

Beyond “Just Sitting There”: Function Addition through Collaborative Balance Sensory Activity with Balance Board Math

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Abstract

Sedentary classroom norms are challenged by both theories of embodied cognition, whereby bodily experiences impact cognitive structures, and of sensory regulation, whereby bodily activity provides sensory input that supports self-regulation. Balance Board Math (BBM), a design-based research project, envisions learning as both movement-based and sensory-regulatory, focusing on the under-activated sense of balance. We evaluated high school students' learning outcomes and pathways and their regulation (enjoyment and arousal) levels upon completing BBM function addition activities on sensor-equipped balance boards. Our findings bear out embodied cognition and sensory regulation perspectives in that students developed new sensorimotor schemes integrating their balance experiences with features of the graphs they generated, yielding both high levels of excitement and improved post-test math scores.

1. Introduction and Theoretical Framework

Balance Board Math (BBM) is motivated by two intersecting problems with the sedentary norms of traditional high school classrooms: these ignore 1) the sensorimotor foundations of cognition; 2) students' need for sensory stimulation to regulate arousal state. The former problem is raised by theories of embodied cognition (e.g., Newen et al., 2018), whereby cognition depends upon our bodily experiences. Embodied cognition has inspired learning sciences work surfacing the role of bodily activity, such as gesture, in learning (e.g., Alibali & Nathan, 2012) and new ways of moving as the basis for conceptual learning (e.g., Abrahamson et al., 2020; Tran et al., 2017) including with the full body (e.g., Gallagher & Lindgren, 2015; Kelton & Ma, 2018; Vogelstein et al., 2019). Drawing upon enactivism (Varela et al., 1991), we theorize cognitive structures and consequently mathematical concepts as arising from repeated patterns of sensorimotor activity (Hutto et al., 2015).

The latter problem is raised by models of sensory processing and self-regulation (e.g., Dunn, 1997; Dahl Reeves, 2001). These models suggest that the intensity and type of sensory inputs, especially within foundational sensory systems like balance, interact with our neurological thresholds to affect arousal, alertness, and affective

experience. When stimulation is insufficient or overwhelming, self-directed activities like fidgeting support sensory regulation. However, these activities can be read as disruptive—a view compounded by ableism (Nolan & McBride, 2015) and racism (Annamma et al., 2013). We investigate what learning might look like if it offered sensorially stimulating, movement-based ways of engaging with concepts toward a more integrative model of classroom sensorimotor experiences.

2. Design-Based Research Context: BBM Function Addition

This study is embedded within a larger BBM design-based research (e.g., Cobb et al., 2003) project¹ that investigates balance-sensory experiences as an integrative means of both embodied self-regulation and grounding mathematical concepts. Learners are invited to engage in rocking, a common form of self-regulatory sensory stimulation, as a means of exploring math concepts. BBM design principles (Tancredi et al., 2022) are as follows:

- 1) **Foster movements that ground mathematical concepts in experiences of balance.** This principle is inspired by evidence the balance system broadly affects cognition (Bigelow & Agrawal, 2015), spatial/proportional reasoning skills (Frick & Möhring, 2016), and conceptual reasoning (Antle et al., 2013). In particular, BBM graphing activities seek to cultivate immersive phenomenologies of graphing, inspired by work showing expert graphers' tendency to gesture as if "being the graph," not merely "seeing the graph" (Gerofsky, 2011).
- 2) **Support learners' discovery and control of dynamical properties.** This principle applies *action-based embodied design* (Abrahamson, 2014), a framework for enactivist pedagogy wherein design fosters students' discovery of dynamical properties in a designed environment as a means of grounding mathematical concepts.
- 3) **Invite embodied self-regulation during and through instructional activities.** This principle reflects the implications of sensory regulation (Dunn, 1997; Dahl Reeves, 2001) for instructional design by allowing students the kinds of sensory stimulation they need to self-regulate as part of the learning experience.
- 4) **Inclusively adapt to different sensory profiles.** This principle reflects a goal of inclusive instructional design wherein learners of various sensory profiles have equitable access to both instructional content and self-regulation opportunities and can participate together.

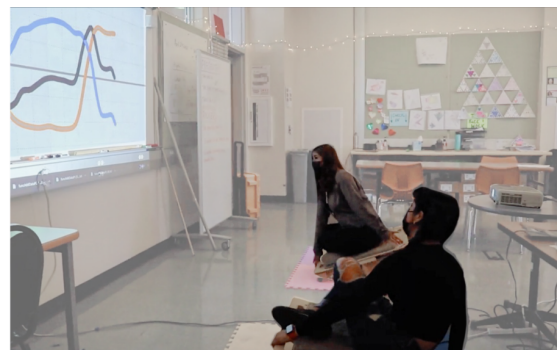
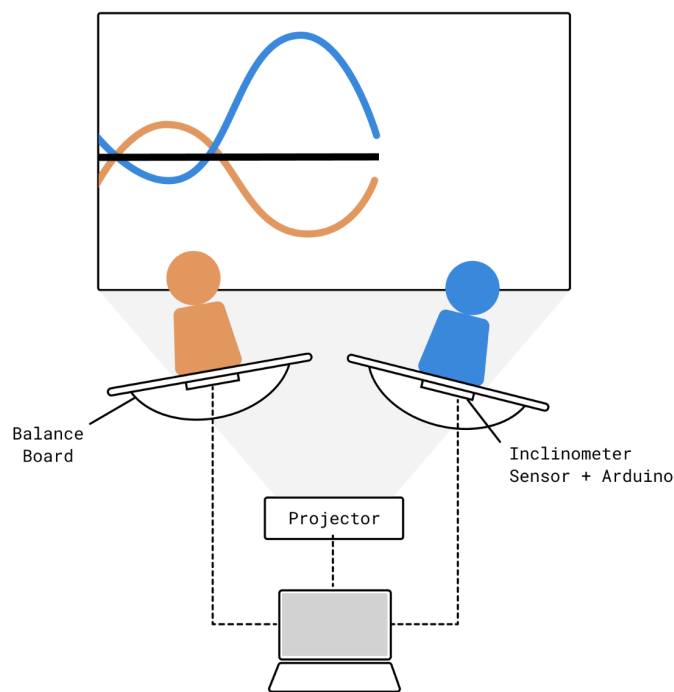
In the present study, we used mixed methods to evaluate one BBM function and graphing activity, Function Addition (Figure 1), with regard to principles 1-3 in the context of high school classroom instruction. In Function Addition, two students rock

¹ To date, BBM has undergone multiple iterations focused on the relationship between rocking for sensory regulation and movement for learning and communication across different sensory profiles, informed by semi-structured clinical interviews with children (see Tancredi et al., 2022).

on balance boards together to generate digital graphs (shown in orange and blue) that affect a sum line (shown in black). Graphs appear from left to right, with the y-value controlled moment-to-moment by the board's angle. Leaning in one direction increases the graph's y-value; leaning the other way decreases it. Users can orient the board left-to-right (left increases y, right decreases it) or front-to-back (forward decreases y and backward increases it) as if riding a surfboard or roller coaster over the contour of the emerging graph, respectively. Students work together to control their sum graph in real-time, and can generate different sum functions and explore multiple ways of summing to a function.

Figure 1

BBM Function Addition Activity: a) Implementation Diagram, b) Two Examples of Students Graphing



Note. The angle of each player's board controls the y position of either the blue or orange graphs. $y=0$ when the board is flat, with angles on one side yielding negative y values and the other side yielding positive values. The upper b image shows students rocking front to back, and the lower b image shows students rocking left to right.

We hypothesized that by creating the conditions to coordinate moving in new ways with the balance boards, Balance Board Math offers students opportunities to develop new conceptual substrates for mathematical ideas such as amplitude (max angle), frequency (rocking rate), slope (steepness), and phase (relation with peer's board angle). These activities render students' experiences of moving their bodies alone and together with others across domains, including sports and daily life, as

resources for thinking about mathematical concepts. Inspired by Gerofsky's findings (2011), we theorized interpreting graphs as a dynamical process of projecting one's body into the graph from an egocentric perspective, drawing upon our existing perceptuomotor capacities for conceptualizing bodily movements in time and space. We hypothesized that students cultivate a "being-the-graph" perspective by controlling dynamical graphs through proximal balance actions, shaping how they come to perceive even static graphs². In this study, we focused on whether and how students regulate and learn with BBM collaboratively with peers in a classroom context.

3. Objectives

Our research objective was to evaluate the effectiveness and learning pathways of balance-based embodied exploration through the following research questions:

1. *What is the effect of BBM on students':*
 - a. *learning, operationalized as their capacity to answer function addition math questions?*
 - b. *sensory regulation, operationalized as self-assessed enjoyment and arousal levels?*

Based on embodied cognition literature, we hypothesized that learning to coordinate movement with a peer to achieve different sum functions would bolster conceptual understanding of function addition (1a). Based on sensory regulation literature, we hypothesized that BBM's amplified opportunities for self-driven vestibular stimulation would increase arousal levels for most sensory profiles without becoming unpleasant (1b).

2. *How did students speak and gesture about how they perceived their activity, their peers' activity, the resulting graphs, and the relations among these?*

Following an enactivist view of learning, we posited that students will develop new perceptual orientations to guide their actions on the board, and that the emergence of these new perceptual orientations is constitutive of their conceptual thinking (Hutto et al., 2015). We further conjectured that movement constitutes a dialogic resource (Enyedy et al., 2015) for students and teachers to access and interact with one another's ways of seeing.

² For analysis of changes in how students gesture about static graphs after using BBM in lab interviews, see Tancredi et al., 2022.

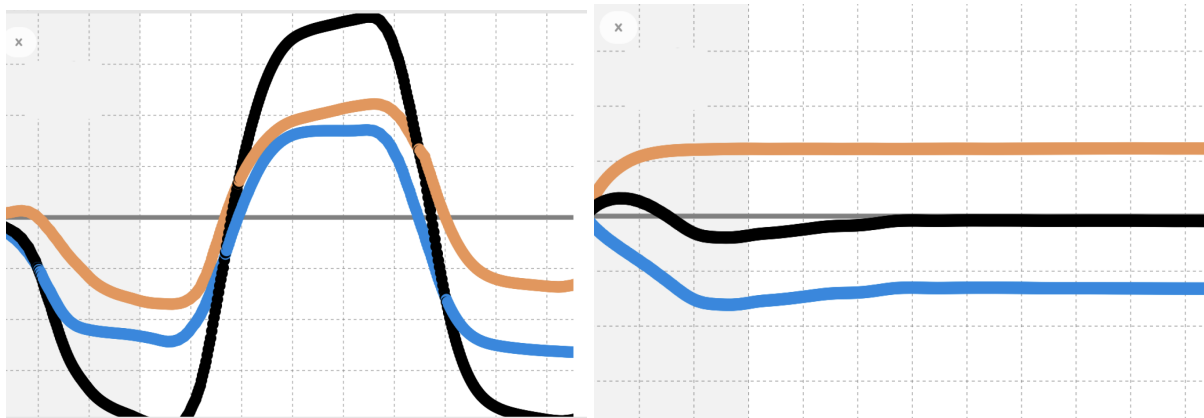
4. Data Sources

We co-planned and co-facilitated a BBM function addition lesson with a public high school teacher in Oakland in four Algebra I and Geometry classes ($N=25$ in groups of 3-8, 55 minutes each)³. Function addition was identified as a novel concept for students that connected to their prior knowledge about graphing and slope. Target learning outcomes included students being able to graphically define the sum of two given functions and identify different functions that sum to a given function. Sessions were audio-video recorded. Students completed 10-minute written assessments prior to and immediately following the lesson (Appendix I).

First, two student volunteers demonstrated the boards⁴, and the class was invited to share initial interpretations of their graphs, specifically the meaning of the x- and y-axes and sum line trajectory. Students were then split into groups and given the challenge to generate a sum function of either a high-amplitude sinusoid (Figure 2a) or $y=0$ (Figure 2b). After brainstorming, groups took turns introducing and testing solution ideas using the boards, with feedback from their peers and prompting questions from the teacher and researcher, for 30 minutes. If multiple solutions were found for each problem before this time elapsed, the class was given additional challenges, such as summing to a diagonal line with a positive slope or an upwards-trending sinusoid. Finally, the class re-grouped for a 10-minute discussion during which relevant disciplinary vocabulary such as “sum” and “function addition” was used to interpret graphs they had generated in the activity.

Figure 2

Example Student Graphs: a) Solution to Challenge A; b) Solution to Challenge B



³ Demographics at this school site are as follows: 50 percent Latino, 30 percent African American, seven percent White, and 13 percent Asian American; over 70 percent of students qualify for free and reduced lunch. 56% of participants were female and 44% were male.

⁴ Half of the classes began with the boards in a front/back configuration, and half in a left/right configuration. Students were permitted to move the boards and change their orientation on the board.

Note. Challenge A was to generate graphs (blue and orange lines) that would sum to a high-amplitude sinusoidal function (black line). Challenge B was to generate graphs (orange and blue lines) that sum to the function $y=0$ (black line).

5. Methods

RQ1: T-Test of Pre/Post Scores and Enjoyment/Regulation Ratings

To answer question 1a, two raters scored test questions using a rubric awarding 0-1 points per question. Question source (pre vs. post-test) was redacted prior to scoring. Inter-rater agreement was 89% for a sample of 36 responses. Cohen's weighted kappa was 0.81, indicating substantial agreement. We conducted a paired, 1-tailed t-test at alpha level 0.05 on students' pre and post sum scores. Pre-assessment scores exhibited a floor effect due to a lack of formal exposure to function addition, yielding a right-skewed distribution. Data were $\log(n+1)$ -transformed to meet t-test assumptions, verified with the Shapiro-Wilk test ($p=0.94$ for pre, $p=0.15$ for post) and Levene's test ($p=0.56$). We ran the t-test with and without two outlier students who scored 5.5/6 on the post test to evaluate their impact on results. We calculated the effect size using Glass's delta since we anticipated the intervention would impact standard deviation (Laken, 2013).

To answer question 1b, we included a simplified version of the Self-Assessment Manikin (Bradley & Lang, 1994) in the post-test to evaluate students' enjoyment and arousal levels, and invited students' written comments on the activities⁵.

RQ2: Qualitative Analysis

We compiled videos from four angles into a single stream, then transcribed and coded these files using ELAN annotation software. To answer the second research question, we coded types of perceptual strategies (*graph-oriented*, such as using gridlines; *board-oriented*, such as monitoring board angle) starting with sub-codes from prior analysis of 1-player BBM activities (Tancredi et al., 2022). We then inductively revised and expanded these codes to account for novel perceptual orientations that arose in this two-player context (for example, peer-oriented strategies such as monitoring temporal synchrony).

6. Results

RQ1a

There was a significant difference in mean scores pre ($M=20\%$, $SD=16\%$) to post ($M=37\%$, $SD=27\%$) ($t(23)=-3.5$, $p=0.001$) and the effect size was moderate (Glass's

⁵ We used the following written prompts: a) When graphing using the board, which picture best corresponds to how you felt (from unhappy to happy)? [Self-Assessment Manikin valence scale]. b) When graphing using the board, which picture best corresponds to how you felt (from sleepy to excited)? [Self-Assessment Manikin arousal scale]. c) Any comments for us on today's activities?

delta=0.75). This held even with the highest-scoring outliers on the posttest removed ($t(21)=-2.29$, $p=0.016$).

RQ1b

100% of students rated their enjoyment at or above 4/5 ($M=4.41/5$, $SD=0.50$). All written student comments were positive, such as “the activity was good, interactive” and “it was interesting,” with 6 students leaving the comments section blank or indicating they had no comments to share. The most common descriptor used by students was “fun,” such as “it was fun to do and learn :),” appearing in 58% of comments. One student compared BBM to typical sedentary learning, noting: “It was coo [sic]. Better than just sitting there. School would be better like this.” All but one student⁶ rated their arousal level on the board as moderate (3/5) to high (5/5) ($M=3.93/5$, $SD=0.85$). Taken together, student ratings suggest most students experienced an energized, pleasant regulatory state during BBM activities.

RQ2

Students’ language during the lesson reflected identification of dynamical properties of both *distal* graphical and *proximal* movement. A salient initial distal orientation was *control*: students initially interpreted visual similarities between one graph and the sum graph, such as shape or amplitude, as that player “overt[aking]” or “win[ning]!” the round. This gave way to describing the sum graph as “following” or “mimicking” *both* functions. Students’ orientation progressively shifted from the distal to the proximal: whereas early utterances focused on graphs’ vertical trajectories (e.g., “The orange went up more, also the black”), later utterances focused on board rocking (e.g., go “right a little bit and then left”). Students also increasingly fused the two frames of reference, as when planning, “if she goes right [on the board], then I’ll go up in the top [on the graph],” pointing up while saying “left,” or describing the sum graph as the “balance” of its component graphs. This integration is consistent with the development of novel perceptuomotor schemes organizing and integrating proximal and distal phenomena (Abrahamson & Bakker, 2016).

Students parameterized their proximal movements with increasing specificity, from the direction of movement, through angle, to board phase relations. Whereas initially, students tended to focus on which direction they were moving, they then came to attend to the specific angle of their board’s balance, discussing posture and board positioning to fine-tune their control. Later discussions emphasized board relations, featuring gestures showing boards’ relative positions. For example, one group’s planning gestures began as indicating a *direction* of travel (Figure 3a), then progressed to deploying a counting *temporal synchrony* strategy (Figure 3b), and finally culminated in the rehearsal of *specific angles* (Figure 3c) (Appendix II). As students identified parameters of graphical control and integrated proximal and distal dimensions of

⁶ The outlier student rated their arousal as low (2/5) paired with high enjoyment. This student may have been an outlier in terms of their vestibular sensory profile.

action, they progressed from qualitative thinking, such as “slower” towards more discrete and quantitative thinking, such as “go 2 down [on the grid]” and planning specific y-value positions such as 10 and -10 to sum to 0 (Figure 4). Here, students spontaneously took up tools such as grids and numbers to refine and communicate about their movements. BBM thus instantiates the design principles of action-based embodied design (Abrahamson, 2014; Alberto et al., 2021) by rendering mathematical disciplinary tools as resources for refining their enactment of solutions to motor control problems.

In the classroom setting, students’ actions on the boards became discursive resources for the group. For example, when trying to generate a diagonal sum line, one group recalled another’s earlier maneuver “when they were just sitting still [trying to each draw $y=0$], but then they like adjusted the line [to correct each board’s balance].” This adjustment resulted in a slight diagonal. Students built upon their peers’ unintended result from earlier to purposefully create a linear function with a non-zero slope. Students were able to make a connection between their peers’ physical movements and graphical outcomes and extrapolate from this relation to generate additional graphs. Additionally, students participated in peers’ actions through verbal and gestural input and reactions such as clapping. These findings suggest that students are able to develop their thinking about graphs from a movement-based activity vicariously. Work on mirror neurons (Rodríguez et al., 2014) suggests that perception and action share neural mechanisms, supporting motor skill improvements from observational motor learning. In a movement-based learning activity, our observations were consistent with students experiencing such vicarious learning.

Figure 3

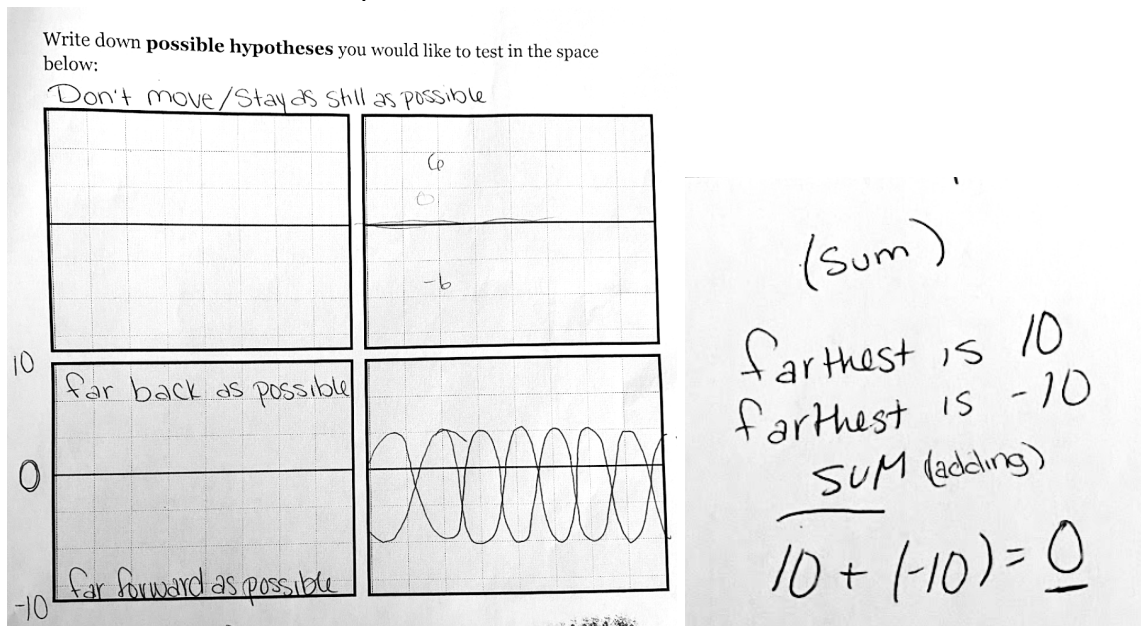
Progression of Preparatory Strategies Across Rounds: a) Gesturing Rocking Direction; b) Rehearsing Counting-Based Timing; c) Practicing Coordinated Angle Positions



Note. These images show how students worked to coordinate their actions prior to initiating a round of graphing: a) Adriana (pseudonym) points to her left to indicate which direction her peer should go. b) Marisa (pseudonym) models leaning left and counting “1, 2, 3” to time changes in direction. c) Marisa rocks to the most extreme rightward angle possible; John (pseudonym) leans to the most extreme leftward angle possible.

Figure 4

Students Brainstorm of Ways to Generate a $y=0$ Graph: a) Sketch of Different Solutions; b) Scratchwork from Same Group



Note. Solutions brainstormed by the students in a) include tracing the x-axis, drawing $y=6$ and $y=-6$, and opposite phase sinusoids. The scratchwork in b) shows students' use of addition. The $f(x)=6$, $g(x)=-6$ graphical solution and $f(x)=10$, $g(x)=-10$ solutions highlight how this group came to discuss a general rule to generate a family of solutions to this problem.

7. Discussion

Through BBM, high school students developed new sensorimotor schemes integrating their proximal experiences of balances with distal graphical features, discussing and controlling these with increasing nuance by drawing upon disciplinary tools such as grids. Students actively guided, responses to, and built upon their peers' actions on the boards, suggesting that movement experiences can engage even observers. Students' sensorimotor experiences yielded statistically significant improvements in their ensuing capacity to solve pencil-and-paper function addition problems as predicted by enactivist theories of learning. Additionally, students expressed unanimous enjoyment and high levels of excitement from the highly vestibular-activating task, bearing out sensory regulation theory.

8. Limitations and Future Directions

This first classroom study exhibited several limitations to be addressed in future work. It would be beneficial to replicate results with a more robust sample size as well as a comparison group receiving sedentary instruction of the same material. Additionally, assessment of longer-term classroom use and retention over time is warranted to evaluate the impact of this instructional approach. We did not measure students' performance on a retest without the intervention in this study; although we do not anticipate that students would have shown significant improvements on a re-test given that they did not show a trend of improvement across questions within the test, nor tended to use the full time allotted for the pre test, and had limited to no exposure to concepts, in future work, measuring any test practice effects can support a causal interpretation of data.

In this study, we evaluate the impact of BBM on proxies of regulation (arousal and enjoyment) as well as learning (math questions), but not yet the relation between these two. Future studies can help to identify to what degree arousal and enjoyment are mediators of learning gains as might be predicted by prior literature (e.g., Obergriesser & Stoeger, 2020). Next steps in the BBM project include a closer study of the relation between balance-activating movement and regulation, focusing on students with marginalized sensory profiles and using physiological measures of arousal. Using more fine-grain indicators such as continuous electrodermal activity sensing, we hope to gain deeper insights into how student arousal levels are affected by sensory experiences and in turn, affect learning. Noting the presence of an outlier student in this study whose arousal rating was low, we propose that further work on

the interaction between sensory profile and regulation is warranted to best inform instructional design.

Expanding the range of students who can engage with BBM is central to its goal of inclusive design. Beyond current features such as graph sonification and seating flexibility, we propose further expansion of options for physical engagement (e.g., rigid seat back for added support, integration of alternative input devices) as well as display access (e.g., validating sonification with users). We envision BBM as a context for studying inclusive learning with regards to both sensory differentiation and collaboration across learners with different sensory profiles, leveraging the shared sense of balance as one among multiple flexible access modalities.

9. Conclusion

This study breaks new ground by showing the learning and sensory regulation impacts of engaging the sense of balance in math classroom instruction. Findings corroborate and elaborate an enactivist view of learning through interaction with peers and instructional media by illustrating how learners come to coordinate proximal and distal phenomena through activity. BBM instantiates embodied design using the sense of balance, finding this sense an untapped conceptual, dialogic, and regulatory classroom resource. Our findings suggest that in lieu of ableist norms of ignoring or disciplining students' self-regulatory movements like rocking and fidgeting, these movement forms can instead serve as fruitful grounds for conceptual learning.

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Appendix I: Pre/Post Question Types and Example

Question Types

The pre and post assessments contained 6 questions total, one of each type:

- 1) addition of two functions (graphical)
- 2) addition of two functions (multiple choice)
- 3) point-based function addition (quantitative)
- 4) functions that sum to a given function (graphical)
- 5) graphing of concrete scenario (multiple choice)
- 6) extrapolation question: function subtraction (graphical)

Table 1

Example Question

Type	Instruction	Pre-Question	Post-Question
6	$h(x)$ (the dotted function) is the sum of $f(x)$ and another function, $g(x)$. Draw $g(x)$ on the graph below.		

Table 2

Rubric for Example Question

0	0.25	0.5	0.75	1
<ul style="list-style-type: none"> - leaves answer blank - writes "I don't know" - draws something other than a sinusoid such as a straight line 	<ul style="list-style-type: none"> - draws a sinusoid but with incorrect amplitude <i>and</i> a different frequency than the ones shown 	<ul style="list-style-type: none"> - draws some correct points on the correct function but does not trace full length 	<ul style="list-style-type: none"> - draws a sinusoid with either the correct amplitude or frequency (but not both); <i>for example, draws the sum of the two functions shown</i> 	<ul style="list-style-type: none"> - draws a sinusoid with correct amplitude and the same frequency and phase as $f(x)$ and $h(x)$

Figure A.1

Example Response with a Score of 0

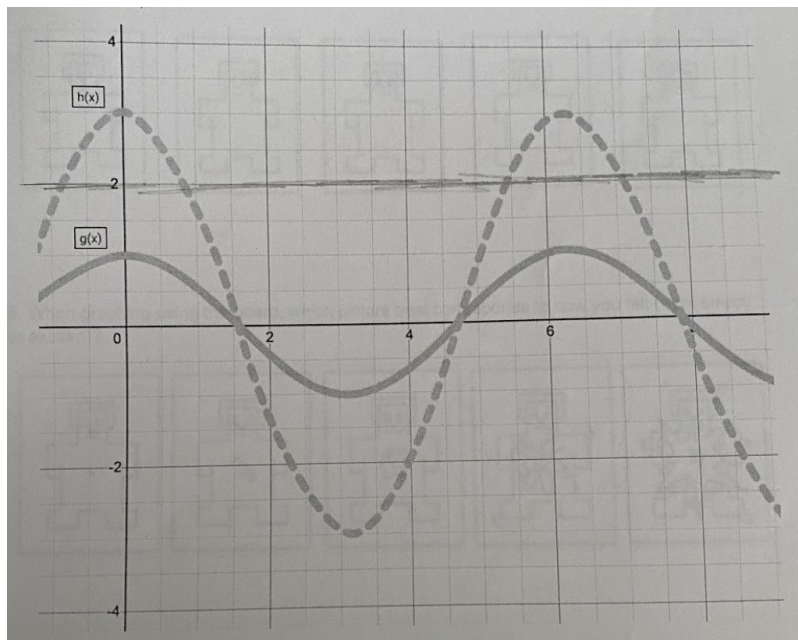


Figure A.2

Example Response with a Score of 0.75

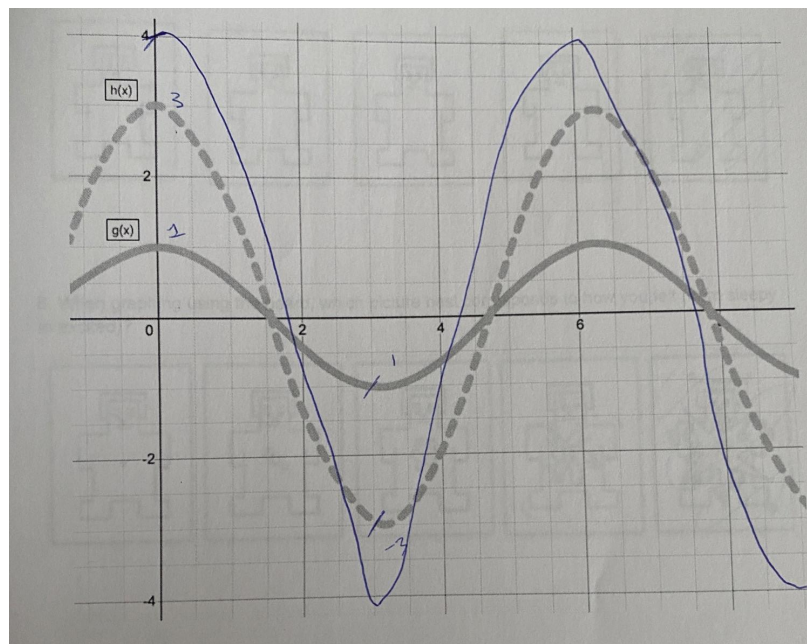
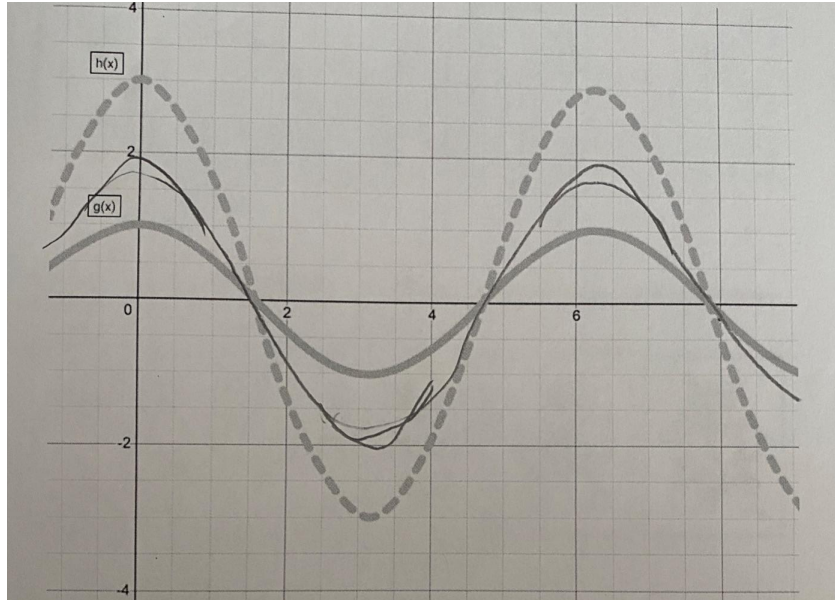


Figure A.3

Example Response with a Score of 1



Appendix II: Transcript Excerpts Accompanying Figure 3

The following transcript excerpts show the evolution of coordination strategies of a group of three students, Marisa, Adriana, and John, highlighted in Figure 3.

Figure 3a: High-amplitude sinusoid challenge, second trial

Prior to getting on the boards, students discuss their plan to have the blue grapher move slower than the orange grapher, inspired by how much they felt the sum line “followed” each graph in their last round. Adriana and Marisa sit down.

- 1 Marisa: Which way should we start?
- 2 Adriana: This way. [Points left]
- 3 Marisa: Left?
- 4 Adriana: Yea, let's go this way first. [Points left].
- 5 Researcher 2: Ok, are y'all ready?
- 6 Adriana: Yeah.
- 7 Marisa: Mhmm.
- 8 Researcher 2: Ready, Three... Two... One... [initiates graphing. Adriana and Marisa rock left together at first. Adriana rocks more slowly, getting out of phase with Marisa.]
- 9 Adriana: That was so much different than the first one.
- 10 Researcher 1: Nice ok. So what- what happened this time?
- 11 Adriana: The black line didn't go as like low or high as we expected it to.

Figure 3b: High-amplitude sinusoid challenge, fourth trial

Adriana and Marisa miscommunicate with each other about the plan for trial number 3. Both rock left, but only one then rocks right, leading to some confusion. The researcher acknowledges that their initial joint rocking to the right led the sum graph to reach the lower limit of the graphing area.

- 12 Researcher 1: So if you wanted to do that, get all the way to the bottom [gestures towards lower graphing area shown] and then get all the way to the top [gestures towards upper graphing area] in the same round, then what do you think you would do?
- 13 Adriana: Go this way? [Rocks left].
- 14 Marisa: I feel like first start left or right [rocks gently right] and then one of us says [starts rocking left towards center, pauses] when to turn to the other side [rocks gently left] so it'll go up?
- 15 Researcher 1: Uh huh
- 16 Teacher: Did you understand what she said?
- 17 Adriana: Not really.
- 18 Teacher: Can you say it again?
- 19 Marisa: Well, I said if you start right, we go right [rocks right] and then me or her, you or me, say "left", we go left [rocks left], so then it goes higher [raises right hand upwards], and then we say "right" and go to the right [rocks right], so then it goes [points downwards with right hand]- so then the black line goes up and then down [traces a sinusoidal shape in the air with right pointer finger]. So if we go at the same time, it goes down. But once we're not going the same way, it starts-
- 20 Teacher: So when you go at the same time what happens?
- 21 Marisa: The black line goes more down [rocks slightly right], and upward?
- 22 Teacher: John, what do you think?
- 23 John: I agree with it, [inaudible]-
- 24 Teacher: You agree?
- 25 Researcher 1: So you're proposing that you both rock right together, and then one of you goes, "switch," then you go left together...is that what you're proposing?
- 26 Marisa: Yeah.
- 27 Researcher 1: What do you think?
- 28 Adriana: That sounds fine.
- 29 Researcher 1: Whose gonna do the-
- 30 Adriana: Like do a count off? Or what. Should we go like this? [Rocks right] One, two, three. [Starts rocking left] and then you go like this then one, two, three. [Rocks right]. I feel like it'll be easier to count off.
- 31 Teacher: Oh!
- 32 Researcher 1: Nice, I like it.
- 33 Adriana: Can you try that? Ok. So we're starting off right. [Marisa rocks slightly right]. Or left? [rocks left].

- 34 Marisa: Umm...
- 35 Adriana: Let's go this way [rocks right]. Ok ready?
- 36 Researcher 2: Three... Two... One... [Starts graph. Adriana and Marisa rock right.]
- 37 Adriana: One, two, three. [Adriana and Marisa rock left together]. One, two, three. [Adriana and Marisa rock right together, generating the graph shown in Figure 2a].
- 38 Researcher 1: Hey!
- 39 Teacher: Woo! Wow, almost off the thing, nice job.
- 40 Researcher 1: Nice work, that's beautiful. Ok, so, did it work how you expected?
- 41 Adriana: Yeah
- 42 Researcher 1: So why do you think you got it so nice and neat this time?
- 43 Adriana: cos maybe it's like... more synchronized, I think?

Figure 3c: $Y=0$ challenge, second attempt

The students successfully generate a first solution, in which both blue and orange keep their boards flat to generate functions of $y=0$, and the sum line "followed" to also make $y=0$. Marisa and John are on the boards.

- 44 Researcher 1: See if you can think of one other way to make the line totally straight other than the one you just did where you both stay still in the middle.
- 45 Adriana: Okay-
- 46 Researcher 1: You have an idea?
- 47 Adriana: Like, um what if you, you go this way [points right with right hand], and he goes this way [points at John and points left with left hand].
- 48 John: You mean like... [rocks left]
- 49 Adriana: You tilt left [points to John], and you tilt right [points to Marisa]. I don't know if that's it, but it's like-
- 50 John: So like tilt all the way to the right? [holding board tilted left].
- 51 Marisa: And then switch sides?
- 52 John: Yeah [rocks right] and then...
- 53 Adriana: Maybe.
- 54 Marisa: So like, just try going like all the way. [John rocks as far as the board can rock].
- 55 Adriana: And just sitting there, cause maybe it'll balance the two out?
- 56 John: [Holds board angle at a left tilt. Raises right hand pointer finger] and so if she goes right [draws finger downwards. Marisa leans slightly right], then I'll go up in the top [draws finger upwards]. And if I go left [moves finger left], then I go in the bottom [draws finger towards central lower graphing area] and then the black like just like moves [positions right palm rightwards, wiggling it up and down slightly]. I think! Well, I'm not really sure. We could try it and see. [Marisa leans farther and farther right].

57 Researcher 1: Ok, so the idea we are going to try is you're going all the way in one direction?

58 John: Yeah, all the way left, and you go all the way to the right. [John is still holding at extreme left position, and Marisa at extreme right position].

59 Researcher 1: Alright! Let's see what happens. [John and Marisa return to balance].

60 Marisa: K ready? Three... Two... One... [Researcher initiates graphing. John rocks to the left and holds the angle. Marisa rocks to the right and holds the angle.]

61 Adriana: It's not...

62 Researcher 1: Ok so, I heard you say it's not exactly what you expected. What's different about it than what you expected?

63 Adriana: I don't think fully expected it to be in the middle, but um...

64 John: I feel like it was somewhere in the middle.

65 Adriana: They're all like, semi-straight.

66 John: Like the black line.

67 Researcher 1: Yeah, it's pretty straight, so that's kind of how you expected it.