Co-constructing Náhookos Bi'ka' constellation with STARR

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ABSTRACT

This is background on the STARR—Students Tracking Angular Rotation Recorder—as a means for students to learn about angles. In an effort to revitalize land-based ethnomathematical practices, this project is framed around Diné cosmology and engages students in a novel astronomical experience where they become immersed in angle creation and measurement. Participants will be 4th & 5th grade (9-12 years) students paired either with a peer or with an intergenerational family member. Working in a simulated planetarium, participants will collaboratively "voyage" along constellations, so as to occasion communications about angles from complementary perceptual perspectives (the first-person "Sensor" and their third-person "Navigator"). STARR investigates the discursive co-construction of situated mathematics to evaluate whether and how students are grounding Euclidean geometry in Diné astronomical knowledge. This tangible cultural interface explores the resonance between Indigenous historical practices and modern pedagogical principles of progressive education focused on situated actions, with potentially broad implications for mathematics education based on critical-pedagogy restorative-justice values.

CCS CONCEPTS

• **Human-centered computing → Activity centered design**; • **Social and professional topics → User characteristics**.

KEYWORDS

Mathematics Education, Embodied Learning Technologies, Multigenerational Learners

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As guided by my grandmother about the meaning of angle, ahináádíí'á, in Diné Bizaad, Diné[1] language, to be about the coming together, connectedness of lines. Since Diné Bizaad is of the land–now partially within the Navajo Reservation–for Diné epistemology it is bounded within the space between the four sacred mountains, and it speaks to the relational perspective of the generational knowledge. To empower the embodied relational perspective of Diné epistemology in mathematics education with angles, is to reify another heterogeneitic example of spatial reasoning. The research project is a first step within a larger research program that aims to rematriate [18] mathematics curriculum by drawing on Diné ethnomathematics to design an embodied activity in dialogue with Eurocentric approaches.

Mainstream mathematics pedagogy and content is founded in Western / Eurocentric objectivist epistemology. By juxtaposition, Indigenous mathematics stems from the individual's understanding of place in their communities and their environment. For the Diné perspective, rooted in its psycholinguistic epistemology, is non-dualistic: there is no philosophical separation between self and environment—you are one with what you are attending to.

> Navajos react to the outside world as if it were primarily constituted of regular phenomena of presumable patterned or 'constant instances' (objects) of processes or fluxes. Consequently this means that if a notion of 'object' is introduced to describe these constant processes, this notion has but a minor kinship with the Western notion of 'object.' The Navajo emphasis is clearly on process in form instead of form as such. [13, p. 162].

The objective of the project is to create, implement, and evaluate a learning environment designed to offer students opportunities to reconcile Diné and Eurocentric perspectives on geometrical features of the environment. The learning environment, which centers on the geometrical concept of angle, is called STARR—Students Tracking Angular Rotation Recorder.

STARR's design is rooted in cultural–historical psychology views of "learning" as transpiring as individuals engage in the socially mediated enactment of heritage practices [20]. Given the central roles that artifacts play in cultural solutions to collective problems [17], learning can be viewed as the process of appropriating available artifacts as means of performing socially valorized tasks. These theoretical groundings of the design bear out directly in the design-based research of students' learning process in STARR. Looking closely at students' interactions with a socio-material environment of our own design, we seek to identify and theorize the students' behaviors, including their actions and utterances, via cultural framings of ontogenetic development.

The study complementarily builds on enactivist philosophy of cognitive science and its pedagogical derivations, whereby learning is implicated as the emergence of new perceptual-motor forms [9] followed by their "languaging" into disciplinary practice [1]. Accordingly, we follow an enactivist design framework to create fields of promoted action [14]: socially facilitated motor-activity spaces, wherein students develop targeted cognitive structures through

[1]Diné, the original name of the people, is used to refer to the people and concepts. Navajo, the colonial term, is used by other authors and by government agencies in exchange for Diné.

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task-oriented interaction with available resources [2].

At the same time, we recognize that designed learning environments are often implicitly modeled on the designers' own epistemologies (whether or not the designers recognize their cultural biases), which is liable to preclude culturally appropriate pedagogy. We therefore deliberately surface, juxtapose, and negotiate the material and linguistic grammars of Diné mathematics vs. Eurocentric forms of thinking, acting, and learning [16]. As such, the project aims to build an epistemologically pluralistic learning environment [19] that restructurates common mathematical practices [21]) as emulating and extending embodied immersion in the natural world [8]. Our data analysis will investigate whether and how Indigenous students come to appreciate the capacity of their heritage cultural practices to serve as epistemological foundations for making sense of hegemonic public discourse [12], specifically of geometrical concepts.

The Common Core State Standard for Grade 4 geometry directs students to draw, identify, and classify shapes by the properties of their lines and angles. Furthermore, students are asked to identify these shapes in two-dimensional figures with right, acute, or obtuse angles. It is only once students have governed 2D shapes that they graduate to studying 3D volumes. Whereas this meta-curricular sequence appears to increase in conceptual complexity, at the same time it may belie our situated phenomenology. That is, a critical Diné position might prefer to begin with natural and cultural 3D voluminous objects of the lived world, only later extracting therefrom unnaturally "flat" 2D objects. Indeed, the Bureau of Indian Affairs (BIA) government-funded American Indian Standards for K–4 mathematics education [6] includes three-dimensional objects early in the units, thus injecting real-life components. Whereas these BIA standards were an important, if modest, start for the American Indian students to ground geometry content in salient features of their natural–cultural environment, these standards never outlined pedagogical approaches to reconciling these Indigenous perspectives with Eurocentric (Euclidean) perspectives on the same features.

This project accomplishes its objectives of grounding Euclidean geometry in Diné onto-epistemology by considering three aspects of Indigenous phenomenology: the land-based perceptual orientation, the modes of perspective, and the enactment of cultural practices.

First, the foundation of the land-based interactions of many Indigenous tribes have allowed them to develop a cosmology and ethical system that is based on intimate and factual knowledge of their environment from their generational participation with the land. The relational connection with the environment through observation and respectful interactions give rise to meaningful lessons of how to live with the land [15]. To live and to travel by the land, provides geographic and spatial knowledge in which their mathematical systems, understandings, and representations of space are within and upon the land that they rely on for meaning and information [11]. In this way, Indigenous cosmology contributes to Indigenous epistemology, creating an embodied ecological awareness within various movement modalities (thinking, acting, interacting, etc.) not reflected in Eurocentric epistemology.

Secondly, the modes of perspective, while Eurocentric epistemology privileges the mind-body separation through Cartesianism thought. Indigenous epistemology is grounded in subject-subject relations in the natural world [12]. Thus, this project works towards extending mathematics education through the cohesion of multiple perspectives or conceptually generative perspectival complementarities (CGPC) [4]. These perspectival complementarities that surface in the Diné conception of space. In the Anthropology of Space: Explorations into the natural philosophy and semantics of the Navajo, [13] introduce a semantic analysis of Diné Bizáád (the Navajo language) around space, created with community members, which provides the basis for the Diné geometry education evolution. The verb-based structures of Indigenous languages bring pedagogical promise to mathematics education and bring to light alternative ways of mathematically orienting to the environment [5]. For Diné, the action-based oral language creates mathematical discourse and "geometrical reference [that] is carried in the verb, rather than the noun" [3, p.29]. For example, a path would be given in terms of movement from landmark to landmark or in terms of (personal and directional) relationship to land and celestial objects. The term angle in Diné, speaks to the coming together of the lines, lines that either come from yourself or from other objects. Thus, a culturally relevant geometry activity will provide students a way to study mathematics in ways that are not estranged to the land, the culture, and the body.

Third, the enactment of cultural practices, such as Diné astronomy. Utilizing the Diné perspective of astronomy and ethnomathematics [7] to ground the Western mathematics content provides an entrance for a new way of orienting to the world.

The project, will focus on the angle's 'open-ness' (i.e., the rotated aperture between the angle's two arms)—study participants are to determine the magnitude of an angle "by measuring the length of the arc that the angle subtends in a circle centered at the angle's vertex" [10, p. 35]; This 'open-ness' has also been referred to as the angle's interior. Hardison [10] shows that angle can be perceptually attended in several different ways, and each perceptual construction may support different actions and understandings under varied contexts of geometrical practice. A Diné first-person perception of angularity may consist in rotating an arm to scan from pointing to one object in the environment to pointing toward another. Given that we wish to privilege a particular perceptual construction that foregrounds open-ness as the meaning of angle, we are to determine what actions this perception might enable and, therefore, which tasks we might create. This design consideration of how a student perceives a situation vis-a-vis what they are tasked to perform in the situation has been developed in the pedagogical paradigm of embodied design [1] as well as in radical constructivism. In particular, the specific perceptual orientation toward performing particular actions has been named an attentional anchor [2]. Here, the 'open-ness' perceptual construction of an angle, which focuses on the angular interiority, could serve as an attentional anchor enabling the performance of arm rotation to complete some relevant task (see below for further details of the activity design).

Once the student discovers this attentional anchor as their perceptual means of accomplishing the task, they can begin to perform operations on the interior of the angle, such as adding two angles together or dividing an angle into four equal parts. Having investigated how students enact angular motions with their hands or with concrete materials like chopsticks, [10] draws on earlier work to

provide a compilation of four attentional motions by which we measure the arc that the angle subtends. Attentional motions are the different (visual) methods by which we dynamically perceive some figural elements in the environment in the context of performing a task. In the case of observing an angle, these attentional motions may include: (a) awareness of experiential length (awareness of the full duration of the movement); (b) awareness of figurative length (internalizing the motion to recreate it); and (c) awareness of operative length (can subject the motion to further operations such as successive iterations of the movement or portions of movement) [10, p. 361]. In particular, a radial sweep (from one angle arm to the other) could serve as an attentional motion for deriving the size of an angle's interior, thus grounding the angle's measure and lending embodied–enactive meaning to its symbolic notation, e.g., as 32°. This restorative-justice project's critical positionality implies a deep re-thinking of curricular materials. Taking a design-based research approach, the project aspires to: (a) identify Diné alternative groundings for curricular content that currently is vested in Eurocentric perspectives; and (b) design activities that occasion opportunities for students to negotiate and reconcile Diné and Western perspectives on this content.

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