DEMO: A Tabletop Action-Based Embodied Design for the Cartesian System

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Abstract: Interactive technology is carving a new frontier in mathematics education by transforming abstract concepts into tangible and collaborative experiences. This DEMO presentation introduces the exhibit, a pedagogical interactive technology wherein visitors learn the coordinate system's basic logico–mathematical principles through collaborative goal-based play. Previous design solutions in this genre of interactive museum exhibits include: Geometris, a geometry learning game that combines elements of *Tetris*™ and *Twister*™ (Rosenbaum et al., 2020); *Math Square*, a multi-user exhibit at the National Museum of Mathematics (MoMath), where visitors collectively navigate mathematical challenges (Shoyfer, 2018); and, closest to home, *Drawing in Motion*, an exhibit developed by the Oregon Museum of Science and Industry, where two participants collaboratively operate a digitized Etch-a-Sketch, one participant per rotating knob, to draw images on a screen (Nemirovsky et al., 2013). Inspired by this interactive exhibit genre, the solution we aim to present offers further empirical context to evaluate the potential educational significance of leveraging technology to foster intuitive embodied understandings of mathematical concepts.

Coordinate systems: Design problems and solutions

Design problem

Coordinate systems such as the Cartesian coordinate plane are used in many subjects in the fields of mathematics and statistics, natural sciences (e.g., geography, physics), or social sciences (e.g., history). Correctly using Cartesian coordinate systems is essential to understanding graphs, maps, and models. Many students have difficulties analyzing graphs (Bell & Janvier, 1981; Kerslake, 1981; Leinhardt et al., 1990;) and reading maps (Downs & Liben, 1991). We were therefore motivated to ideate the design of activities that would occasion opportunities for students to: (a) become oriented in the Cartesian coordinate system, thus perceiving a point in two-dimensional space as cross-plotted at the intersection of two intersecting orthogonal lines that emanate, respectively, from horizontal and vertical reference axes; and (b) understand the diagrammatic semiotic methodology of ordered-pair notation, for example, being able to encode a given point on the plane as "(5, 3)" and, vice versa, to locate a point marked as such.

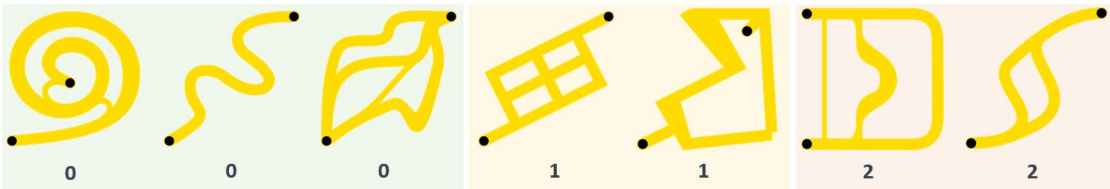
Design solution

Working in the embodied-design paradigm (Abrahamson, 2014; Abrahamson et al., in press), we strived to model the activity on two ideation heuristics: (1) the enactivist tenet that individuals' cognitive structures emerge from recurring task-effective sensorimotor patterns discovered through explorative perceptuomotor activity (Varela et al., 1991); and (2) cognitive-anthropological theorizations of shared ontologies as emerging through multimodal social interaction to facilitate the coordinated enactment of joint action (Goodwin, 2013). The design resulted in an interactive two-player game, where participants collaboratively control a virtual ball's coordinates to navigate it through maze-like paths, with the objective of completing the course in the shortest time and with the fewest moves. Earlier generations of this design's general mechanics—distributed orthogonal control of a digital object's movement across a Cartesian field—can be found in the Mathematics Imagery Trainer research at the Embodied Design Research Laboratory, University of California Berkeley (Lee et al., 2013) and with their collaborators at Utrecht University (Shayan et al., 2015).

Technical configuration and data collection

The game integrates an interactive module equipped with a Naos+ 86" Multi-Touch Display and operates on Microsoft Windows 11 Pro PL 64-bit OEM, marked by product ID FQC-10544. Intended for educational exhibition at museum learning stations, the unit utilizes software designed to enhance user interaction while gathering data on engagement metrics. Data are collected from each interaction and compiled into files from all interactions throughout the day, with the files in CSV format being named after the date. The exhibit interaction data are composed of several elements that reflect participants' activity. Data entries are initiated when participants consent to the study. Information about this selection, team identification, and consent status are systematically recorded. The dataset includes the date of participation, the team's code—a combination of the creation date and a unique identifier—and the language preference indicated by the participant. Timestamps are logged for button presses during the study with precision to milliseconds (hour:minute:second.milliseconds). The dataset indicates the team's completion status as true if all three levels are completed successfully. It also records the method of study conclusion, denoting completion either by pressing the "finish" button, returning to the main screen, or resetting after inactivity is detected. Seven track identifiers and associated three levels are noted (see Figure 1), with timestamps marking the start and end of each level attempt, along with the duration in milliseconds. Seven sample paths items groups into three levels (0, 1, 2) that correspond to increasingly mathematized game mechanics.

Figure 1
Seven Sample Paths Items Groups into Three Levels



Data on track completion, errors (e.g., the ball driven off the track), and level completion are tracked. The times at which specific buttons are pressed and the visibility status of the crosshair are recorded. The position of the ball, recorded as X and Y coordinates for each frame, provides insight into the interaction dynamics. The position is measured in pixels, where the origin point (0, 0) in the coordinate system corresponds to the value 0, 0 in pixels, and the point (15, 15) in the coordinate system corresponds to the value 1800, 1800 in pixels. The dataset reflects the ball's status, indicating whether it is in its initial position, has been relocated correctly, has reached the track's end, is off the track due to an error, or has been manually repositioned to a new coordinate following a mistake. This structured data capture allows for detailed subsequent analysis.

The learning with exhibit

At the outset of the interaction, participants are provided with information regarding the game and the ongoing research. Participants are asked to consent to participate in the research study. Following this, we request participants share some basic information about themselves—their age and the nature of their relationship (whether they are familiar with each other). The gameplay then commences. Upon completing a level, players can either attempt the same level again for further practice/better results or advance to the next level. The solution offers multiple maze layouts for each level, which are randomly generated. Each level offers a distinct experience, gradually increasing complexity and requiring players to adapt and collaborate effectively. Here, we provide an overview of the gameplay progression through each levels: 1) Cooperative Coordination, 2) Introduction of Coordinates—Transitioning from Manual to Numerical Control, 3) Full Numerical Control.

Summary: Design challenges for the science center context

The placement of the exhibit within the context of the science center exhibition, unlike school or other educator-facilitated activities, presented distinct challenges that influenced the initial design criteria: 1) Self-Contained Information, 2) Engagement,3), Minimal Text. Several aspects contribute to promoting equal opportunities within the planned design:

1. Accessibility: Technology designed with accessibility in mind, considering diverse needs such as physical disabilities, language barriers, or cognitive differences. The exhibit's design, minimizing text and relying on visual cues, fosters accessibility for individuals with varying reading abilities or language backgrounds. Additionally, considering accessibility features for individuals with physical impairments ensures that everyone can engage with the exhibit.

2. Inclusivity: The collaborative nature of the exhibit encourages teamwork and engagement among diverse pairs of visitors, promoting social interaction and collaborative learning experiences.

3. Research and Evaluation: Continuous research and evaluation of the effectiveness and impact of interactive technology in education, like the exhibit, should encompass diverse populations. Analyzing how different groups engage with and benefit from such tools helps in identifying potential biases or gaps in learning experiences and ensures that they cater to a wide array of learners.

By integrating these aspects into the design and implementation of educational technology like the OЯTHO exhibit, educational institutions can take significant steps toward promoting equal opportunities and fostering inclusive learning environments for all students.

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