy had a much more integrated approach. She taught at a private school with relaxed content requirements and a department that focused on problem solving. “It allowed us to do whatever we wanted for as long as we wanted, so every time I did content I did a problem with it," to the best of her ability. "I feel like you can take any content and turn it into a problem, I don't think you need to say 'oh we're doing content' vs. 'oh we're doing practices' but this is not a resource that's around for a lot of people. So when I got to other things in algebra we didn't have something set up." She did admit that some content seemed "beyond" making a problem for, "like dividing fractions." The lack of content requirements and collaborative approach to curriculum allowed her to frequently integrate open problems. Though she seemed to have used open problems frequently, she had a difficult time articulating specific goals of usage. She did have the general goal of working "towards establishing a classroom culture (norms) of mathematical curiosity, inquiry, and argumentation" (post-survey). Her focus was much more on the classroom than on the specific problems that rooted it.

Representing the center of this spectrum, Jason admitted to the challenges of integrating open problems within the public school curriculum, though seemed to have implemented open problems on a more regular basis than Hannah was able to (he had also spent more years in the classroom). He shared this perspective on integrating open problems into the classroom:

> *Problem solving lessons are where it's much easier or possible to build in opportunities for students to work with mathematical practices. And come up against interesting content related problems, things they have to kind of solve, and lead to big ideas, like in exponential growth, each step multiplies, so if we're doubling we're going twice as tall! That's a really big idea! Versus linear growth by 1 or 5, to come to an understanding of that content concept, in a context where you're playing and you're coming up with an explanation, that's rich. And you're addressing both practices and content, so I don't know a balance, it depends on time of year, how comfortable students feel in problem solving situations, which is not necessarily a function of how I feel, totally depends on students.*

Therefore, the goals he sets for open problem based lessons include content, the Common Core Mathematical Practices, and, most importantly, students' comfort in working through them. This was an indescribable balance, and, though rooted in a problem, not explicitly dependent on it. The involved set up seemed to involve a thorough gauge of students' present confidence, an analysis of the skills they'd utilize and decisions about how to scaffold these skills.

These varying agendas suggest that settings, expectations, and resources specific to the classroom at hand have direct implications for the design and integration of open problems.

FACILITATION

This part of the discussion was rooted in "task features," which describe the "presence of multiple solution strategies" or "the extent to which the problem lends itself to multiple representations" (Stein, Grover, & Henningsen, 1996, p. 461). This feature is central to the definition of open. When discussing the different possible strategies, teachers listed out the methodologies: "combinatorics (ways can you) solve by creating lists, like the Staircase, Three Boxes is more of a tree diagram, Water Lily is different but you can start by creating a table," and physical modeling with the Three Curves. The study participant teachers felt that each of these problems were characterized by a specific way to solve, however, in doing so, this seems to be directing students to a 'right' solution method. There were no other solution methods discussed.

A key feature of tasks is their presentation. A concern that Amy brought to the interview was 'wordiness' of the problems I presented. "All of these problems are defined with a lot of words, and they are also very challenging." She argued that word problems often use a 'real world' story to obscure the strategies necessary for solving. An opposing example is finding the area of a trapezoid. Amy reasoned that if she "gave a trapezoid with some measurements to 10th graders it would be a different type of problem than if (she) gave it to 3rd graders. For 3rd graders it would be much more difficult, and for them it might be on par with something like this, where the strategies are not obvious." This line of thinking is represented in literature with the idea of *novelty*, and the definition of open problem as one in which the individual encounters information in a *novel* way *for that student*. Amy's argument is that this novelty does not have to arise from the fact that the mathematical idea is embedded within a story, rather, if students are comfortable enough, students can engage in the same way with a problem presented in mathematical terms.

The two features of methodology and presentation are customizable to facilitate and scaffold for particular group of students, revealing the flexibility of problems to fit for multiple age ranges.

DISCUSSION

The goal of this interview was to determine how teachers contemplate, scrutinize, and plan for mathematical problems; what I found through the interview shifts my perspective and raises more questions than I began with. The literary foundation for this research is augmented by the many insights on the practical aspects of integrating open problems into the classroom as well as how the classroom in turn acutely impacts the way a problem is approached by students.

The feature of open problems as being started without specific knowledge and having multiple solution strategies is central to the definition of *open*. I found it interesting that the study participant teachers felt that each of the problems was characterized by a specific method for solution solve, while still maintaining that the problems were open. To me this speaks to the systemization of problem-based lessons; teachers expect and direct students towards a specific methodology, without planning for or acknowledging the use of other strategies. The ease with which the teachers listed out the 'solving method' of each problem made me suppose that there were no other ways to solve them. I was surprised that the participants never brought up multiple solution methods for these problems.

The study participants frequently integrated classroom climate (student habits and dispositions, classroom norms) as a consideration of problem implementation, to the point that it was difficult to isolate factors *of the problem* from factors *of the classroom*. One way this connection was illustrated was in the discussion of access. The need for students to visualize or imagine a first step was seen as an intimidation factor; a factor of students' comfort in the mess of problem solving. This comfort is in turn a product of the teacher's curricular integration and facilitation of problem solving. Though this idea of students' orientation to the problem is the basis for many definitions of problem solving (Pehkonen et al., 2013; Schoenfeld 1985), I did not anticipate this notion to so define the conversation of problems in their initial, textual form. The practical aspects the teachers described of integrating open problems into the classroom consistently outweighed the problem content per se. Problems were not addressed as stand alone tasks, but instead as a way of orienting towards the pragmatics of implementation. Teachers' classification of problems is an implementation-oriented pragmatic taxonomy.

Finally, I would like to discuss the different affordances and challenges inherent to implementing open problems in various types of schools. Implementing open problems in public schools with their set curriculums (and state testing) seemed to be a great challenge. This challenge for Hannah was further compounded by her perception that implementing problem solving with a content learning goal was rare. Amy, however, who taught in a private school without specific mathematical standards and with a cooperative team, felt that almost any content could be made into an open problem. I believe that her opinion of any content as a possible problem is rooted in the fact that she did not have a strict schedule for implementing problems even as she had the support and guidance of the math department. There are multiple possible reasons for why Jason seemed to have greater success with implementing open problems than Hannah; he might have had a department that focused on problem solving, or could have developed the practice over his greater number of years teaching. The discussion possible through collaboration could yield recognition and attention to the intricacies of problem solving, making way for greater problem solving experiences for students.

CONCLUSION

Educational researchers may approach open mathematical problems with an eye on content and solution process, while classroom teachers focus less on the problem and more on the pragmatics of implementation. Teachers' conceptualization and taxonomy of problems is implementation-oriented and pragmatic. This insight that I gained through conducting the study may be pivotal in supporting my own transition into the teaching practice. Teachers can wear different hats -- as mathematicians, as “simulated students,” and as action researchers -- but their foremost concern is with selecting, adapting, and customizing their lesson resources per the learning objectives they craft and monitor for their students.

LIMITATIONS

The findings discussed in this study reflect a small and particular group of teachers’ experiences, and are thus explorative findings. The involvement of all of the participants with the Graduate School of Education at the University of California, Berkeley may reduce the generalizability of this study. Noteworthy is the progressive educational approach at this school, which claims to be “challenging and changing the way people think about education” (GSE of UC Berkeley, 2016). Their affiliation with Berkeley might imply that these teachers do not reflect traditional teaching styles and instead reflect more innovative and recent approaches. In addition, a significant amount of time (2 years) has passed between this interview and the teachers’ last classroom position. Though I cannot claim that these findings are broadly generalizable, I do think they will help to inform my own and fellow practitioners’ teaching. Current practitioners of open problems will probably recognize and relate to the issues that these teachers bring to light.

FURTHER RESEARCH

There are many possible avenues for further research, including multiple research questions and settings. These paths would illuminate more fully both the specific skills required to engage effectively in the activity of solving specific open problems and, consequently, how teachers determine and plan for these factors. This study has revealed many questions about this process and, given its modest nature, has revealed just the tip of the iceberg. Therefore, future research needs to analyze open problems in their initial instructional materials phase, could interview developers of open problem resources, and observe teachers in the planning phase. Another possibility would be a “blog dive” in which research analyzes the many blogs and internet resources from expert teachers who present their ideas and implementations of open problems. Those studies would look to answer the following questions:

- How can learning goals be determined for a specific problem? Are these learning goals inherent to the problem, regardless of how the teacher implements it, or does the classroom determine these possibilities?
- How do developers of general curriculum and open problem based lessons create open problem based lessons?
- How do teachers imagine and develop learning goals for open problems?

In furthering educational research on mathematics teachers’ use of open problems, researchers and teachers can develop optimal ways of implementing open problems in the classroom. This deeper understanding would reveal valuable decision making around selection, creating agendas, and facilitation of open problems. Through this understanding, teachers could be better prepared to implement open problems.

REFLECTION

Through this research I have been exposed to the complexity of open problems and the influence of teacher identity on implementation. This has once again affirmed my conception of teaching as a comprehensive and intricate process of decision-making. A key insight I have gained of the implementation process is the impact of the teacher's perspective. Problems that I had initially determined to be decidedly closed due to the simple solution were viewed with a very different perspective by the participant teachers and instead considered open. I hope to take the varying possibilities for *open* into my teaching career and attempt to redesign and reimagine closed problems as open. I also want to make sure to seek and affirm students' multiple solution paths for open problems instead of steering students only to a prefixed set of paths.

This research experience was novel to me, in that I had not before conducted a study of this scale. I enjoyed the mental stimulation of becoming firmly acquainted with an area of research, though I was challenged by the process of distilling and analyzing the literature as well as my empirical findings. This presentation of ideas in a formal and academically appropriate manner was a particular challenge to me. I am thrilled to have completed this task and to continue to develop my ideas of open problems through my own implementation in a high school classroom this coming fall.

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APPENDIX

PROBLEMS AND QUESTIONS FROM THE INITIAL SURVEY:

- Say there is a rubber band is stretched all the way around the equator of the Earth. How much would the rubber band lengthen if made to hover 1 foot above the Earth at all points?
- One morning, exactly at 6 A.M., a monk began to climb a tall mountain. The narrow path, no more than a foot or two wide, spiraled around the mountain to a glittering temple at the summit. The monk ascended the path at varying rates of speed, stopping many times along the way to rest and to eat the dried fruit he carried with him. He reached the temple precisely at 6 P.M.After several days of fasting and meditation, he began his journey back along the same path, starting at 6 A.M. and again walking at varying speeds with many pauses along the way. He reached the bottom at precisely 6 P.M. I assert that there is at least one spot along the path the monk occupied at precisely the same time of day on both trips. Is my assertion true? How do you decide?
- Find a 6-inch segment using only a letter-sized piece of paper.
- If you have used open-ended mathematical problems in your classroom, how often did you use them?
- At what point in a unit or lesson sequence did you use open-ended problems, and how did you decided this timing?
- Where do you find open-ended questions?
- Do you have any favorite open-ended questions for the classroom? Why are these your favorite?

PROBLEMS USED IN GROUP INTERVIEW:

- Imagine that you have three boxes, one containing two black marbles, one containing two white marbles, and the third, one black marble and one white marble. The boxes were labeled for their contents - BB, WW and BW - but someone has switched the labels so that every box is now incorrectly labeled. You are allowed to take one marble at a time out of any box, without looking inside, and by this process of sampling you are to determine the contents of all three boxes. What is the smallest number of drawings needed to do this?

- To move a safe, two cylindrical steel bars 7 inches in diameter are used as rollers. How far will the same have moved forward when the rollers have made one revolution?
- What three curves are produced by making sections of a right circular cone in directions other than parallel to the base?
- A water lily doubles itself in size each day. From the time its first leaf appeared to the time when the surface of the pond was completely covered took forty days. How long did it take for the pond to be half covered?
- In a far away land, it is desired by families to have one male child to pass the family's inheritance on. That means, after a family has had a male child, they will cease to have children. One birth does not impact the next. What is the ratio of boys to girls in the country?
- There is a staircase with 10 steps. A child runs up the steps, either 1 step or 2 steps with each stride. How many different ways can the stairs be climbed?
- In a warehouse you obtain 20% discount but you must pay a 15% sales tax. Which would you prefer to have calculated first: discount or tax?
- Two trains are on the same track a distance 100 m apart heading towards one another, each at a speed of 50 mph. A fly starting out at the front of one train, flies towards the other at a speed of 60 mph.. Upon reaching the other train, the fly turns around and continues towards the first train. How many miles does the fly travel before getting squashed in the collision of the two trains?

FULL PROCEDURE

Dialogue:

20 minutes: Card Sort; Describe characteristics

- Each member individually sorts cards, grouping however they want.
- Then, as a group, take turns placing one problem at a time until every problem is in the group
- As group places cards, members must explain why, and then describe formed groupings.
- Why did you organize the problems like this?
- If you disagree, how do you see it differently?
- How are these problems alike, how are those problems different?
- What are the characteristics you sorted by?
- How would students approach each problem / group?
- How would you characterize the learning goals of each?
- When would you use each group of problem in the classroom? (For what purpose, what timing?)

20 minutes questions: specific to open ended problems

- What kind of planning is involved in using an open-ended problem in the classroom?
- What do you do beforehand?
- How do you set a learning goal for a problem you haven't done with students? (Then do it as a group!)

30 minutes: Go through planning process

- An 'open-ended' problem; A 'closed' problem (go through the process of 'opening' it)
- Planning, perceived student reactions, student responses

10 minutes: closure

- Summarize process
- Develop an open ended problem; Have you ever made an open ended problem?

Follow Up Survey:

- Briefly share the problem
    - Your answer
- Describe the context you used it in (ex. algebra 1, 7th grade)
    - Your answer
- At what point in the year? At what point in a unit?
    - Very beginning of the year
    - Sometime in the middle
    - Towards the end
    - Introductory
    - Formative
    - Summative
    - Other:
- What was the content objective?
    - Establish problem solving skills
    - Continue to develop problem solving skills
    - A specific content objective (please describe below!)
    - Other:
- Describe the features of this problem
    - "The task features refer to aspects of tasks that mathematics educators have identified as important considerations for the engagement of student thinking, reasoning, and sense-making." Please feel free to elaborate!
    - The existence of multiple solution strategies
    - The extent to which the task lends itself to multiple representations
    - The extent to which the task demands explanations and/or justifications from the students
    - Other:
- Describe the cognitive demands of this problem
    - "The cognitive demands of the task during the task-set-up phase refer to the kind of thinking processes entailed in solving the task AS

ANNOUNCED BY THE TEACHER" (i.e., different from those employed by students in actuality)
    - Memorization
    - Use of procedures or algorithms
    - Employment of complex thinking / reasoning strategies that would be typical of "doing mathematics"
    - Other:
- Describe your subject matter knowledge related to this particular problem
    - Your answer
- Describe your knowledge of students as related to this problem
    - How did you anticipate they would approach it? Were they particularly interested in this context?
    - Your answer
- Were you hoping to establish anything else by implementing this problem?
    - Your answer
- What was involved in your planning for this problem?
    - Your answer
- Anything else that I haven't covered?
    - Your answer