Balance Board Math: "Being the graph" through the sense of balance for embodied self-regulation and learning

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ABSTRACT

Balance Board Math (BBM) is a new balance-based interface for math instruction. BBM integrates disparate work on embodied cognition and on sensory regulation to offer learners integrated opportunities to both self-regulate through movement and to use their sense of balance as a resource for conceptual understanding. This approach imagines beyond common views that self-initiated background activity, such as fidgeting, is unproductive for education. With a sensor-equipped balance board and dynamic real time display, BBM's Balance Graphing activities offer users opportunities to playfully explore and embody different aspects of functions and graphs such as frequency and amplitude. We conducted an in-depth study with 6 school-aged children to examine how their movement and personal sense of balance were used for both self-regulation and to make sense of mathematical concepts through BBM. By inviting learners' regulatory movements to serve as an interaction resource for exploring mathematical concepts, BBM offers a new genre of sensory-responsive design that could better serve instructional differentiation.

CCS CONCEPTS

• **Human-centered computing**; • **Human computer interaction (HCI)**; • **HCI design and evaluation methods**;

KEYWORDS

Balance, Mathematics, Graphing, Sensory regulation, Vestibular, Embodied Interactions, Balance boards

ACM Reference Format:
Sofia, M., Tancredi, Julia, X., Wang, Helen, T., Li, Carissa, J., Yao, Genna, L., Macfarlan, and Kimiko, Ryokai. 2022. Balance Board Math: "Being the graph" through the sense of balance for embodied self-regulation and learning. In *Interaction Design and Children (IDC '22), June 27–30, 2022, Braga, Portugal.* ACM, New York, NY, USA, 13 pages. https://doi.org/10.1145/3501712.3529743

IDC '22, June 27–30, 2022, Braga, Portugal

ACM ISBN 978-1-4503-9197-9/22/06.
https://doi.org/10.1145/3501712.3529743

1 INTRODUCTION

Many children crave movement, yet traditional classrooms demand they sit still. Research in occupational therapy and psychology suggests that movement such as rocking and fidgeting can have positive effects on self-regulation and sustained attention by supporting an underlying need for sensory stimulation [1]. What if instead of suppressing spontaneous movements, we made them *part of* mathematics instruction? What if children were empowered to engage their own sense of balance in exploring mathematical concepts? *Balance Board Math (BBM)* aims to cultivate a context where sensory regulation and learning activity are integrated. With the BBM system, children generate dynamic graphs on a large display by rocking on a wooden balance board (Figure 1). BBM offers a variety of activities where children can playfully explore the relationship between their physical movements and graphical visualization of their movements in real time. By controlling mathematical forms through movement, BBM users can reflect and refine their thinking about how different parameters of graphs such as amplitude, frequency, and function work as a system. We contribute our design and evaluation of BBM and discuss implications for sensory accessibility, investigating the balance sensory system as an untapped regulatory and conceptual resource in educational research.

1.1 Background

From different theoretical perspectives, movement in learning contexts is modeled as functioning for cognition and regulation. Embodied cognition theory views the body as playing a central role in cognitive processes. Far from one cohesive theoretical stance, embodied cognition enfolds a range of perspectives from more conservative [2] to more radical (e.g., [3]), spanning a range of foci, notably the "four Es": mind as embodied, enacted, embedded, and/or extended [4]. *Embodied* perspectives have been fruitfully applied to education [e.g., 5], notably to the role of gesture in thinking and learning [6, 7]. Gesture research in mathematics has found gesture to reflect students' thought processes [8] and even their mathematical expertise: skilled graphers were found to describe graphs from a different perspective than unskilled graphers, immersively "being the graph" with their bodies, not merely "seeing the graph" and tracing it as if on a page [9]. *Enactive* perspectives conceptualize cognition as emerging from sensorimotor activity: per [10], "(1) perception consists in perceptually guided action and (2) cognitive structures emerge from the recurrent sensorimotor

Figure 1: The Balance Board Math system. The child shifts their balance on the balance board, and these shifts dynamically change the height of a graph being generated on a large display in front of them. The display highlights different features of their graph in different activities such as amplitude, function, and frequency explorations, leading the child to explore different parameters of their graphing.

patterns that enable action to be perceptually guided" (p.173). Enactivist views of mathematics analyze mathematical thinking as overtly and covertly embodied, highlighting how even purportedly "abstract" symbols objectify what are in fact multimodal notions (for example, the "+" symbol denotes the process of grouping together), and are manipulated as if they were objects [11]. From an enactivist perspective, sensorimotor activity forms the basis of math thinking and learning.

Separately, theories of sensory modulation and integration highlight the role of movement in self-regulation. Prominent models in occupational therapy assert that individuals require different intensities of sensory stimuli to support optimal functioning [1], with neurodivergent populations more likely to fall at the poles. One individual's felt experience of something might differ from another's due to the sensitivity of their neurological thresholds, yielding different sensory preferences and requiring different amounts of stimulation for self-regulation [1]. *Sensation seeking*, or *self-regulatory movement* describes active attempts to meet one's sensory needs through activities such as rocking, pacing, and fidgeting that fall outside of the direct scope of the task at hand [1]. The implications of this model for education are that mismatch between sensory needs and task/environment can adversely affect learning. Research on neurodivergent populations offers some evidence of this: sensory processing differences in sensation seeking and auditory processing explain 47% of academic achievement variance among children on the autism spectrum [12], and among children with ADHD, higher levels of gross motor activity has been correlated with improved working memory performance [13]. From a sensory regulation perspective, sensorimotor activity forms the foundation of self-regulation that supports academic performance.

Bringing together embodied/enactive cognition and sensory regulation perspectives, and with an eye towards the full sensory spectrum, especially those at the most sensory seeking end who are at odds with common educational practices, we ask: If cognition is a sensorimotor activity, and regulation through sensorimotor activity is ongoing, how might these processes interact? Are sensory-regulatory and cognitive processes indeed always distinct, or might regulatory activity interact with *or even participate* in cognitive processes? A prime context for exploring movement for regulation and cognition is the vestibular sensory system in the inner ear, which detects balance, orientation, and acceleration. The vestibular sense has been implicated in both self-regulation [14] and cognitive development, including spatial and numerical cognition and arithmetic [15], and has been largely neglected in educational research. In this work, we seek to integrate regulatory and conceptual views of sensorimotor activity in learning and explore their intersections through a design that offers learners vestibular activity with both conceptual and regulatory affordances.

1.2 Related work

Prior work in Human-Computer Interaction has explored movement as a regulatory and learning resource, separately. Work on movement-for-regulation examines embodied self-regulation in the margins of computing space [16] and the effect of physical exertion on enjoyment (e.g., [17]). Self-stimulatory movements such as hand-flapping have been explored as inputs for game locomotion with autistic participants [18]. The sense of balance has been highlighted as a design resource [19], drawing upon balance boards for exercise (e.g., [20, 21]), therapy, rehabilitation [21–24], co-regulation [25], and recreation [26, 27]. With regards to movement and conceptualization, [28] examined the impact of a a balance-based input device abstract reasoning about questions of justice, applying theory on body-based conceptual metaphors as the foundations for abstract thought [29]. [30] used a balance board input device for seesaw physics problems. We build upon these related works by integrating regulatory and conceptual affordances of embodied, balance-based interactions in the context of math instruction.

Common solutions in schools to accommodate learners' sensory needs offer stimulation outside of, or in parallel to instructional activities: breaks, recess, sensory therapies [31], and/or specialized tools such as fidget toys or alternative seating [32]. We propose that these solutions fall short of their full potential for supporting classroom conceptual learning in that there is limited attention to how completing regulatory activities in parallel might affect the cognitive load [33] imposed on students, or to the regulatory impact of instructional activities themselves. Looking to the sensory qualities of math instructional designs, work inspired by embodied cognition has expanded upon a history of bodily involvement in early education [34, 35] to call upon the body as a learning resource across ages and contexts (e.g., [36, 36–39]). Recent technologies such as motion sensors have further expanded whole-body interaction possibilities [40, 41]. The types and intensities of sensory stimulation offered in embodied design activities range from small movements such as finger-gestures (e.g., [42]) to immersive whole-body activities such as gym-scale number lines [43]. Very few offer substantive vestibular stimulation. The field does not yet expressly attend to sensory dimensions of these designs as impacting regulation.

Movement-based instructional designs present a spectrum from explicit instruction of specific movements (e.g., [44]) to creative movements (e.g., [45]). The *action-based embodied design* genre falls between these poles [46]: learners figure out for themselves how to move in a new way given a task and set of resources. An example design is the Mathematics Imagery Trainer for Proportion (MIT-P), wherein learners control the height of two points on a display and try to figure out how to make the display turn green, where green is associated with their two points standing in a target ratio (for example, 1:2). With exploration and practice, users learn to move their hands continuously while maintaining the target ratio, generating a new movement pattern wherein the gap between the two points grows as the points move upwards, a basis for proportional reasoning [47]. The present project instantiates action-based embodied design with a focus on actions that stimulate the vestibular sense.

2 BALANCE BOARD MATH

2.1 Design Principles

To integrate regulatory and conceptual views of sensorimotor activity in mathematics learning, we defined the following design principles, described further in subsequent section 2.1.1-2.1.4:

- Foster movements that ground mathematical concepts in experiences of balance
- Support learners' discovery and control of dynamical properties
- Invite embodied self-regulation during and through instructional activities
- Inclusively adapt to different sensory profiles

2.1.1 Foster movements that ground mathematical concepts in experiences of balance. Following action-based embodied design [28], we look to ground mathematical concepts by leveraging children's capacity to enact complex movements (see also [46, 47]). BBM aims to invite children to explore graphs through their movement and sense of balance. A balance board input device stimulates the vestibular sense as part of multimodal experience. The child's movement on the balance board is translated into a real-time sinusoidal graph on a large display in front of them so that the child creates a mathematical representation of their board movement. During one round, the graph progresses steadily across the screen in the x axis, and the board's angle controls the graph's y value. Rocking left/back dynamically increases the y value; rocking right/down decreases it. This real-time relationship between the children's whole-body rocking and the digital visualization in front of them creates a dynamic environment within which to investigate whether children can truly embody and, therefore, "*be* the graph" by rocking its form [9]. The goal is to encourage multimodal exploration of graphs, mapping different aspects of rocking to graphical features.

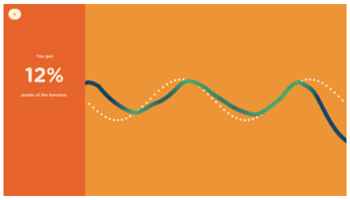
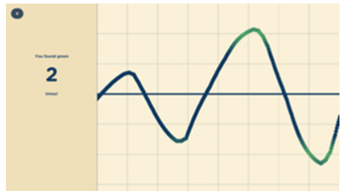
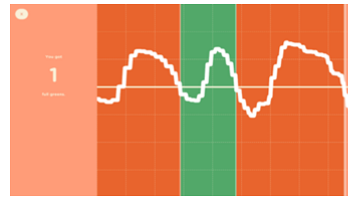
2.1.2 Support learners' discovery and control of dynamical properties. In action-based embodied design, instructional designs create a context of goal-directed activity within which focal concepts are detectable by children as consistent dynamical properties [28]. This entails inviting children to enact continuous, exploratory movement in an environment where feedback is consistently associated with a property of interest. In the example discussed in section 1.2 of the MIT-P, children are invited to try to "make green," where green color is associated with the invariant property of ratio [28]. This approach shares some foundations with constructivist [48], discovery-based [49], and constructionist pedagogies [50] that view learning as learner-driven exploration.

In BBM, participants receive open-ended prompts, such as "try to draw as much green as possible." In each activity, children figure out how to dynamically conserve a focal property such as the amplitude of their rocking. Conservation of these properties is reflected in consistent color and sound feedback. To make green, the child's movements must generate a graph that aligns with either a focal *function, amplitude,* or *frequency,* each of which can be modified (Table 1). For videos of each activity in action, see Appendix A.1. While maintaining the focal parameter, children can vary other aspects, such as varying amplitude while maintaining frequency. After Function Exploration, the other activities introduce a grid and numbers to support children in refining and discussing rocking. For example, the gridlines in the Frequency Exploration activity can help evenly subdivide the x-axis of the graph, a prospective support for rocking at a consistent frequency (Table 1). This follows action-based embodied design guidelines whereby cultural tools are introduced as resources that children can appropriate for their situated action [28].

Children are invited to attempt activities multiple times–once reaching the end of the screen, they receive feedback on what percent or number of times they made green and can reflect and try again. Using this feedback, children can generate and test their own hypotheses for how to maximize the target feedback. Drawing inspiration from ethnomethodological conversation analysis perspectives on inclusive instruction [51], we see collaboration as a context for children to find shared ways to make sense of and communicate about their movements together. Collaborative modes in BBM include taking turns on a balance board and 2-board split-screen mode.

2.1.3 Invite embodied self-regulation during and through instructional activities. Meeting one's sensory needs underlies learners' capacity to self-regulate and attend [12]. Children often do so through moving their bodies, as with fidgeting and rocking [1]. HCI work

Table 1: BBM Base Activities

Activity	Function Exploration	Amplitude Exploration	Frequency Exploration
Visual Output			
Conceptual mapping	A sinusoidal function is experienced as stable and symmetrical rocking.	A function's amplitude is experienced as degree of lean to the left or right	A function's frequency is experienced as the speed of rocking and number of rocks in a given time interval.
Goal	To trace a given function (white dotted line) by rocking	To rock at a certain amplitude	To rock at a certain frequency
Color feedback	A blue-green line color gradient corresponds with the user's accuracy or inaccuracy in tracing the predefined function.	The parts of the user's function that meet the target maximum and minimum turn green, while the rest of the function remains blue.	A green-red background color gradient for each period corresponds with how closely that period matches the expected rocking frequency.
Numerical feedback	Percent match between their graph and the function sin(x).	Number of times reaching the target amplitude.	Number of times matching the target frequency.
Sound feedback	Presence/ absence of a harmonic tone corresponds with the user's accuracy in tracing the predefined function.	Changes in pitch correspond with changes in the user's tilt. A "counter" noise plays each time the user meets the target maximum or minimum.	Different tones play for each local maximum, minimum, and passage through the x-axis. A chord plays each time the user completes a period, which is progressively more major and less minor as the user's period matches that of the target frequency.

suggests that embodied self-regulation is ubiquitous in the general population and that self-regulation tools are a fruitful design space in their own right [16]. We are committed to welcoming children's regulatory activity into the learning space. Unlike previous work, BBM invites self-regulation directly into the learning process. Rocking, a common regulatory activity, especially for sensory-seeking learners, directly participates in the learning process because BBM uses rocking as the central form of interaction, in contrast to traditional input devices such as a keyboard, mouse, or touchscreen. The activities are designed to offer the kind of repetitive, stimulating, and self-modulated dynamics prevalent in self-regulatory movements like fidgeting. Additionally, by sitting on a balance board, a common regulatory device, children are implicitly welcomed to rock on the board during and between activities as a means of self-regulation. In this way, rocking serves multiple purposes in BBM: it allows for children to physically embody sinusoidal graphs while *also* allowing for self-regulation.

2.1.4 Inclusively adapt to different sensory profiles. BBM follows inclusive design [52], seeking to expand the spectrum of sensory modulation profiles accommodated by instructional designs. We offer amplified stimulation opportunities for *sensory-seeking* students who crave movement. At the same time, we aim to design learning contexts where students across the sensory spectrum can tune to their preferences. To do so, the activities are programmed with adjustable graphing speed and sensitivity so that participants can rock in ways commensurate with their sensory profile. BBM also offers feedback through vision and sound. Additionally, BBM allows flexibility of body position: children may plant their feet on the floor for a more controlled rock, or sit with their legs crossed for more range, may rock left-to-right or front-to-back, and may hold the board's rim or interior handles.

2.2 Technical Implementation

Children interact with BBM primarily via a wooden balance board. We use Southpaw's Rocker Balance Board (28" L x 25" W x 7.5" H), which is made of wood and has a carpeted mat on top (Figure 2). The large surface and gently curved base allow children to rock on the board with stability and comfort. We attached foam balls to the board's sides to prevent learners from injuring their fingers and cushion extreme rocking. A wooden compartment affixed to the bottom of the balance board houses a WitMotion Inclinometer sensor to sense board angle and an Arduino Mega 2560 to send sensor data to a custom BBM web application via USB-connection. The BBM web application is projected in front of the user.

3 STUDY

3.1 Research questions

We evaluated whether children's interactions with BBM fulfilled its core design principles by investigating the following research questions:

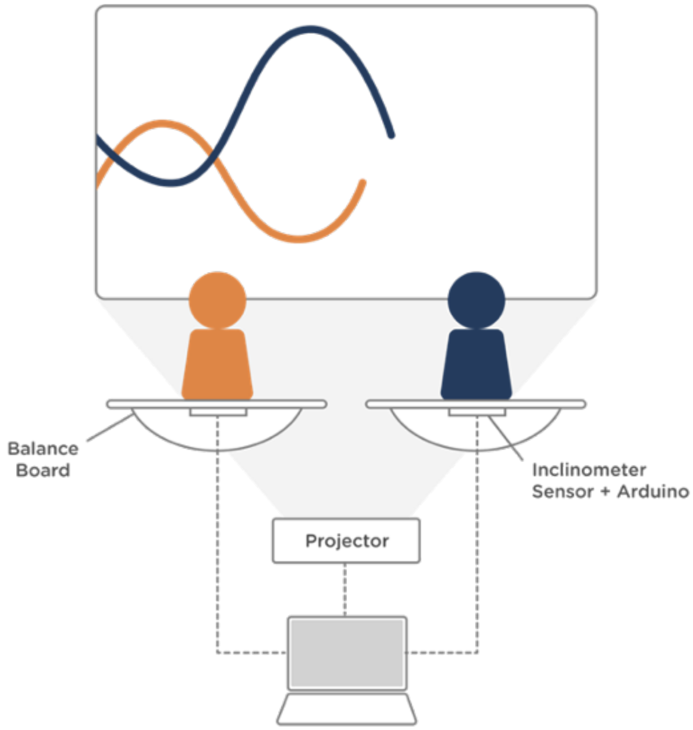


Figure 2: BBM system diagram

- How do BBM's unique interaction design features facilitate children's whole-body embodied explorations of functions and graphs?
- How do children engage in discovery-based learning of math concepts when using BBM?
- How do children engage in embodied self-regulation when they are invited into the BBM learning space?
- How do children's different sensory modulation profiles interact with the BBM platform?

3.2 Participants

Children in kindergarten through grade 12 were eligible for this study. 6 participants in grades 2 through 6, two girls and four boys, participated. Interviews were conducted in a lab setting and at a learning center. In the lab setting, only the participants, the interviewers, and the participants' guardians were present. The interviews at the learning center took place in a partitioned cubicle within a larger, multipurpose indoor space. Following Covid-19 guidelines at the time of data collection, masks were worn by all present. Two participated individually (pseudonyms: Maria and Ben), and four participated in pairs (pseudonyms: Kyle and Cory, Lyla and Arthur). Four participants stayed for post interviews. We ran participants until at least 3 students identified as sensory-seeking and 3 not were included. Learning center instructors with training in sensory profiles identified three participants as sensory seeking (Maria, Ben, and Arthur). We asked parents and students informal sensory questions to corroborate that the other three participants did not exhibit sensory-seeking tendencies. Participants' special education status was not collected in this study.

3.3 Methods

We conducted task-based, semi-structured interviews centered around the different Balance Board Math activities. The interview began with a calibration phase where participants were invited to rock on the board. Children were invited to rock fast and slow, big and small, and to try out different positions of their bodies and the boards until they felt comfortable. Next, for each activity, the on-screen prompt such as "see how much green you can find on the screen" was stated aloud, and children were told that they could try each activity as many times as they liked. Between tries while seated on the balance board, when the screen stopped at the graph from their previous attempt, the children were asked questions about their thinking such as "what do you think makes green?". In later tries, children were also asked to reflect and perhaps iterate on their brainstormed solutions. Children in pairs were also encouraged to take turns on the board and discuss their ideas with each other. These sessions were analyzed as a dyad in interaction. In each session, participants tried out the Function Exploration, Amplitude Exploration, and Frequency Exploration activities, except for Cory, who did not try the Frequency Exploration activity. All participants started out with Function Exploration, differing in whether they did Amplitude Exploration or Frequency Exploration next. Each session lasted between 14 and 57 minutes with an average length of 28 minutes. The interviews were audio and video recorded when

Table 2: "Being-the-graph" codes for pre/post graph gestures

Category [53]	Range	Description
being-the-graph index	0-100	An equally weighted average was taken for all 7 "being-the-graph" variables below and converted to a percentage.
placement of the x-axis against the gesture's body	0-1	X-axis placement was binary coded with "throat and above" marked as 0 and "below throat" marked as 1.
acceleration/ deceleration in gestural movement	0-2	Gestural movement acceleration was coded as 0 (no acceleration/deceleration), 1 (some acceleration/deceleration), or 2 (significant acceleration/deceleration).
presence or absence of eye tracking	0-2	Eye tracking was coded as absent (0), partial (1), or consistent (2).
engagement of the spine in movement	0-2	Spinal engagement was coded as absent (0), partial (1), or significant (2).
distal or proximal nature of the gesture	1-3	Gestures were coded as (1) further than elbow distance, (2) within elbow distance from the body, and (3) close to the body (less than elbow distance).
hand usage	0-1	Hand usage was coded 0 if the participant traced with a single finger and 1 if they used their full hand.
parts of the body used to create the gesture	0-8	Each body part type involved in creating the graph's shape counted as 1 point from the following list: hands, wrists, elbows, shoulders, torso/spine, head, hips, legs.

approved by children's guardians and children (five of six participants), and the research team took field notes during interviews. In the lab, one video camera was set up behind the participants to capture the digital output on the screen as well as participants' backs, and one in front of participants to capture their gestures and facial expressions. For sessions in the learning center, only the front camera was functional. All recorded interviews were transcribed.

Before and after participants engaged with the platform, we conducted structured pre and post interviews at a desk in which children were invited to describe three sinusoidal graphs printed on paper using only gesture, then only language. We used questions like "Can you show my friend who can't see it what the graph is like using your hands?" and "how would you describe it to her in words?" We adapted this approach from prior work on children's conceptions of graphing that prompted children to gesture visual graphs [9], and added the secondary verbal description to accommodate kids' desire to verbalize. When children combined both gesture and words, we asked them to repeat only in gesture, then only in words. Pre and post graphs featured one reference function (sin(x) or cos(x)), one higher amplitude function, and one higher frequency function (Appendix A.2). Participants spent between 14 and 49 seconds on each pre interview graph with the average length of 33 seconds per graph. Participants spent between 26 and 71 seconds on each post interview graph with the average length of 38 seconds per graph. All but one of the participants were seated for pre and post interviews. One participant, Ben, preferred to remain on the balance board for his post interview. Children who participated as dyads were split up and interviewed separately.

Video recordings of children's completion of and reflections on the activities were iteratively coded for instances of different movement functions such as *activity-related, regulatory, expressive,* and *cognitive* using the ELAN annotation software. Following fidgeting literature, movements that exhibited no direct salience to the activity at hand, such as tapping feet while waiting, were coded as regulatory. Each movement sequence could receive multiple codes. We compared types of board engagement among participants, and between sensory-seeking and non-sensory-seeking subgroups. Interview transcripts were coded for children's perceptual orientation towards the task using inductive coding. Video recordings of children's pre and post graph interviews were coded deductively using 6 hallmark "being-the-graph" criteria from Susan Gerofsky's 2010 paper (Table 2), which we then compiled into one index variable to compare pre to post. Coding consensus was reached using two videos, then two coders coded all videos. Inter-rater agreement was 86.4%. Videos were blinded for pre/post order to mitigate potential biases. Being-the-graph pre and post index scores were square-transformed to meet the normal distribution assumption, checked with the Shapiro-Wilk test. An F-Test Two-Sample for Variances found no significant difference in variances between pre and post scores, so a paired, two-tailed t-test at the 5% level assuming equal variances was used to compare the mean being-the-graph index pre and post intervention.

3.4 Results

3.4.1 Grounding mathematical concepts in experiences of balance: Towards being-the-graph. In BBM, to achieve desired graphical outcomes, children must coordinate rocking the board with the resulting dynamic graph. We observed that children controlled and transformed their rocking to achieve desired graphical effects. In talking about the activities, all children spoke explicitly about features of their rocking, their produced graphs, and display feedback, frequently making connections between them using deictic gestures (Figure 3). For example, Kyle described the result of a Frequency Exploration round as follows: "I went a lot slower and I only have like three [...] hills, like cycles." Here, he connects his felt experience of rocking *slower* with the *frequency* of his generated graph. Over the course of the activities, children gradually developed more coordinated ways of interacting with and talking about their bodily movement and graphical features.

Figure 3: Children pointing to features of interest in the graphs they have generated

Through coordination of board and display, the children came to experience the graphs themselves multimodally. Their explanations of how to make *green* often came to combine both board-oriented notions such as *speed of rocking* or *angle*, and graph-oriented notions such as *period* or *y-axis location.* For example, Lyla described her Amplitude Exploration strategy as: "try to find the greens but keep the balance." Here, she coordinates the display's color feedback with her embodied experience of rocking evenly. Her solution indicates that she has integrated her personal sensorimotor experiences of rocking evenly in each direction with the graphical form of a sinusoid with a given amplitude. She enters into the concepts multimodally, drawing upon her active sensorimotor experiences. Our findings suggest that by bringing together their vestibular and proprioceptive experiences with the board and the graphical display, children came to draw upon their bodily experience to think about mathematical properties of graphs.

We found that actively "being the graph" on the board transferred to how children made sense of static, printed graphs afterwards. For example, Kyle, who had connected his rocking speed with graph frequency earlier during the Frequency Exploration activity, described pre-given graphs on paper using movement-based language: a lower frequency graph was "slower" and a higher frequency one "quicker;" a high amplitude graph was "rocking a lot farther." As he spoke, Kyle physically rocked his body in his chair in ways that mirrored his descriptions. This suggests that the coordination developed between rocking and features of graphs became a resource for sense-making of static 2-dimensional representations. Each child showed different idiosyncratic changes from pre to post. For example, Kyle, who initially gestured with only one hand from right to left in his pre interview, gestured with two hands from left to right after the intervention (Figure 4, upper). Kyle's gesture direction changed to follow mathematical convention of reading graphical data left to right. His two-handed gesture distinguished motion above the x-axis, gestured with one hand, from motion below the x-axis, gestured with the other, reflecting a newfound attention to vertical symmetry. Ben went from raising his hand for high amplitude graphs to standing up and reaching skywards (Figure 4, lower). The scale of Ben's graph expanded after participation, making more use of his whole body. Ben's expression of high amplitude became more prominent, and the scale of his gestural graph expanded. Despite the variety of graphing gestures, looking

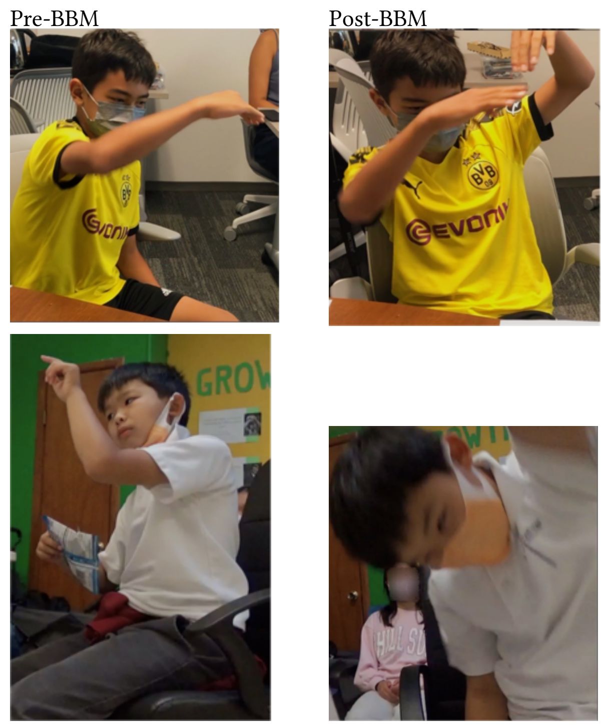


Figure 4: Examples of two children gesturally depicting graphs pre (left) and post (right) participation in the BBM activities. Kyle (upper) moved from gesturing with one hand (pre) to producing larger gestures with two hands (post). Ben (lower) moved from a small deictic gesture to a large movement involving his entire body.

across children, there was a significant increase in mean being-the-graph gesture index scores from pre to post ($t = 2.37$, d.f.=11, $p < 0.05$). This suggests that overall, children's gestures shifted from *seeing* towards ***being*** the graph, consistent with the gestures of more expert graphers [9].

3.4.2 Discovery of focal concepts through dynamical conservation. BBM highlights aspects of graphs with color and quantitative information. Children took up the display's color and quantitative

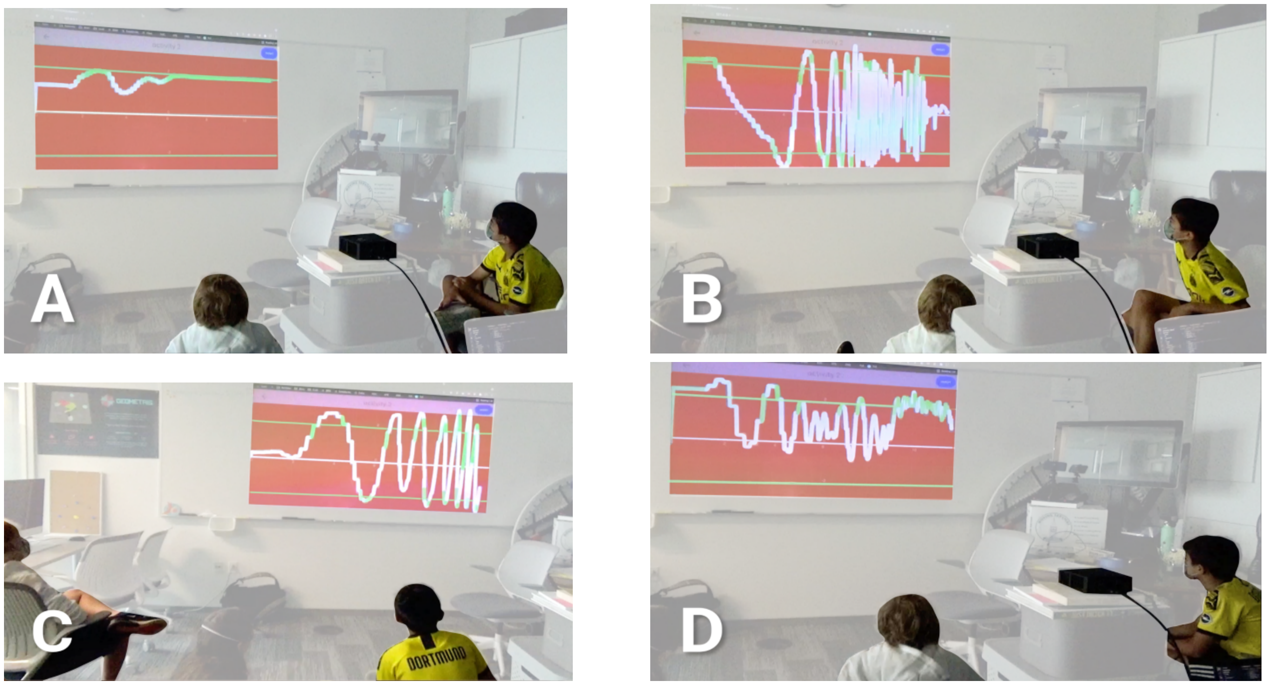


Figure 5: Kyle and Cory jointly explore different hypotheses in the Amplitude Exploration activity.

feedback to formulate, test, reject, and refine their hypotheses about each activity. All children connected observations about their graph with reflections on why certain parts of their graphs were *green*. Kyle and Cory's initial engagement of Amplitude Exploration exemplifies children's open-ended progression of reasoning within activities. First, Cory explores the activity by leaning to one side without returning to the x axis (Figure 5a). He discovers that his graph stays flat and green. However, because Cory's graph never completes a full period, BBM displays, "you met the amplitude 0 times." Cory comments at this stage, "If you stay on the line, it's all green. I don't know if it is the *amplitude*." While he has identified the y=3 reference line, he notices that simply making green in this way does not increase amplitude count Cory then proposes that "you have to *go back and forth between lines*" which he and Kyle decide to test out with another round. This time, Cory rocks from y=3 down to y=-3 (Figure 5b). Cory confirms from this that "every time I hit the bottom it turned green, and every time I hit the top it turned green;" "it makes green if you go straight *from line to line*." This time he reaches the amplitude 32 times. He proposes that the count reflects "it go[ing] *to the top and bottom* as many times as possible." Kyle then takes a turn (Figure 5c), which sparks a new idea about getting a greater number of greens by going "faster with less movement." This idea begins to highlight graphical frequency ("faster") and amplitude ("less movement") (tested in Figure 5d). Observing the resulting graph, Kyle offers that "I feel like it's kinda just the same thing except it's smaller." Here, Kyle is coming to recognize similarity in the overall shape of the graph (still roughly sinusoidal) even as the amplitude differs ("smaller"). Through exploration, Kyle and Cory progressively differentiate features such as amplitude and frequency that draw them closer to math disciplinary practice.

After further explorations, Kyle and Cory analyze another round (Figure 6), blocking the projector light with their fingers to point out, "every peak when you go up and down, there are 13. Every spike going up and down." At this stage, Kyle and Cory have identified the vertically symmetrical nature of amplitude and have tuned into each local maximum and minimum. In this sequence, Kyle and Cory generated hypotheses from analyzing color and numerical feedback, informing each attempt with new ideas to explore, and arriving at a way of seeing that more closely reflects math disciplinary practice. BBM's open-ended prompts and visual feedback on the children's personally generated graphs supported their discovery-based math conceptual learning.

3.4.3 Embodied self-regulation during and through instructional activities. Unlike a desk chair, BBM's balance board celebrates the child's own sense of bodily balance and movements by making readily available means of vestibular stimulation, a common aspect of sensory self-regulation [12]. The experience of self-guided vestibular stimulation sparked joy for sensory-seeking 2nd-grader Maria, who commented: "I wish I had one of these [balance boards] at home!" Consistent with a sensory regulation perspective (e.g., [1]), most participants also rocked on the board in between BBM activities in ways that serve a regulatory function. These instances occurred most frequently during conversation, when peers or interviewers were speaking, during quiet reflection, and sometimes when the child him or herself was speaking. During activities like listening that entail less inherent vestibular stimulation, the children were able to seek sensory stimulation with the board to support

Figure 6: Kyle and Cory highlight features of a graph by blocking the projector light with their finger.

their engagement. Sensory seeking participants engaged in a higher frequency and intensity of regulatory rocking both during and between activities, suggesting that BBM indeed offers a learning space where vestibular self-regulation is always implicitly welcomed as an accessible means to seek regulatory vestibular stimulation within and beyond the activities.

In the BBM setting where rocking is part of both activities and background regulatory activity, we found that these two processes sometimes converged. Participants' regulatory rocking was influenced by and fluidly blended with their thinking and talking about the learning activities. For example, Kyle switched from using the board in the front/back configuration (leaning forwards and backwards) to using it in the left/right configuration (leaning left to right). Prior to the switch, when sitting in a swivel chair, his regulatory rocking was consistently a rhythmic front/back rocking. However, after the switch, Kyle's regulatory movement in the chair switched to twisting left to right. In another instance, Kyle rocks side to side, apparently for regulation. As the interviewer comments, "you mentioned earlier something about the distance between each hill," Kyle's rocking suddenly shifts to slow, sweeping rocks, corresponding to how he had earlier created spaced out features on the graph. Kyle's bodily movements reacted to the interviewer's speech before he had yet verbalized a response. This moment is consistent with embodied cognition literature suggesting that language comprehension is a spatial, sensorimotor process [54] and work on gesture as part of conceptualization [55]. Kyle acted out the conjecture raised by his interlocutor. Movement-for-regulation fluidly transformed into movement-as-thinking, intertwining and interacting rather than unfolding as separate processes.

3.5 Accommodating diverse sensory modulation profiles

The children exhibited a range of sensory preferences with regards to rocking on the board, which BBM accommodated through custom speed and sensitivity settings and the multiple solutions possible within each activity. For example, Lyla rocked gently with limited amplitude (Figure 7, left), whereas sensory seeking Arthur and Ben rocked intensely, using the board's full rocking range (Figure 7, right). Other participants like Cory, Maria, and Kyle varied their intensity across different activities. Participants also chose to position their feet differently, with participants who preferred gentle rocking tending to keep their feet grounded on the floor, and participants who preferred intense rocking tending to pull their legs onto the board, crossing their legs or kneeling. BBM's flexibility enabled all participants with diverse sensory modulation profiles to successfully explore all activities and "find green." Lyla and Ben, a pair of participants with opposite sensory preferences, were able to communicate effectively with each other about their hypotheses and enact learnings from observing each other's activity. As predicted, different sensory profiles led to different engagement styles, all of which led to successful interactions with the platform.

4 DISCUSSION

Our findings enter into dialogue with embodied learning and sensory regulation literatures and chart novel research directions at their intersection. With regards to embodied learning, we found that children were able to form new mathematical ideas through the achievement and analysis of dynamical consistencies in their movements in an action-based embodied design environment. These findings brought children closer to mathematical professional vision [56] in that they came to attend to graphs in ways that reflect disciplinary practice, such as identifying one full cycle above and below the midline (mathematical period) as a meaningful unit and attending to the amplitude of sinusoidal graphs. BBM corroborates principles of embodied design, demonstrating that they obtain effectively in a new content area (see also [58]).

Prior work in embodied mathematical cognition has highlighted that experts view static graphs dynamically. From an enactivist pedagogical perspective, "one objective of education is to enculturate students into understanding static images as offering opportunities for action" (p.7) [11]. This study sheds light on one pathway towards such enculturation: we found that dynamic, whole-body graphing experiences led even static, 2d images of graphs to evoke dynamic experiences for children, reflected in a greater degree of "being-the-graph" in their gestural expressions. By interacting with designed pedagogical tools, the children changed their perception [57]; the participants came to draw upon their own sense of balance and movement to think about the mathematical meanings of graphs' abstract features.

That children used their balance experiences to conceptualize is particularly novel, as vestibular input has not historically been the explicit basis for mathematics instruction. Whereas vestibular

Figure 7: Examples of two participants' different rocking styles. Left: Lyla rocks gently with feet planted on the floor. Right: Arthur rocks heavily with legs crossed on the board.

research has implicated the system in skills affecting math such as spatial reasoning, and conceptual metaphor work has suggested a role for balance experiences in abstract reasoning, this study explicitly introduces mathematical concepts through vestibular experiences. That students can draw upon these experiences to perceive and think about math concepts suggests the epistemological viability of different perceptual modalities in mathematics, offering means to challenge the discipline's oculocentric history that has marginalized learners with disabilities [51, 59]. BBM offers a proof of concept for engaging senses traditionally muted in classrooms, particularly the sense of balance, in mathematical learning.

With regards to sensory regulation literature, our findings are consistent with models of sensory modulation that suggest that children are tuned to require different stimulation intensities [1]. As expected, children identified as sensory seeking exhibited more intense and frequent movements. These movements occurred not just in the background of activities. Also as expected, children showed spontaneous rocking activity during tasks that were not as sensorially stimulating, such as listening or waiting. Our findings extrapolate beyond sensory regulation models: firstly, we found that when offered the chance to select the sensory intensity of a learning task, children differentiated according to their sensory profile. Second, we found that participants with opposite sensory preferences were able to communicate effectively with each other about their activity despite notable differences in the actual movements they enacted.

This study offers some preliminary glimpses at how sensory regulation and embodied learning theory might interact. Contrary to prevailing research assumptions that these two processes are separate, we found instances where activity traditionally interpreted as sensory regulatory behavior was dynamically taken up as a structure to think with, influenced by the discussion of different proto-mathematical ideas, and where activity movement affected ensuing regulatory movements. Our findings suggest that movement for exploration and for regulation may affect each other. At minimum, we identify opportunities to take up regulatory forms such as fidgeting as resources for richer, more integrative learning experiences within which these actions take on new meanings. More boldly, we suggest that background activity might be thought of as part of ongoing cognitive activity, participating ongoingly, if generally covertly, in embodied cognition.

4.1 Limitations and Future work

The present study is limited by its small sample size. We plan to run more participants in future studies and investigate two-board, collaborative interactions. We are also working with classroom teachers to design classroom interaction models and activities for further concepts such as function addition, and plan to use written pre/post math assessments to assess learning outcomes. We are still working to expand BBM's accessibility through sound/sonification and visual design to improve accessibility for colorblind and visually impaired participants. We hope to investigate inclusive learning by working with and across participants with different sensory profiles. Methodologically, we noted that having children sit during their pre/post interviews prospectively limited the range and bodily engagement possible for them and will conduct pre/post interviews in a standing position for future sessions. The informal identification of sensory profiles in this study is not highly replicable, so we plan to collect more structured sensory profiles using questionnaires, as well as continuous regulation data (EDA).

5 CONCLUSION

We presented the design and evaluation of Balance Board Math, which offers children integrated opportunities to self-regulate through movement and reach conceptual understanding using their sense of balance. Our study showed that BBM fostered exploratory movements offering a new entry point into mathematical practice of graphing. BBM activities impacted how children made sense of graphs, eliciting an increase in markers of ***being**-the-graph* as opposed to ***seeing**-the-graph*. BBM celebrates using movement to explore activities, to support self-regulation, to communicate ideas, to reflect, and to comprehend, with these different forms and functions of rocking fluidly combining and intertwining. Our findings suggest that sensory self-regulation and cognition interact and thus, should not be treated as entirely distinct aspects of activity. BBM's flexible balance board interface accommodating a broad range of physical movements combined with a dynamic display of sinusoidal graph visualizations invited children with different sensory preferences to interact naturally and playfully with the activities, suggesting that sensory-flexible instructional resources can support expanded instructional accessibility. We offer BBM as one such promising technology as an instructional and research tool.

ACKNOWLEDGMENTS

The Balance Board Math project is supported by the National Science Foundation Graduate Research Fellowship Program under Grant No. 1938055 as well as the Jacobs Institute Innovation Catalyst grant. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. We also thank Ashton Morris for his work on BBM sound and sonification design.

SELECTION AND PARTICIPATION OF CHILDREN

School-age (K-12) children were eligible for participation in this study. A convenience sample of 6 participants in grades 2 through 6, two girls and four boys, participated. Participants and their parents were informed of all activities, the focus of this research, and that participation would be recorded. All completed consent, assent, and media release forms approved by the Institutional Review Board prior to the children’s participation, with the option to discontinue at any time. Breaks were offered to participants between activities as desired. An adult known to the child such as their instructor or parent was always present on site. Media release forms allowed parents to indicate their choices regarding recording their child’s participation as well as usage of the recordings in different contexts such as publications and presentations. We also honor here the wishes of child participants regarding the sharing of their identifiable data.

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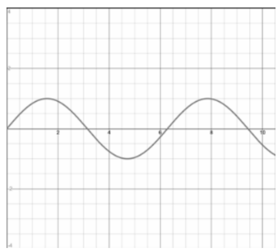
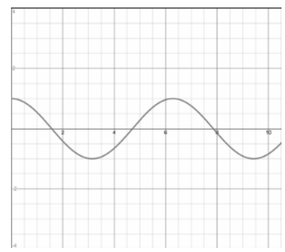
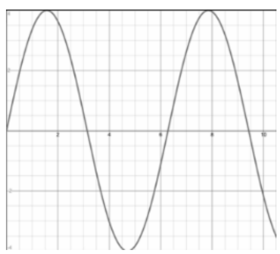
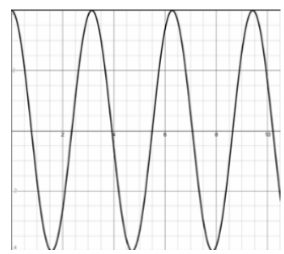
A APPENDICES

A.1 Demonstration videos of exploration activities

- Function Exploration Demo Video (00:00)
- Amplitude Exploration Demo Video (00:13)
- Frequency Exploration Demo Video (00:25)

A.2 Graph Description Prompts

	Pre	Post
Basic sinusoid	sin(x)	cos(x)
		
High-amplitude sinusoid	4 sin (x)	4 cos (2x)
		
High-frequency sinusoid	2 sin (6x)	½ cos (10x)