Walking the Number Line: Enactive Understanding of Integer Arithmetic

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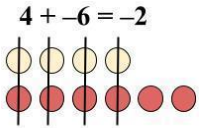
Objective: Improving Number Fluency for All Learners

Basic number sense is a high predictor of children's future mathematical achievement (Lyons & Beilock, 2011). Yet, students often struggle to accurately add and subtract positive and negative integers.

Background: Blocks & Counters vs. Number Lines

Bossè et al. (2017) surveyed the various concrete representations teachers employ to teach integer operations. They found that these representations fall under three categories: (1) isomorphic blocks; (2) colored counters; and (3) number lines:

Table 1. Previous Solutions

Isomorphic Blocks	Colored Counters	Number Lines
		
Pros: helpful manipulatives for adding and subtracting positive integers **Cons:** cannot accurately portray negative integers	**Pros:** provide students with clear algorithm for adding and subtracting **Cons:** tokenizes negative integers as concrete "things;" fails to address polysemy of "–" sign	**Pros:** represents integers as ratio scale comprising consistent spatial intervals **Cons:** relies heavily on symbolic notation and iconic signs to represent integers

Children often struggle to adequately use number lines to add and subtract positive and negative integers, possibly because number lines include diagrammatic and iconic signs signifying negative quantities, which are unrealistic entities, that is, they are ungrounded in students' lived experiences and skills. Moreover, the "-" sign ambiguously means both the arithmetic operation "subtract" and the value's negative polarity, an ambiguity exacerbated by symbolic sequences such as "-1 - (-3)".

Design: Egocentric Walking Number Line & Allocentric Small Number Line

The Walking Number Line was designed to occasion opportunities for students to unambiguously ground negative-integer arithmetic in whole-body enactment. First, students solve basic addition and subtraction problems on a body-scale number line (egocentric experience; Fig. 1). Next, they simulate their full-body actions by manipulating a "mini-me" figure on a desk-scale number line (allocentric orientation; Fig. 2). This bi-perspectival activity sequence may enable students to coordinate the two perspectives (Benally et al., 2022), so that they can ground a normative allocentric use of the standard number line in their egocentric experience walking the body-scale number line. The activity was piloted with 2 pairs of G. 8 low-achieving students.

Figure 1. Student walking on the body-scale NL.

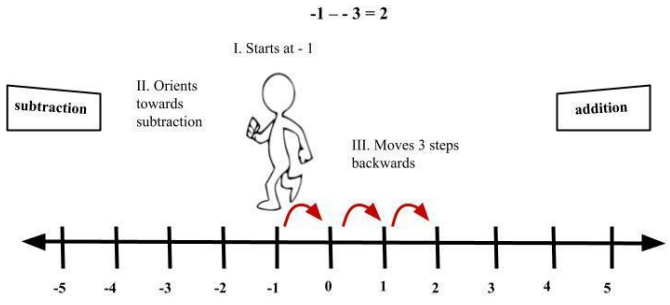
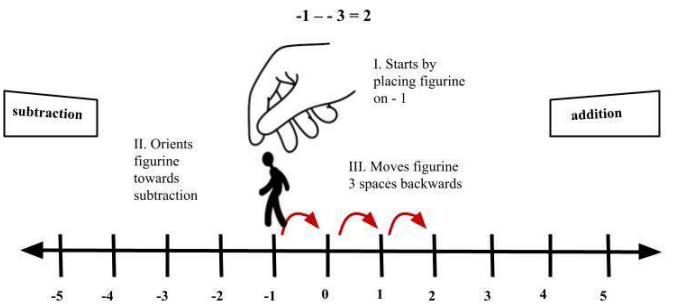


Figure 2. Student walking a figurine along a desk-scale NL.



Results and Conclusions

Non-canonical enaction of arithmetic procedures elicits and productively disrupts students' mathematically inappropriate solution routines, letting them re-engage with core concepts. Students can learn math concepts by coordinating 1st-person situated enactment of solution procedures and 3rd-person canonical diagrammatic operations on conventional media.

References

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