

The icosahedron in the room: Revisiting pedagogical dilemmas of individual learning in collaborative activities

Dor Abrahamson^{1,2}, Catalina Lomos³ and Alik Palatnik⁴

¹University of California Berkeley, USA, ²Korea University, Korea, dor@berkeley.edu

³Luxembourg Institute of Socio-Economic Research, Luxembourg

⁴The Hebrew University of Jerusalem, Israel

Four adult study participants successfully built together a body-scale polyhedron with minimal instructions. Preliminary observation of their rapid construction process would suggest that they were generally “on the same page” and, therefore, perhaps all entertained the same geometrical ideas. However, deeper analysis of their multimodal behaviours and utterances, both during the activity and in post-intervention stimulated-recall individual interviews, suggests acute divergence in perceptual Gestalts, even when two or more participants referred in speech and gesture ostensibly to the precise same material element, voicing operational consensus. As such, our findings query cognitive–anthropological theory of organised social activity, e.g., Goodwin’s Co-Operative Action, by producing empirical evidence of productive joint action on indexically referenced co-attended features of the environment yet in the absence of conceptual cohesion.

Keywords: Collaboration, co-operative action, geometry, groupworthy tasks, joint action

Let us imagine a language ...The language is meant to serve for communication between a builder A and an assistant B. A is building with building-stones; there are blocks, pillars, slabs and beams. B has to pass the stones, and that in the order in which A needs them. For this purpose they use a language consisting of the words ‘block’, ‘pillar’, ‘slab’, ‘beam’. A calls them out; — B brings the stone which he has learnt to bring at such-and-such a call. — Conceive this as a complete primitive language. (Wittgenstein, 1953, §2)

Design problem: learning despite collaboration

It is not simple to facilitate equitable collaborative learning. When a group of students works on a challenging task, they may not enjoy equal opportunities to engage with the task’s core cognitive challenges, as evidenced in process analyses and individual assessments. Instead, group members may spontaneously distribute the labour so as to maximise the group’s productivity and ultimate success. Consequently, members best pre-equipped to tackle conceptual problems lead others down the production line, who design, manufacture, and assemble artifacts, thus engendering what Abrahamson and Wilensky (2005a) diagnose as a *stratified learning zone*. As such, group activities may not always hoist the lower-achieving students as hoped. Worse, teachers are liable to both congratulate students on their successful achievement as a group and critique struggling group

members for their unequal achievement on individual assessments. Something is not working in group work.

Well aware of this pedagogical tension between the quality of collaboration versus individual learning, Cohen (1986) developed the *complex design* framework, which included: (1) a set of heuristics for creating *groupworthy* tasks—tasks that authentically require a number of contributing participants, each with a distinct role, who must consistently coordinate their actions; along with (2) facilitation guidelines, such as rotation schedules, to ensure that each participant fills each of the roles. Approaching the same design tension from a constructionist perspective, Abrahamson and Wilensky (2005b) explored digitally enabled groupworthy tasks that create equitable participation, without prescribing rotation regimens, by ensuring that all students were operating similarly on the same types of objects (see also White & Pea, 2011).

Still, when students engaged in a shared task are handling the same objects together, are they necessarily perceiving and understanding these objects in one and the same way, such that, by and large, they are learning the same notions?

Sharing perception through the social enactment of cultural practice

According to the Theory of Co-Operative Action (Goodwin, 2013), human activity is intensely co-operative, emerging from interpersonally elaborative actions. New forms of co-operative action by a social group of common interest rely on their developing a *substrate*—a cumulative amalgam of iteratively laminated heterogeneous semiotic contributions respecting a shared domain of scrutiny. The substrate is the group's ever-growing resource of endorsed notions, routines, and perspectives for fluently managing their practice. Drawing on the substrate, co-operative action is coordinated through: (1) participants physically orientating their bodies towards each other and their shared domain of scrutiny and manipulation, creating a focus of perceptual attention and a sense of collective endeavor; (2) a spatial arena that includes phenomena of shared interest; (3) production and regulation of consensual linguistic tokens organising mutual interactions; and (4) gestures—deictic, iconic, and emblematic—for indexing features of the shared environment and imbuing them with contextual meanings.

Moreover, theories of joint action often underscore the vital function of joint intentionality, attention, and frames of reference, as well as semiotic interaction, in facilitating participants' coordination of effective simultaneous actions (Marschner et al., 2024). These theories of co-operative and joint action could be read as suggesting that when several students engaged in a groupworthy task appear to be doing literally the same manual actions, looking at the same objects, and even discussing these same objects, we could quite safely assume that they are developing similar conceptual understandings. But is that necessarily true? Our motto quotation from Wittgenstein's *Philosophical Investigations* suggests the possibility of joint action absent of shared meanings. We now look at resonant voices from systemic paradigms.

Critiques of joint-action and joint-attention theory

Alternative theorisations of coordinated action are rooted in scientific paradigms such as cybernetics, as well as phenomenological philosophy that promote a different fundamental sensibility of order, one that can embrace the complexity of dynamical systems in flux (Kelso, 2021). Embodiment, a rising exemplar of such post-Cartesian perspectives, has been evaluated as bearing dramatic implications for the learning sciences (Hutto et al., 2015; Macrine & Fugate, 2022; Nathan, 2021; Shapiro & Stolz, 2019). We now overview some recent cognitive science work in this vein that queries traditional models of joint action.

Rejecting reductive theories of collective intentionality, philosopher Seemann (2024) develops an *enacted theory of joint attention*, where, “The openness of joint agents’ intentions and meshing subplans is explained by appeal to their practical knowledge of how to maintain the process by way of which they pursue the collective intention” (p. 259). The would-be golden link between joint attention and joint action is further interrogated by questioning the inter-cultural universality of previous assertions that were based on empirical work within WEIRD societies (i.e., Western, educated, industrialised, rich, and democratic; see Henrich et al., 2010). Investigating parent–child naturalistic interactions in non-industrialized societies, Taverna et al. (2024) demonstrated joint action in the absence of joint attention, thus posing questions for ontogenetic theories that foreground joint attention as a critical developmental marker (Tomasello, 2019). In a resonant view from the philosophy of sports, Birse (2024) rejects representationalist assumptions that what is shared in joint attention is mental content. Instead, he sides with relationalist interpretations that look to include individuals’ aspectual perceptual perspectives as well as the actions that perception inherently enables (see also Story, 2021, on the emergence of joint action without a theory of mind). Finally, Loehr (2022) problematizes what she views as facile models of individuals’ sense of agency as mediating joint action.

What might all this mean for pedagogical frameworks and, more generally, anthropological theories of sociogenesis and semiosis? We wish to leverage these alternative theorisations of joint action in the study of individual learning in groupworthy tasks. As such, we ask:

1. Can individuals engaged in the successful collective enactment of a group task entertain different perceptions of the objects they are jointly handling?
2. How would productive collective enactment be possible when individuals perceive differently the key objects they are jointly handling?
3. What causes such hetero-perception? What are its educational implications?

Methods

Our empirical context is a groupworthy task, in which 4 adults, several of whom had worked as math teachers, are tasked to construct and analyse a body-scale polyhedron. Whereas we had used this activity extensively in researching geometry education (Benally et al., 2022; Palatnik & Abrahamson, 2022, 2024; Rosenbaum et al., 2024), we had yet to closely examine individual

experiences. An auspicious opportunity emerged during our workshop on learning analytics (EDaMMLA 2024), where a team built a large icosahedron (see illustration in Figure 1, left) from long wood dowels and silicon-pipe joiners, with no instructions for construction.

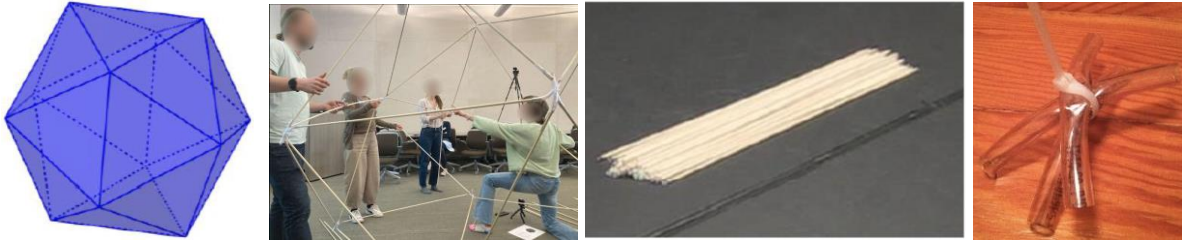


Figure 1: Building a body-scale icosahedron from wood dowels and joiner links

The session was audio–video recorded. Our data further comprise post-intervention individual stimulated-recall interviews, where participants (hence, e.g., P3) discussed their experience of collaboratively constructing the form. We analysed mutual physicality, language, gestures, and the surrounding phenomena. The video was coded by marking key moments in the construction process to capture elements of co-operative action. Utterances were transcribed and coded, marking aspects of perception and action. We later juxtaposed key video moments in the construction process with relevant testimonials from stimulated-recall interviews (hence, SRI).



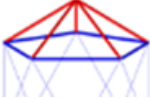



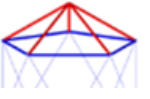
Results

To address our research questions, we examine 3 vignettes (see Table 1) that triangulate the activity’s transcribed videography with the post-intervention individual SRIs.

- Vignette 1 shows four participants engaged in a groupworthy task, performing the same manual actions, looking at and discussing the same objects, while the SRIs of P1 and P2 indicate that they are not developing similar conceptual understandings, implying the possibility of *joint action in the absence of shared meanings*.
- Vignette 2 suggests that what is shared in joint attention is not mental content, as P1 and P2 evidence hetero-perceptual perspectives and actions that nevertheless support *joint action without a shared theory of mind*.
- Vignette 3 illustrates that individuals can engage in successful collective enactment of a group task while entertaining different perceptions of the objects they are jointly handling, such that they manifest and *learn different notions*.

In building the icosahedron, P1 and P2 followed different conceptual perceptions: P1 counted the five edges in each vertex, while P2 tried to build “the roof.” Meanwhile, P3 deliberated over these two strategies—he would later admit that the activity’s hardest aspect was “knowing who[m] to listen to in such moments of confusion.” P4 followed on, attentive to the various framings.

Table 1. Collective enactment through hetero-perceptions in joint action

The vignettes (60 seconds)	The transcript (selected utterances)	P1: SRI	P2: SRI
<p>Vignette 1 (98 s to 111 s)</p> <p style="text-align: center;">P2 P3</p>  <p style="text-align: center;">P1 P4</p>	<p>P3 [leaning in, centre right]: Ok, I'm making... [attempts to connect a sixth edge to a vertex]</p> <p>P1 [kneeling, centre]: But there is only..., is it..., should be...</p> <p>P2: But there's, yeah...</p> <p>P1: I think we're... is there five [points to a vertex]? I think there should be five vertices or five edges meeting.</p> <p>P2 [on left]: But then this one is extra, yeah.</p> <p>P1: Yeah....</p> <p>P2: ...we need to take this off [holds the edge going down in the middle of P1's vertex]</p>	<p>I don't know... I think it's just like "I was just doing the 5s patrol." I was just like "I have to say something now because this is a thing that...". Maybe I should have vocalized it earlier: "Oh, there's five," but I think in my head I was like, "Oh, there's five!"</p> 	<p>Now I can see that this is not how I imagined "the roof"—the roof is not working well. (...) Everyone is working on a different side, so I can see that it is not symmetrical.</p> 
<p>Vignette 2 (112 s to 121 s)</p> <p style="text-align: center;">P2 P1 P3</p> 	<p>P2 [on left]: [to herself] This extra one in the middle...</p> <p>[Instructor intervenes] (...)</p> <p>P2: Yeah...</p> <p>P1 [leaning in, centre]: [to herself] Oh, there should be... [looks carefully at what P2, on left, is doing]</p>	<p>(...) That was the first time that I thought, "We don't need to add any more of these, because there are five." (...) It seems like that was the first time that I actually felt like I had to say what I was thinking for the past two minutes.</p>	<p>(...) It came from "the roof" idea, because there was something in the middle, [an edge], that's why I thought that, "Okay, it's not the roof yet."</p>
<p>Vignette 3 (122 s to 160 s)</p> <p style="text-align: center;">P2 P1 P3 P4</p> 	<p>P2 [on left]: (...) [removes an edge from one side of the "top" vertex,] joins it into opposite side, rendering the top vertex visible] Yeah, this is the now "top" [raising the vertex].</p> <p>P1: Oh, cool.</p> <p>P2: Yeah, we can think like this and then...</p> <p>P1: Yeah.</p> <p>P3 [right]: Yeah.</p> <p>P2: ...we go from here.</p> <p>P3: Alright</p> <p>P1: Ok</p> <p>P4: [silent, looks at "top"]</p>	<p>I don't think I noticed this ["the roof"] in the activity itself, but watching it now, I see this shape of the five triangles at the very top. It is interesting that we didn't even finish that shape first—we started going into the next...</p> 	<p>Now, the idea of "the roof" is becoming more and more urgent for me, so now I'm...yeah...yeah...ah..."</p> 

Our four “Rashomon” participants experienced different learning opportunities, pursuing self-assigned purviews (“5s patrol” versus “the roof”) or following the group’s actions. P1, who “just kept an eye on the five,” maintained her role as “5 patrol”; she did not perceive the whole polyhedron “until the very end, when it all came together.” P2, per the SRI, drew on prior knowledge that the shape comprises congruent triangles to form her “roof.” P3 grasped the form as a whole only towards the end of the activity, “when the shape was already correct and there was very little missing.” Even though P4 did not always understand the task at hand and was comfortable following the group, per her report, the “5s rule” prompted her sense-making. She stated, “OK, if I have to repeat it, I know how to do it.” In sum, the vignettes described task participants who, while engaged in task-effective co-operative action, nevertheless: bear different *perceptual orientations* on the situation; highlight *different properties* of the environment as action affordances; entertain different *construction objectives*; and consider different *task constraints* which they announce, monitor, and enforce. We thus witness a set of individuals who, though inhabiting different environments (Shvarts & Abrahamson, 2023), nevertheless successfully perform an apparently well-coordinated collective task.

Discussion

Regarding RQ1, we have told the tale of four persons who, though successful in building together a complex geometric structure, each perceived it differently (Borovska et al., 2024) and, later, retained different conceptual notions about its geometry (Zamecnik et al., 2024). This tale demonstrates that participants in collaborative activities may not enjoy equal opportunities to engage with the task’s core cognitive challenge, highlighting the pedagogical tension between the quality of collaboration and individual learning (Abrahamson & Wilensky, 2005a).

Regarding RQ2—the issue of how productive collective enactment is possible given a group’s hetero-perceptions—our findings pose new questions to cognitive–anthropological theory of organised social activity, such as Goodwin’s Co-Operative Action. Namely, we have produced empirical evidence of productive joint action on indexically referenced, co-attended features of the environment yet despite the absence of conceptual cohesion: When task members make heterogeneous semiotic contributions that are apparently endorsed by others, what may actually be endorsed are practical operations on designated *local* features, while colleagues’ idiosyncratic *global* perceptual constructions of these features are filtered out in communication. Co-operation does not imply, and may not require, co-perception. When teachers conduct group activities in their classrooms, they should remain vigilant that effective coordination may nevertheless conceal hetero-perception and divergent understandings (Abdu & Schwarz, 2020). These divergent perspectives could be gainfully leveraged through facilitated post-activity reflection.

Regarding RQ3, we thus rekindle long-standing research concerns over the pedagogical optimisation of groupworthy assignments. Efficacious joint action may result not from explicitly negotiated engineering, distribution, and monitoring of individual roles, which would occasion pedagogical opportunities for engaging in disciplinary discourse around a lesson’s target content. Instead, it may result from the emergence and stabilisation of multiple, loosely recurring, ad hoc

local transactions among spatially proximate participants, each with their self-assigned purviews, who temporarily and unwittingly become functionally co-dependent, thus requiring only brief, cursory coordination.

Developing interpersonally heterogeneous perceptions of shared objects may be an adaptive evolutionary social heuristic for collaborative problem solving in novel contexts under conditions of uncertainty, namely, disaggregating exploration so as to maximise discovery opportunities (Richardson et al., 2008). Educational designers, however, ultimately are charged with converging students towards more-or-less homogeneous understanding of the subject content. We believe that educators should keep deliberating how to utilize humans' atavistic genetic inclinations for hetero-perception—leveraging it, rather than inhibiting it—nevertheless while also ensuring equitable instruction. As such, we propose, if indeed we are to foster the skill of collaboration, we should appreciate, understand, embrace, and support humans' soft-assembly routines, including tinkering, divergence, emergence, and loose coordination, rather than impose regimens for the arbitrary distribution of labour that are liable to suppress students' powerful evolutionary proclivities. At the same time, we should surface the phenomenon of hetero-perception for the students themselves, so that they can learn through coordinating their divergent visualisations.

Further investigations of equitable learning in collaborative activities are called for, because our speculations are inferred from a single group of adult participants working in a unique social context (an international workshop) on a rather irregular task, using unusual media.

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