Embodied Icosahedron Activities[1]

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Goals

- Learners will construct icosahedra of varying scales and of various materials.
- They will then use these various models to think through questions about the geometric properties of icosahedra (number of edges and vertices, cross-sectional shapes, etc.).
- Learners will reflect on how properties of scale and materiality shape our thinking and reasoning processes.

This activity collection is motivated by theories of distributed and embodied cognition, which highlight the role of physical experiences with the world in thinking and learning processes. At the extreme, stepping inside an icosahedron affords different perspectives than holding one in your hand.

Activity Stations

Each station gets an icosahedron die.

A) Paper nets* – 2-4 people
B) Golden rectangle icosahedra* – 2-4 people
C) Snap-together triangles* – 3 people
D) Foam balls & toothpicks* – 2 people
E) Magnetic sticks and balls* – 3 people
F) Small wooden rod construction – 3 people
G) Large wooden rod construction – 5 people

*With smaller groups of people, these stations are optional. Try to include at least one for a stronger diversity of experience.

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[1] Abrahamson, D., & Rosenbaum, L. F. (2016). Embodied Icosahedron Activities. EMIC Working Group, PME-NA.

A) Paper Nets (2-4 people)

Materials:

1. Printed icosahedra templates from https://www.polyhedra.net/pdf/icosahedron.pdf (Appendix A)
2. Scissors
3. Clear tape or glue stick

Preparation:

Print out the nets on card stock or heavy paper.

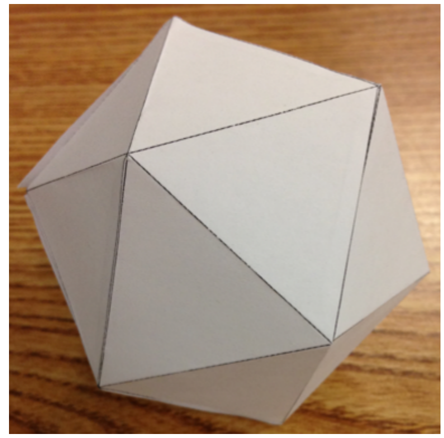


Figure 1. A paper net icosahedron.

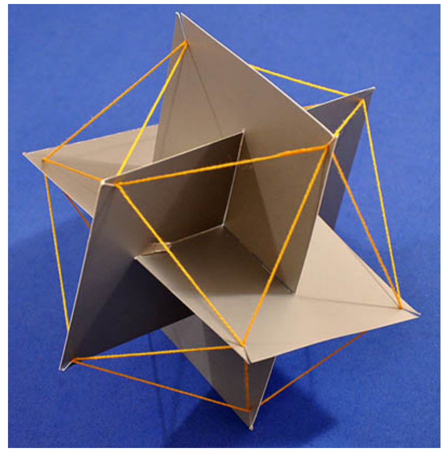
Assembly Instructions:

1. Cut the net out from the paper.
2. Fold along the lines.
3. Tape/glue the trapezoid tabs to adjacent triangles, always making raised pentagons (Figure 1).

B) Golden rectangle Icosahedra (2-4 people)[2]

Materials:

1. Printed golden rectangle templates, 3 for each icosahedron (see Appendix B)
2. Heavy card stock or box board, as from a cereal or tissue box
3. Scissors
4. String such as embroidery thread



Preparation:

Print out the templates. Use them to cut 3 golden rectangles, with corner and midline slits, for each icosahedron. Cut the second slit on only 1 rectangle (see template).

Assembly Instructions:

See http://cutoutfoldup.com/805-icosahedron-from-three-golden-rectangles.php for an instructional video.

1. Slip the three rectangles together along the midline slits.
2. Pass craft thread through all the corner slits, connecting them in any order.

The thread and the rectangles' short edges form an icosahedron!

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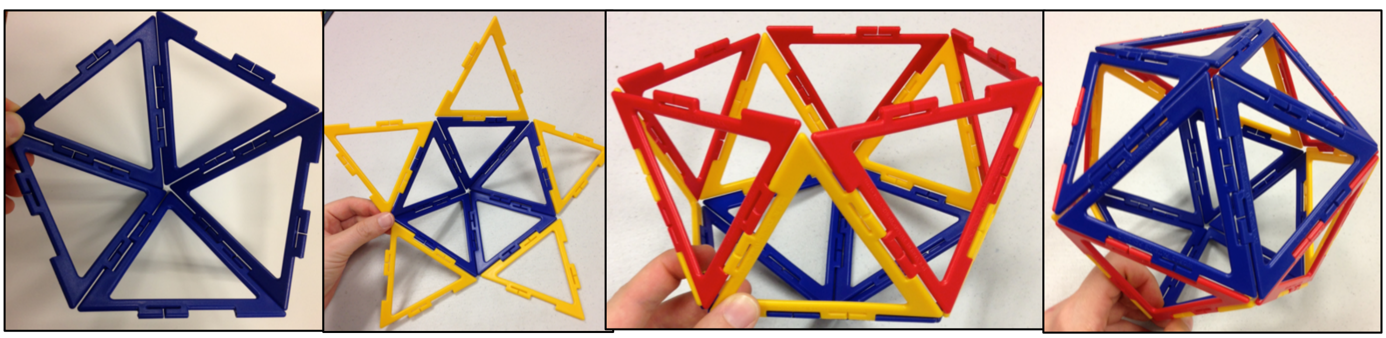
[2] This activity was created by coutoutfoldup.com

C) Snap-together Triangles (3 people)

Materials: Equilateral triangles that snap together, such as Polydron Frameworks (see Figure 2 below).

Assembly Instructions:

1) Connect 5 triangles to form a raised pentagon (Figure 2a).
2) Connect a triangle to each edge of that pentagon, forming 5 “teeth” (Figure 2b).
3) Insert a triangle between each pair of teeth (Figure 2c).
4) Create a second raised pentagon. Use it to close the icosahedron by attaching to the triangles from Step 3 (Figure 2d).



a b c d

Figure 2. a) Raised pentagon. b) “Teeth” off the raised pentagon. c) Triangles between the teeth. d) Close the icosahedron with a second raised pentagon.

D) Foam Balls and Toothpicks* (2 people)

*Note: this is *the most difficult* of the materials to work with and will likely be very challenging for all age groups/experience levels.

Materials (Figure 3):
1) Toothpicks, at least 50
2) Small foam balls, at least 50

Assembly Instructions: see Vertex-Edge Assembly Instructions below

Figure 3. Toothpicks and foam balls.

E) Magnetic Sticks and Balls (3 people)

Materials: Small magnetic sticks and balls (Figure 4a) as in the PlayMaty Magnetic Toy Building Rod kit (Figures 4b, 4c).

Assembly Instructions: see Vertex-Edge Assembly Instructions below

Figure 4. a) Magnetic rods and balls. b) The PlayMaty magnet kit. c) Contents of the magnet kit.

F) & G) Small and Large wooden rod constructions (3 people for small; 5 people for large)

Materials, Preparation, and Vertex-Edge Assembly instructions provided by B. Cheng (personal communication, May 25, 2016).

Materials:

Clear vinyl tubing, wooden rods, and at least 12 standard sized cable ties (aka zip ties).

Size	Wooden Rods			Vinyl Tubing (Vertices)			
	Length	Diameter	Count (2 extra)	Inner Diameter (ID)	Outer Diameter (OD)	Total length	Segment length
Large (stand inside)	48 inches	5/16th inch	32	5/16th inch	7/16th inch	16 feet	6 inches
Small (head inside)	12 inches	¼ inch	32	¼ inch	3/8th inch	15 feet	5 inches

Preparation:

1) Cut tubing into segments of indicated length using a pair of good scissors or a large wire cutter.
2) Take 3 tube segments and hold them together with the ends flush. Secure with a cable tie around their middles and pull super tight (see Figure 5). Trim extra cable tie. Repeat to create 12 total bundles.

Disassembly tip: Tugging the pipe off the rod results in the famous "Chinese Finger Trap" effect. Instead, *pry the tube slightly away from the rod* with your fingernails while pulling and *twisting the rod out of the tube*.

Figure 5. One bundle of tubing

Vertex-Edge Assembly Instructions (for D, E, F, and G):

1) Create a 5-spoked star by connecting 5 toothpicks/sticks to one ball/magnet/bundle of tubes (Figure 6a).

2) Connect the spokes of the star with 5 more toothpicks/sticks, creating a some-what raised pentagon (Figure 6a).

3) Add 2 toothpicks/sticks to each edge of the pentagon and connect them with a ball/joint. This should result in a triangular "tooth" coming off each edge of the pentagon. Two such teeth are illustrated in Figure 6b. Create a tooth off each edge of the pentagon for a total of 5 teeth.

Figure 6a. The 5-spoked star with edges connected from Step 1.

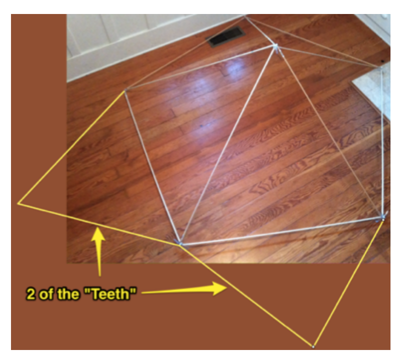


Figure 6b. Two teeth off the 5-spoked star. Create 5 such teeth.

4) Connect the teeth tips with 5 more toothpicks/sticks. You may need to turn the original pentagon on its side and close in the teeth slightly. These new sticks connecting the teeth tips will form a new pentagon rim.

5) Finally, make another 5-spoked star (without a rim), and use it to close off the icosahedron from the new pentagon rim formed in Step 4.

Questions for construction-and-analysis session:

1) How many sides/faces does an icosahedron have? [20]

2) How many vertices does an icosahedron have? [12]

3) How many edges does an icosahedron have? [30]

4) Any parallel edges in an icosahedron? [Yes, 15 pairs.]

5) If the icosahedron were half full with water, what would be the shape of the surface of the water when the icosahedron is on a triangular base (Figure 3a)? What about when the icosahedron is tilted onto a single vertex (Figure 3b)? [The answers are 12 and 10, respectively]

Figure 7a. Icosahedron on triangle base.

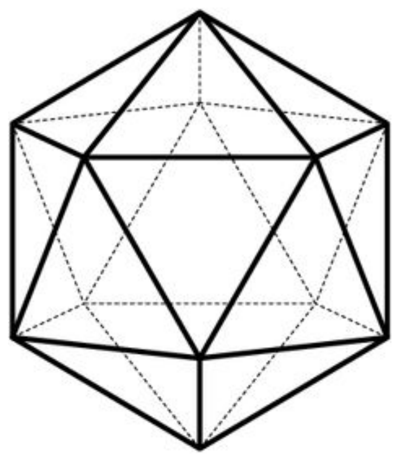


Figure 7b. Icosahedron on a single vertex.

Simulator available here: https://www.geogebra.org/m/hKsYEf73

Questions about relation between media and concepts.

1) Which media were easier or harder to use, and why?

2) Which, if any, media, were easier or harder for addressing some of the questions above?

3) What, if any, different behaviors did you see around you as people were interacting with the media to solve the questions (e.g., opaque vs. transparent, individual vs. group work, counting vs. calculating, etc.)?

Emergent Research Themes for Researchers, Practitioners, and Graduate Seminars

1. Construction, Transparency, and Cognition: What you don't build, you don't know. What comes ready made, you don't need to figure out. See:

Chase, K., & Abrahamson, D. (2015). Reverse-scaffolding algebra: Empirical evaluation of design architecture. In A. Bakker, J. Smit, & R. Wegerif (Eds.), Scaffolding and dialogic teaching in mathematics education [Special issue]. *ZDM Mathematics Education, 47*(7), 1195-1209.

Meira, L. (1998). Making sense of instructional devices: The emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education, 29*(2), 129-142.

Other Fun Facts

The icosahedron is the dual of the dodecahedron (Figure 8). Mark the middle of each face of one of them, connect the dots, you get the other!

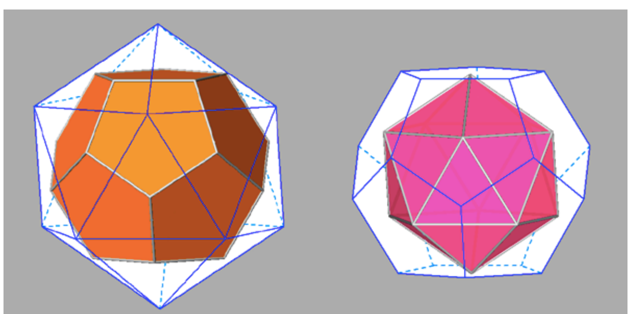


Figure 8. A dodecahedron created by connecting the faces of an icosahedron (left). An icosahedron created by connecting the faces of a dodecahedron (right).

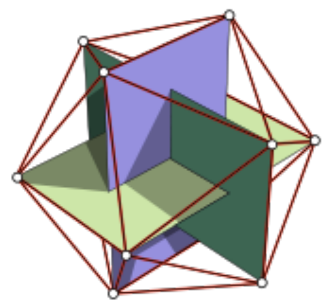


Figure 9. An icosahedron formed from golden-ratio rectangles.

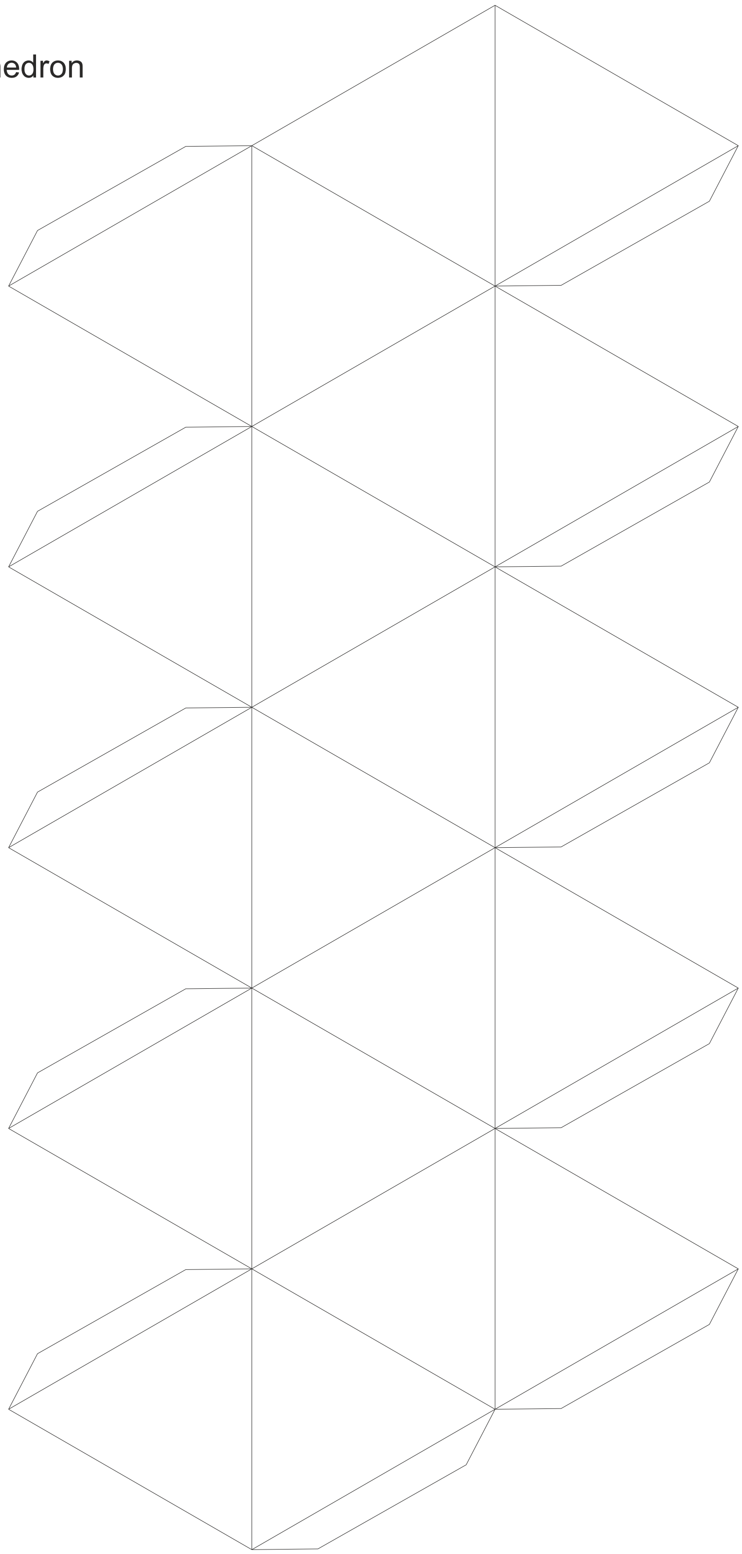
An icosahedron can be created by connecting the 12 vertices of three orthogonal, identical, golden-ratio rectangles as in Activity B (Figure 9).

Rudolf Laban developed a spatial theory framework based on the icosahedron. Part of his practice involved movement scales in which dancers performed choreographed exercises in a large icosahedron (Figure 10).

Figure 10. A dancer following Laban practice in an icosahedron.

Appendix A – Paper Net Templates

Icosahedron



Appendix B – Golden Rectangle Template (x3)

3 ×