Graduate School of Education Research Day, April 2, 2021

Linear Algebra Conceptions as Movement and Perception in Motor-Control Tasks

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Project for EDUC 222C: Design Based Research Forum
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Objective: Addressing Difficulties That Novices Face When Learning Linear Algebra

Linear algebra unifies and formalizes methods for solving linear problems in a variety of domains (e.g., functional analysis and geometry). However, these advantages of the formal theory in terms of generalization and simplification are evident only to experts. For novices, linear algebra brings with it a number of difficulties, including the "obstacle of formalism" related to a heavy emphasis on logical expositions and formal definitions, the challenges associated with navigating different registers and representational systems, and prevalent tendency to prefer procedural over conceptual approaches (Dorier & Sierpinska, 2001). These difficulties motivate the development and evaluation of new instructional approaches.

Background: Using Dynamic Mathematics Environments to Introduce Linear Algebra Concepts

One type of instructional approach for addressing students' difficulties in undergraduate linear algebra courses leverages the subject's affinity with geometric representations by using dynamic geometry software. However, previous work on this approach (Gol Tabaghi & Sinclair, 2013; Sierpinska et al., 1999) does not entirely escape symbolic representations' ontological imperialism (i.e., ignoring intuitive *know-how's* conceptual potentials, Bamberger & diSessa, 2003); and does not fully embrace these environments' visual–kinesthetic affordances as fundamental starting points for mathematical inquiry.

Design: Creating Motor-Control Problems Whose Sensorimotor Solutions Anticipate Concepts

Departing from previous approaches, the present design project draws on embodied design (Abrahamson, 2009), a pedagogical framework for creating and evaluating dynamic mathematics environments centered on motor-control tasks. Using the enactive approach to cognitive science (Varela et al., 1991) as a guiding perspective, embodied-design activities seek to ground disciplinary concepts and representations in perceptual solutions to motor-control problems. Figure 1 shows part of the present design: participants are tasked with turning green the draggable Vector *u*, and to find all areas in the plane corresponding to a green *u*. As it turns out, *u* turns green whenever it aligns with its linearly transformed image, *T(u)*. Visual and kinesthetic explorations elicit attentional anchors (e.g., two green lines corresponding to the eigenvectors of *T*) to coordinate the enactment of "conceptual choreographies" (e.g., moving *u* along the lines); these, in turn, make salient latent relational features of the virtual environment (e.g., the ratio of *u* and *T(u)* is invariant along a green line), thus anticipating normative notions, such as eigenvectors and eigenvalues, which later become elaborated through further discourse and representation.

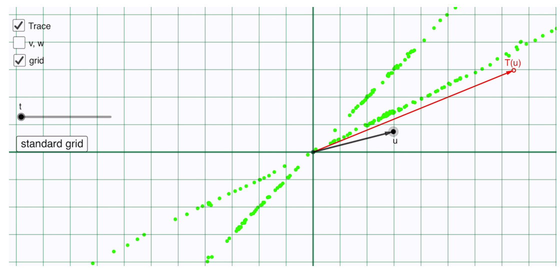

Figure 1: Eigenvector design; Vector u *turns green whenever it aligns with its linearly transformed image T(u), leaving a trace of eigenvectors.*

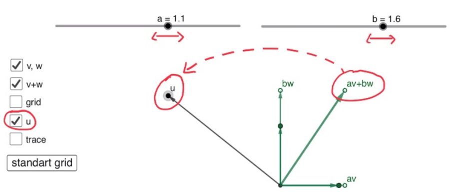

Figure 2: Linear combination design

The above task is part of a sequence of activities, implemented in Geogebra, that do not require prior experience with linear algebra. In another task from this sequence (see Figure 2), participants scale Vectors *v* and *w* using Sliders *a* and *b* to make their linear combination *av+bw* congruent with pre-given Vector *u*. One intended takeaway is that any Vector *u* can be reproduced by *av+bw*, as long as *v* and *w* are not collinear. Such insights anticipate the disciplinary notion of span. We look forward to re-implementing this task as bimanual.

Results and Conclusions

Tasks such as those described above were implemented as part of a semi-structured clinical interview with a single volunteering participant who self-identified as unfamiliar with the targeted mathematical content. Microgenetic analysis of her multimodal behaviors suggested that pre-symbolic notions relevant for conceptual learning in linear algebra can be indirectly fostered through visual and kinesthetic engagement with dynamic mathematics environments, without *a priori* introduction of formal definitions. The analysis also highlighted the epistemic role that prior experience with Cartesian coordinates can play with respect to the conceptual learning of linear combinations and related notions.

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