CHAPTER 6

Intercorporeal Dynamic Functional System

A Dual Eye-Tracking Study of Student-Tutor Collaboration on a Mathematics Embodied Design

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Abstract

We evaluate the viability of modeling the phenomenon of mathematics teaching/learning from the combined theoretical perspectives of radical embodied cognition and the cultural–historical approach. In particular, we illuminate an apparent deep affinity between two notions, respectively: (1) a *complex dynamic system* is a self-organizing coordination of distributed processes within a set of constraints; and (2) a *functional system* is a spontaneous organization of cognitive processes oriented on a subjective task. Our thesis is grounded empirically in detailed analyses of several student–tutor dyads working on an action-based embodied design for parabolas. Micro-analysis of audio–video and dual-eye-tracking data reveals the dyad operating tacitly in *inter*corporeal sensory–motor coupling, giving rise to joint visual attention to the manipulated material. In so doing, tutor and student form an *intercorporeal dynamic functional system*. Contingent scaffolding within the system steers the student to coordinate the learning material with cultural ways of referring. Later, the tutor's scaffolding operations are transformed into the student's self-regulation of her attention as extended by co-thought gestures. Learning is thus the transformation of an *inter*corporeal system into new *intra*corporeal dynamics generating the student's new functional capacity.

Keywords

functional dynamic system – cultural-historical approach – embodied design – eye-tracking – forward model – intentionality – intercorporeal (coupling) – joint attention – micro-zone of proximal development – mathematics education

1 Introduction

Embodied approaches to mathematics teaching and learning vary from rather moderate consideration of abstract mathematical concepts being grounded

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in sensory-motor metaphors (Lakoff & Núñez, 2000) to radical positions, such as the new materialistic approach, which claims that mathematical concepts expand far beyond the human mind or even body (de Freitas & Sinclair, 2013).[1] Within this variety, our point of departure lies in a rather radical position emanating from seriously considering the epistemological tenet of cultural–historical theory. Mathematical cognition, as a species of higher psychological functions (Vygotsky, 1978), appears to be constituted within joint action space (Radford & Roth, 2011), where a student and a tutor form an intercorporeal system. We investigate empirically student–tutor intercorporeal systems to document and model their dynamic transformation into functional systems with new mathematical capabilities. This transformation is contingent on the participants collaboratively and iteratively establishing micro-zones of proximal development (mZPD).

To build our thesis, we theoretically elaborate the cultural–historical approach through notions of dynamic systems, self-organization, coupling, and anticipation, all of which evolved in embodied cognitive science, such as in coordination dynamics (Bernstein, 1967; Fogel & Garvey, 2007; Kelso, 1982; Stephen et al., 2009; Stepp & Turvey, 2010) and enactivism (Gallagher & Miyahara, 2012; Jurgens & Kirchhoff, 2019). Contemporary experimental work on joint attention and joint action substantiate our theoretical statements with empirical evidence.

Finally, a micro-ethnographical analysis of empirical dual-eye-tracking data demonstrates the intercorporeal coupling between a student and a tutor, as their eye-movements are coordinated either with the student's performance or the tutor's explanation. In this coordinated system, the dyad progresses through a sequence of micro-zones of proximal development, from establishing new sensory-motor patterns to labeling these verbally and, further, to understanding a mathematical formula as a coordination of actions upon visible mathematical inscriptions.

2 Theoretical Framework: Embodied Mechanisms of the Teaching/Learning Collaboration

This theoretical part of the paper elaborates on the culture–historical approach to teaching/learning by focusing on embodied mechanisms that ground collaboration between student and teacher. We take a radical perspective on the embodied roots of mathematical knowledge and aim to establish a joint theory that would allow seeing mathematical cognition as a direct continuation of the human ability to act in the material world. The cultural–historical framework elucidates teacher–student embodied communication as explicating a student's progress from initial (or natural) to cultural forms of action. Drawing on embodied cognitive science models of motor-skill development, we explain how mathematical cognition, a high-order ability, is formed from sensory-motor experience.

In line with the theoretical claim that higher-order understanding is based on sensory motor experience, we will offer the reader "intuitive pumps," namely anecdotal yet paradigmatic examples of sensory-motor experience that the readers might consider with as they engage with our presentation of theoretical concepts.

2.1 *The Functional-System and Complex-Dynamic-Systems Approaches: Coupling and Anticipation*

Intuitive pump. I [AS] remember myself at 6 years old, running alone on a narrow trail between a forest and a steep precipice down to the sea. Making my way quickly along the path, I recall looking carefully at the path, as my sandals took wide steps. And then I clearly remember myself at the end of the path, by my parents, very frightened. What happened is that there had been a snake lying across that trail. I did not stop as I was running, nor did I even slow down. My body, having grasped some strongly repulsive object, somehow adjusted its steps—I remember experiencing a really long jump—to clearly avoid the obstacle.

Approaching my sensory perception and motor system as assembled in a functional system, we stress that the function of initial behavior (running to get to parents) was maintained, given and despite the new circumstances, by introducing a subtle adjustment within the sensory-motor routine. My body did it in a way that I remember not as "Me, jumping over a snake," but, rather, as "It happened, my legs did it."

The idea of a functional system was first introduced in physiological investigations of behavior by Anokhin in the 1930s, in the USSR (Anokhin, 1935/1975). It is defined as "a complex of neural elements and corresponding executive organs that are coupled in performing defined and specific functions of an organism" (Anokhin, 1935/1975 as cited in Kazansky, 2015, p. 105). As he developed the science of reflexology, Anokhin conducted experiments on the rehabilitation of cut nerves, thus demonstrating the adaptive qualities inherent to the functional organization of the neurosystem (Anokhin, 1935/1975). His thesis essentially replaced the classical vision of the brain system as producing fixed reactions to external stimuli with the idea of biological reorganization through behavioral tryouts. The mind is not a fixed assembly of predisposed modules. Rather, it is a functional system characterized as "self-organizing

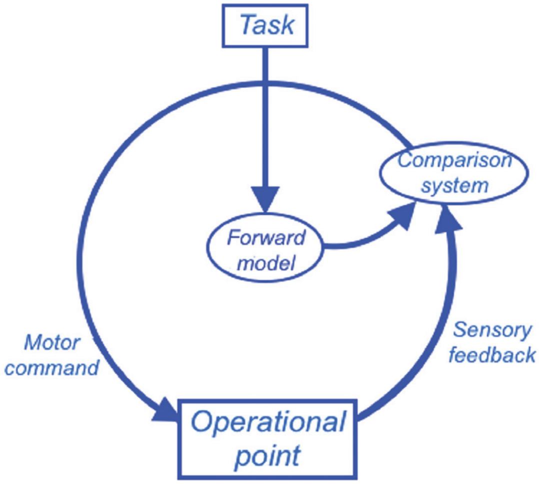


FIGURE 6.1
A reflex circle: the coupling between sensory afferentation and anticipatory motor efference as a unit of the complex functional system. (BASED ON Bernstein, 1947, p. 384)

non-linear systems composed of synchronized distributed elements" (Kazansky, 2015, p. 106).

The functional-system approach to the budding field of coordination dynamics took another major step forward through the scholarship of Bernstein (1947/1967), who would later be considered as a key founder of coordination dynamics research (Kelso, 1982; Thelen, 2000; Turvey, 1977). Per Bernstein, functional systems operate by means of synergies, namely relatively stable spontaneous peripheral coordinations that diminish degrees of freedom in any motor problem. This thesis avoids the determinism of a strictly invariant motor scheme, instead positing the flexible constitution and readjustment of each motor act in accord with local circumstances (Turvey, 1977). Each movement act appears to solve a motor problem by assembling multi-level regulatory mechanisms, in which motor and sensory processes are essentially coupled and interrelated (Bernstein, 1967; Mechsner et al., 2001; Turvey, 1977). A highly simplified model of such coupling is presented in Figure 6.1. This "reflex circle" is one of many that self-organize to enable the enactment of a complex movement. The reflex circle is triggered by a task to specify iterative corrections via ongoing comparison between movement anticipation (forward model) and sensory feedback. Thus, anticipation serves the function of prospective control, not of the representation of some statistically valid information. By way of application let us return to the snake story. Here, when the visual system detects a snake, its image is sent not to a central regulatory mechanism to elaborate appropriate reactions; rather, it is treated at the periphery by way of comparison with the anticipatory sensory image (of a trail without a snake), precipitating correction. Most probably the snake was not even recognized as such, but just seen as an obstacle for a step. The awareness of danger arrived only post facto.

Importantly, the forward model should be understood not as some form of abstract brain-driven anticipation. Many peripherally constituted forward models are supposed to serve the realization of a task via multiple, independent, and yet functionally assembled oscillating circles. Unlike the description of predictive coding by Bayesian (e.g., Körding & Wolpert, 2004) or quantum (de Freitas & Sinclair, 2018) statistics, we do not mean anticipation as being constructed independent of the behavior system, but as emerging from mere coupling between processes, as captured by the notion of a strong anticipation (Latash, 2015; Stepp & Turvey, 2010).

A following simplification of a well-known problem of catching a baseball (Chapman, 1968) might help the reader to grasp a process of strong anticipation mathematically modeled by Stepp and Turvey (2010). Imagine you are trying to catch a ball that has been thrown by a person nearby you. Weak anticipation would mean that you have some calculating system that estimates where the ball will drop. You run there and catch the ball. Strong anticipation would mean that, instead, you just run directly below the ball, adjusting your speed according to the ball's horizontal location. Inevitably, the ball will drop into your hands. In this case, you would not need to create any complicated internal representation of the ball trajectory, but just couple with the ball into a dynamic functional system. We believe that this type of anticipation is what was noticed by Anokhin (1935/1975) and Bernstein (1967), despite the sometimes misleading semantics, such as "forward model" or "an image of desired future," which may falsely imply internal representations.

Analysis of clinical data similar to Anokhin's led Mearleau-Ponty (1942/1963) to the elaboration of Husserl's idea of operative intentionality "which produces the natural and antepredicative unity of the world and of our life" (Merleau-Ponty, 1945/2002, p. xx) by unique presence, not a judgment upon its meaning. The construct of operative intentionality plays an essential role in phenomenological analysis of the constitution of mathematical objects (Zagorianakos & Shvarts, 2015; Zagorianakos, 2015) and is now reformulated as enactive intentionality within the contemporary enactivist trend (Gallagher & Miyahara, 2012). The construct of enactive intentionality captures the notion that recognizing an act possibility does not require any mental processing, any judgment: it is an immediate actualization of the enactment itself, when an organism becomes able to approach the world in a particular way.

An idea of motor problem is critical to expand the notion of functional system from typical organismic functions, such as breathing, chewing or walking, to the problem-solving processes. As soon as we follow Bernstein (1947/1967) in acknowledging that any movement is constituted on the spot in accord with task and environmental constraints, and as soon as we accept that enactive intentionality imposes forward models in the world to regulate performance, we understand any movement solely as a creative process that relies on previously established coordinations and engages previously established synergies of micro-processes or structural organizations. A functional system, then, is understood as a permanently adjusting dynamic "complex of neural elements and corresponding executive organs coupled in performing" (Kazansky, 2015, p. 105) a particular task under environmental constraints.

We may now describe the functional system as it emerges to fulfill the demand of "understanding," which ostensibly differentiates the solution of mathematical vs. motor problems.

From a complex-dynamic-systems approach, the process of solving a new task and further mastering this solution is characterized as innovation emerging in the dynamics and structure of coupled processes. Stephen, Dixon, and Isenhower (2009) asked participants to solve a gear-systems problem, that is to determine if a set of interconnected gears would turn and in which direction a target gear would rotate. One way of solving the problem is to follow the rotation of each gear. However, this solution can be substantially compressed by, instead, detecting whether the number of gears is odd or even. The researchers recorded participants' finger motions as they were solving the series of problems. Based on analyzing the complex dynamics of the fingers' motion, the researchers were able to predict the study participants' subjective experience of insight. Thus, the functional system of solving the gear problem switched from motor enactment to a mode that we might consider as a mental solution: an even parity check. However, the mental solution emerged as the spontaneous self-organization of a motor enactment that brought a new strategy to the surface of consciousness.

Phenomenologically, this new-strategy achievement within the functional system of problem solving may be described in Merleau-Ponty's words:

> At the decisive moment of learning, a 'now' stands out from the series of 'nows,' acquires a particular value and summarizes the groupings which have preceded it as it engages and anticipates the future of the behavior; this 'now' transforms the singular situation of the experience into a typical situation and the effective reaction into an aptitude. (Merleau-Ponty, 1942/1963, p. 125)

A possibility of solving the problem by imaginarily enacting its rotation is now compressed into a strategy that supplements the motor solution with simple calculation. That said, if participants were to argue for their condensed solution, they would likely move their fingers once again.

A similar idea of *functional complex dynamic systems* motivated Vygotsky's late rethinking of higher psychological functions, as sketched out in the neuroscientific project of localizing psychological functions:

> At the basis of such a theory lies a theory of the systemic and semantic structure of human consciousness. This theory proceeds from the paramount importance of (a) the mutability of the interfunctional connections and relations; (b) the formation of complex dynamic systems which integrate quite a number of elementary functions; and (c) the generalized reflection of activity in consciousness. (Vygotsky, 1987, p. 140)

The "elementary functions" (namely the previously established capacities) do not just jointly and independently work to serve the "complex functions" (as Vygotsky refers to the higher processes) but form a "united system" (p. 141) that fulfills the required task.

These early ideas were elaborated by neuropsychological analysis of brain injuries during World War II (Luria, 1973), and they are now confirmed by contemporary neuroimaging data that ground the neural reuse approach (Andersen, 2010). Multiple research evidences dynamic reorganization of the resources distributed in the brain into new "functional complexes" (Anderson, 2010) or "neuronal coalitions" (Jones, 2018), as they are reused in fulfilling new functional demands of the cultural environment. Thus, cultural arithmetic abilities recruit old regions, responsible for such ancient systems such as approximate number estimation, but these regions are connected with other regions, for example, those responsible for symbolic representations (Jones, 2018). This dynamic functional reorganization matches the radical enactivist theory of cognition (Hutto, 2019).

2.2 *Functional Dynamic Systems in Mathematics Learning: Action-Based Embodied Design*

Applying this theoretical lens to educational phenomena, in this paper we focus on the teaching–learning process that implements the pedagogical principle of teaching content as facilitating discovery. In particular, it follows Abrahamson's (2014) action-based embodied-design genre, which specifies the development and facilitation of educational activities, in which students: (1) first learn to enact a new goal-oriented movement that instantiates a target

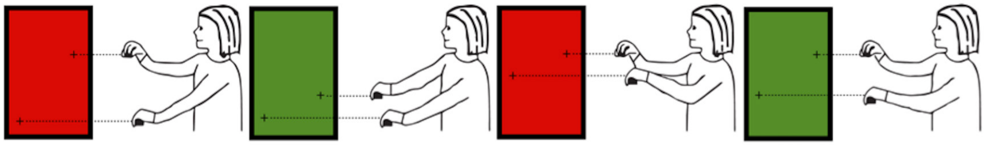


FIGURE 6.2 The Mathematics imagery trainer for proportion. Here the screen is green when one cursor is two times higher above the bottom of the screen than another cursor, for a 1:2 ratio. Art acknowledgment: Virginia J. Flood.

mathematical concept; and only then (2) are guided to analyze these movements using mathematical artifacts and discourse.

The Mathematics Imagery Trainer for proportions (Abrahamson & Trninic, 2011) is an environment that produces green feedback on a screen when a student's hands are placed on it at heights corresponding to an unknown ratio. Otherwise, the screen is red (Figure 6.2). For example, when the technology is set at a 1:2 ratio, the screen will be green when the student's left and right hands are 10 and 20 cm above the bottom of the monitor, respectively. The student needs to learn to keep the screen green while moving the hands, which means that the hands must move at different speeds.

Abrahamson and his collaborators have repeatedly observed students transition from the initial form of interaction, in which the hands move at the same speed, to the cultural form, in which the hands move at different speeds. The transition is characterized by the emergence of a new functional system where task-effective motor action is coupled with stable patterns of eye movements (Duijzer et al., 2017). This stable performance co-occurs with the emergence of attentional anchors.

An attentional anchor (AA) is an imaginary perceptual construction that emerges for individuals engaged in a demanding, ecologically coupled movement problem as their means of facilitating the coordination of motor actions, such as two hands moving simultaneously (Abrahamson & Sánchez–García, 2016; Hutto & Sánchez–García, 2015). From the perspective of coordination dynamics (Kostrubiec et al., 2012), the emergence of an AA can be implicated by state changes in an order parameter that marks a complex system's phase transition to a new, dynamically stable constitution. The AA has been compared to Piaget's construct of reflective abstraction, in the sense that it constitutes a new psychological structure that coordinates and subsumes existing schemes to cope with a new class of situations (Abrahamson et al., 2016), thus facilitating the performance of the functional system.

In turn, the AA, as a subjective phenomenological entity, may bear what Bartolini Bussi and Mariotti (2008) call "semiotic potential," that is, it may come to constitute a mathematically meaningful, cognitively accessible, and discursively articulable system of relations, similar to the emergence of the new strategy in the gear-systems problem (Stephen et al., 2009). As such, the learning process with the Mathematics Imagery Trainer is explicated as the emergence of a new functional system, which originates in the form of sensory–motor coordination and is later sedimented by cultural semiotic means in the process of conceptualization.

We have now introduced the notion of a functional dynamic system as a complex of embodied processes, including neuronal activity and executive organs, that are coupled in fulfilling a goal within environmental and task constraints. Following the action-based embodied design paradigm, students' transition from solving a movement problem to learning mathematics is a seamless phenomenological continuity, even as a new ontology comes forth that can be signified with technical disciplinary forms. Gravitating towards stable solutions, their new functional system stabilizes its performance and, at the same time, generates and maintains a new dynamic entity—the AA—that is phenomenologically experienced as a new strategy or a new object in the external world.

2.3 *An Intercorporeal Functional Dynamic System: A Goal-Oriented Coupling of Two Bodies*

Intuitive pump. Let us, this time, consider a child attempting his first rides on a bicycle. The auxiliary wheels have been just removed, but the child is still very unstable, so a parent is helping the child by holding the back-rack. At first, the child needs quite a lot of support. If the parent supports only "after" the child begins falling, it would be very strenuous to raise the bike, with the child still sitting on it, back to the stable position. Instead, parents anticipate, from tiny movements, the moment when the child is about to fall and provide the necessary corrective adjustments. Parents do not react to the fall, but restabilize the child before the fall through additional pressure to a particular side. In this embodied collaboration of riding a bicycle, the perception–action systems of child and parent are intercorporeally coupled as one system: it is this intercorporeal functional dynamic system, not child per se or parent per se, that fulfills the task of riding a bicycle.

We have stressed before that higher psychological functions are systemic entities that are functionally organized in accordance with the tasks that a child meets. Discussing their development, Vygotsky stresses its social origins: "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological)" (Vygotsky, 1978, p. 57).

These new cultural forms of behavior originally emerge in ontogenesis through "collaboration with more capable peers" (Vygotsky, 1978, p. 96) that creates a *zone of proximal development* (ZPD), namely a range of problems that can be solved only with help from a more knowledgeable other—not by a child or a student solely (Vygotsky, 1978). In the case of the teaching/learning process, this inherent collaboration on a problem, a student and a teacher form within the ZPD a dyadic *interpersonal functional system* (Newman et al., 1989). Historical analysis shows that the well-known term *scaffolding* (see Van de Pol et al., 2010 for a review) refers exactly to the collaborative process within this functional system of two people; this process resembles the functional organization of different brain and body sub-systems in acquiring new motor skills (Shvarts & Bakker, 2019).

Importantly, the "collaboration," which in both Russian and English literally means "collective labor," implies dual-agent activity, "the uninterrupted permanent cooperation of thought" (Vygotsky's Notebooks, Zavershneva & van der Veer, 2018, p. 354) that is deeply vested in both a child's and an adult's participatory interactive experience, including their sensory and physical experience. It is not the direct involvement of a child into a ready-made social practice; rather, it is considering the child as an active, constituting participant who negotiates and thus determines how the practice will develop no less than the adult does. Expanding Vygotsky's approach to education, Stetsenko writes: "Mind is always made in co-acting, together with other people, in shared collaborative activities that are part and parcel of wider social practices and collaborative projects" (Stetsenko, 2017, p. 319).

While from the wide cultural perspective this collaboration is understood as a part of *joint labor* (Radford, 2016), here we focus on the details of collaborative embodied activity, when student and teacher/tutor are immersed in a *space of joint action*, which is "made up of bodily resonance and intercorporeal coordination accomplished at different levels: speech, posture, gestures, artifact- and sign-mediated actions, joint perception, etc." (Radford & Roth, 2011, p. 232). What are the embodied processes that make tight collaboration between two people possible?

2.3.1 Cognitive Mechanisms of Intercorporeal Coupling: Joint Action and Joint Attention

Numerous studies have demonstrated how people engaged in joint action coordinate their bodily dynamics (Knoblich et al., 2011). Whereas some of these coordinations are planned, others spontaneously emerge during the joint action, such as coordination in the phase of iterative leg movements (Schmidt et al., 1990) or the synchronicity of two people observing each other tapping

(Oullier et al., 2008) or rocking on rocking chairs (Richardson et al., 2007). The dynamics between two bodies appear to be similar to coordination within one person (Schmidt et al., 1990). Jointly acting partners can also anticipate and endorse other properties of the co-actor's circumstances, such as destination, trajectory, and speed (Schmitz et al., 2017). A recent paradigm of hyperscanning (Dumas et al., 2011) provides a quickly growing set of evidence on inter-brain synchronization in joint action. For example, effective joint visual search is associated with similar wavelength of brain activity between subjects (Szymanski et al., 2017), and coordination of brains is achieved together with the achievement of coordinated finger movement (Fuchs & Kelso, 2018). Trivially, there are no literal neuronal connections between the brains; however, environmental affordances and task constraints give rise to the coupling of brain activities.

Consider further empirical demonstrations of intercorporeal dynamical coupling. When two people discuss objects in a shared visual sensory field, they tend to coordinate their visual attention on the objects, even when each person has no concrete cues with regards to the direction of the other person's attention (Richardson et al., 2007; Spivey et al., 2009). When a learner follows a facilitator's demonstration of manipulating a puzzle, their gazes coincide at the puzzle, without the learner following the gaze of the facilitator (Pagnotta, 2018). When an imaginary object mediates joint action (e.g., an imaginary number line mediating the comparison of two numbers), reaction times of two people will resemble the reaction times of one person, as though they share a joint imaginary space of action (Atmaca et al., 2008).

These and other case of interactions leading to joint attention suggest a model of joint attention as resulting from self-organizing multimodal coupling between co-acting partners performing an "exquisite real-time 'dance' of social interactions, in which effective adjustments within the dyad happen in fractions of seconds" (Yu & Smith, 2016, p. 3). Joint visual attention emerges as an outcome of joint object-oriented actions and leads to intercorporeal coupling of attending to a jointly manipulated object.

All these pieces of evidence support the idea of intercorporeal coupling between perception–action systems (Spivey et al., 2009) as constituting an inter-corporeal dynamic complex system in a self-organization process of aligning multiple resources, such as rhythm, intonations, and joint space (Dale et al., 2014). Thus, teacher and student form an interindividual system, in which they "temporarily lose their 'individual' identities, thereby forming cooperative units, or coordinative structures, that have unique properties that transcend the individual components" (Fogel & Garvey, 2007, p. 252).

2.3.2 Coupling of Two Bodies into a Functional System

When one is investigating two collaborating bodies in action, one may discern their emergent perceptuomotor synchronization. At the same time, one should bear in mind the functional character of this emerging system: Parent and child do not synchronize per se in their movements—they synchronize so as to realize the riding of a bicycle. As such, this teaching/learning system surfaces as a functional system, because it is the complex of two bodily systems coupled in fulfilling a task at stake. This view bears implications for planning and analyzing teaching strategies.

An important factor in the emergence of dyadic functional systems is humans' ability for intentional synthesis[2] (Shvarts & Zagorianakos, 2016). Intentional synthesis, we propose, emerges between student and tutor whose sensory-motor systems are coupled by joint anticipation of possible actions. Importantly, the construct of intentional synthesis refers not to the conscious understanding of what the other person is doing, as the phrase shared intentionality might suggest (e.g. Tenenberg et al., 2016; Tomasello & Carpenter, 2007) but to an ability of one party to directly respond to the other's actions in the context of shared activity. Intentions and dispositions are seen directly "in the embodied behaviors, and movements, and facial expressions, and gestures, and actions of others" (Gallagher, 2011, p. 298; see also Meyer & Von Wedelstaedt, 2017) and can be experienced without reconstructing others' mental states via "mind minding" (Hutto, 2011). Enactive intentionality plays a key role in establishing a primary experience of the other engaged in intentional synthesis (Gallagher & Miyahara, 2012). That is, enactive intentional synthesis connects two bodies in action, not just two cognitions.

Experimental data support the thesis that humans immediately experience the other's directedness: by observing someone's movement, humans are able to anticipate the arc of their intentionalities, even from early kinematic information (Becchio et al., 2012; Pesquita et al., 2016). Moreover, as humans observe somebody's actions, they do not follow current moves but predict the next ones (Flanagan & Johansson, 2003; Gredebäck & Falck-Ytter, 2015).

Anticipating the other's actions relies on visual cues from different parts of the actors' bodies (Vaziri–Pashkam et al., 2017), thus exemplifying the complexity of embodied interaction between two co-actors. Taking into account each other's goals also appears to be critical for inter-brain synchronization, which was observed in case of cooperative and obstructive interaction, but not in parallel play (Liu et al., 2016).

These results might be attributed to simulative activity of the observer's mirror neurons system (e.g., Rizzolatti et al., 1996); however, theoretically they evidence strong anticipation (Stepp & Tuvey, 2010) of each other's behavior within some specific task, just like it was theorized for coupling between perception and action. Two people do not need to model each other's behavior, but instead respond directly to it within a particular anticipatory result, namely a task solution. In our example, it is not that parents think that a child is "planning" to fall from a bicycle and decide to catch him. Rather, it is a direct intentional co-action of bodily systems.

Finally, our theoretical elaboration suggests that two collaborating people form an intercorporeal dynamic functional system, namely a complex of two bodies, including brains, that are tightly and yet dynamically coupled to each other while performing a joint task.

2.4 *Teaching/Learning as the Development of an Intercorporeal Functional Dynamic System: Micro-Zones of Proximal Development*

2.4.1 On How the Dynamics of Intercorporeal Systems Develop

We have described embodied mechanisms that allow two bodies to couple in one intercorporeal functional system, as they perform joint actions and approach objects in joint attention towards a particular goal.

The body of literature that proposes the analysis of interpersonal coordination in teaching and learning is growing (see, e.g., Okazaki et al., 2019), including synchronization of a student's and a tutor's brain activity in the scaffolding process (Pan et al., 2020).

This system of two bodies is characterized by mutual transformation of strategies, where not only the student is influenced by a tutor, but also the tutor is influenced by the student. The tutor's motor actions in showing how to solve a Tower of Hanoi puzzle are influenced by the way the student imitates the tutor's actions (Okazaki et al., 2019). In the case of a joint performance in a Tetris game by an expert and a novice, the perceptual actions of both become more similar to each other, compared to the corresponding eye movements of participants in expert–expert or novice–novice pairs (Jermann et al., 2010). Analysis of infant–adult collaboration shows that joint attention can be initiated by both a child and an adult (Yu & Smith, 2016), and both patterns are present in student–tutor collaboration (Shvarts, 2018a). In mathematics-education literature, this mutual transformation of strategies by a student and a tutor, as they collaborate, has been theorized as symmetrical positions in a zone of proximal development. In turn, these symmetrical co-adjustments enable asymmetrical influences from the teacher (Roth & Radford, 2010).

Let's focus on the way a student–tutor functional system evolves through the teaching/learning process. How do changes emerge in an intercorporeal dynamic system, such that initially the adult must support the child substantially as she sits on a bicycle, yet at the end the child is capable of riding on her own?

Analysis of the infant–caregiver interaction suggests three stages of a dynamic transformation that can be easily applied to educational situations (Fogel, 2006): the first stage is that of ordinary variability, the variability of a new pattern that has been established between two individuals (e.g., a parent re-supporting a child each time she is about to fall). The second stage is called innovation, as it appears novel to the participants in the situation (e.g., a scaffolding process, where a parent occasionally lets the child teeter a bit more, before the parent instigates corrective movements). This innovation might lead to transformation in the third stage, which is called developmental change, when a new pattern of interaction that emerged in the innovation stage is established and becomes an essential part of the system (e.g., fluently riding a bike).

An illustration of this view in teaching/learning processes, modeled as transformations in complex dynamic systems, has been described in relation to the improvisation-driven approach in teaching music. In four-hands piano improvisation, first, tutor and student establish a coupled intercorporeal system within "spontaneous achievement zones" (Laroche & Kaddouch, 2014, p. 2); next, the teacher suggests an improvising pattern beyond these already-explored zones, thus inviting the student to develop dynamics that are beyond the student's unassisted effort. In response to the invitation, a new pattern is established between two improvising partners, wherein the student transcends her previously stable zone of achievement. We align this moment of invitation with Vygotsky's notion of the zone of proximal development, which was classically introduced in relation to the problems that lie beyond the student's current capabilities yet are accessible together with a teacher (Vygotsky, 1978). However, bearing in mind that the tutor's invitation and the student's response are happening in a short moment within a long teaching/learning interaction, we suggest referring to it as a micro-zone of proximal development (mZPD) (see also Kimmel et al., 2014; Shvarts & Abrahamson, 2019) that emerges as a momentary dynamic possibility within extended teaching/learning collaboration.

From the perspective of functional systems, the tutor's invitation to develop new dynamics makes evident a development of intentionality within the functional system. The teacher attempts not only to accomplish the current task, but always intends beyond, toward further teaching goals. Stable dynamics do not change, unless a teacher perturbs the system, such that it reconfigures to meet new goals.

2.4.2 An Object of Joint Attention Develops

That said, achieving positive change in piano-improvisation dynamics is very different from acquiring new understandings of mathematical objects.

As a result of the mathematical teaching/learning process, the student and a tutor's *shared action space* needs to become structured in a similar way (Radford & Roth, 2011). Note, however, the complexity of educational situations with respect to the ontologies composing this shared action space. The shared space per se is highly ambiguous, and in the course of engaging in activities within this space, students come to distinguish meaningful mathematical objects and structures from what have been perceived as arbitrary lines and points (Abrahamson et al., 2014; Arcavi, 2003; Radford, 2010; Shvarts, 2018a; Zagorianakos, 2016; Zagorianakos & Shvarts, 2015). As such, it becomes problematic to state simply that joint attention involves two (or more) people focused on th*e same object*, as if an object is an objective ontology. Rather, student and tutor *come* to distinguish the same object in a shared domain of scrutiny (Goodwin, 1994). That is, through the process of joint attention, student and teacher develop shared theoretical perception (Radford, 2010).

With this insight, we ask in turn, what is the scaffolding process that can bring about theoretical/professional perception of visual materials in mathematics?

A straightforward answer—that an expert guides the student's attention along the visual materials, "showing" the mathematical structures—has been shown unsuccessful for the mathematical domain (e.g., Schneider et al., 2018; van Gog et al., 2009). Indeed, studies of adult–infant joint attention have demonstrated that working *with* a child's focus of attention bears greater impact on learning than initiating a new locus of joint attention (Tomasello & Farrar, 1986). This finding, we believe, obtains in the case of a tutor supporting a student's work with an interactive virtual learning environment.

In order to understand how mathematical objects come to be distinguished in student–tutor collaboration, we built on enactive intentionality of any active behavior. An intended object is embedded in a functional system's anticipatory cycle. As a consequence, when two people are not engaged in joint actions, objects composing a shared action space bear for different people different opportunities for action. A tutor might see a parabola graph as depicting a quadratic function, while the student sees it merely as a U-shaped form that can be copied. Only once they are coupled in a functional system within intentional synthesis, thus aiming at the same activity objective, will they come to attend a jointly manipulated intended object.

Embodied action-based designs provide rich opportunities to witness how an object, which emerges as an intended invariant of sensory-motor bi-manual coordination, then further emerges as instantiating a normative conceptualization of proportional relation between the heights of the two hands. We witness how a tutor and a student mutually transform each other's perception of the domain of scrutiny, as they develop verbal labeling for previously established sensory-motor coordination. The student and the tutor begin with vague references to yet-unarticulated emerging functional entities (Flood et al., 2016). Tutors' tactics are demonstrated as comprising repeating, revoicing, and elaborating on the student's multimodal utterances, both verbally and gesturally (Abrahamson et al., 2012; Flood, 2018; Flood & Abrahamson, 2015). In each case, the tutor does not offer the student a ready-made explanation but, instead, adjusts his multimodal flow to the student's ad hoc expressions of her own experience. In cases where tutor and student co-operate the device, they enter "intersubjective sensorimotor coordination by anticipating and closely tracking each other's actions" (Abrahamson & Sánchez-García, 2016, p. 230).

Theorizing this process from the perspective of intercorporeal functional systems, we see the student–tutor system as bootstrapping its new dynamics. The bootstrapping is mobilized by the tutor iteratively initiating change in one modality, such as speech, while preserving coordination in another modality, such as gesture (Flood, 2018). Preserving intercorporeal coupling and anticipation, this iterative process progresses through multiple mZPDs, namely the moments in collaborative activity where a tutor's move introduces a change in the dynamics of student–tutor coupled system. Finally, this collaboration leads to the constitution of new forms of behavior that were not present in the student's (and perhaps the tutor's) repertoire (see also Fogel, 2006; Jurgens & Kirchhoff, 2019; Laroche & Kaddouch, 2014). These newly constituted forms immediately appear to the participants as meaningful, since they emerge within the participant's web of intentionality.

The teaching/learning process does not end with joint actions, because the ultimate educational objective is the student's solo mathematical performance. For accomplishing this ultimate objective, a newly acquired dynamics of the intercorporeal system needs to differentiate from the coupled system into a solo functional system of a student. With this theoretical notion of a student differentiating from a coupled student–teacher system, the latter Vygotsky substitutes his own earlier idea of interiorization (Vygotsky's Notebooks, Zavershneva & van der Veer, 2018). However, this final part of the learning process is beyond the scope of this paper, which focuses on the intercorporeal system and its transformations in mZPDs.

2.5 *Research Questions*

The aim of this paper is to describe the process of mathematics teaching and learning by applying and coherently integrating the radical embodied approach, as based on dynamic systems theory, and the cultural–historical approach. In the theoretical sections of this paper, we have demonstrated historical congruency as well as epistemological grounds for bridging these approaches, and we explicated central notions for the analysis of teaching/learning from this integrated perspective, namely: the *inter*corporeal *dynamic functional system* and *micro-zone of proximal development* (*mZPD*). In the empirical section, we aim to provide evidence that these theoretical concepts form a coherent and workable lens. Our research questions concern:

1. What kinds of evidence for *intercorporeal coupling* between a student's and a tutor's perception–action systems could researchers discern in multimodal data that include dual eye-tracking, video, and audio records?
2. How does a *mZPD* emerge in the student–tutor collaborative educational process? What embodied opportunities does this coupling provide the tutor for detecting a *mZPD,* namely, identifying the best moment when a teaching prompt would move a student towards new understanding?

3 Methodology

The study utilized eye-tracking instruments to investigate teacher–student dyadic collaboration in a mathematics tutorial centered on an action-based embodied-design module for parabolas. Below, we warrant these methodological choices as constituting viable operationalizations for a Vygotskian investigation into the hypothesized mechanisms of teaching–learning as collaboration in mZPDs.

In applying his cultural–historical approach as a guiding philosophy for empirical studies into developmental psychology, Vygotsky sought to design an "experimental–genetic method" (Vygotsky, 1978), whereby particular conditions are created that enable researchers to witness the development process. Regularly, we are not able to observe the constitutive parts of a higher psychological function, because its functioning process is quick, internal, and automatized. Vygotsky proposed to investigate higher psychological functions in their earlier, social form by creating special cultural conditions for a new function to appear. This method therefore involves active participation of a researcher qua 'more knowledgeable other' in co-constituting a student's future abilities. As such, unlike the natural sciences, where the researcher constitutes a "sterile" agent, who administers the experiment exterior to the phenomenon under inquiry, in Vygotsky's genetic developmental experiments the researcher is an integral component of the phenomenon, part of the basic unit of analysis—the researcher is conceptualized as a critical party to the genesis of the student's higher psychological functions. Accordingly, the experimenter in our study interacts as a tutor with the student; later, this just-instructed participant acts, in turn, as the tutor to a new "student" participant, so that the developmental phenomena would not be limited to an intervention conducted by a particular person with privileged information and theoretical biases (the researcher). Later, the comprehensive data set from this exchange is analyzed as transformations in complex dynamic systems. Specifically, we look to represent and investigate multimodal empirical traces of the higher psychological functions as shaped by, and shaping, the emergent systemic dynamics of the teacher–student collaboration.

Our methodological instruments and educational design are chosen to make explicit the dynamics of transformation. Embodied action-based design (Abrahamson, 2014) serves as our theoretically grounded platform for surfacing in spatially articulated form the student's transformation dynamics through the teaching/learning process. Students' dynamic sensory-motor innovations are traced through eye-tracking. Given that we aspire to document the emergence of new psychological functions in the student–*teacher* intercorporeal dynamics, we use *dual* eye-tracking technology (DUET) to also trace the teacher's optical behavior and to analyze relations between teacher and student eye-tracking data (see Section 3.1).

In sum, a combination of selected theoretical, methodological, and design considerations led us to establishing a congruent theory–methodology–design bundle (Shvarts, 2019) that is particularly sensible to the phenomenon in question, the coupling between student and tutor perception–action systems and the emergence of a distributed *intercorporeal dynamic functional system.*

3.1 *Dual Eye-Tracking Technology*

Dual eye-tracking (DUET) is an emerging instrument in educational research. DUET allows synchronous tracking of the eye movements of two people. DUET holds potential for the analysis of ongoing teaching/learning collaborations by capturing participants' synchronous attention to features of the visual scene. In our research, we use two Pupil-Labs (Kassner et al., 2014) eye-tracking goggles, which are head-mounted eye-trackers. Head-mounted eye-trackers allow freedom of movement in ecological settings, where two people share a common space and discuss manipulations, such as on a shared monitor (Figure 6.3). Thus, DUET supplements traditional audio and video recording of two people engaged in an activity by synchronously capturing each participant's perception process as it is disclosed through the directions of their gazes. The novelty of our DUET equipment (compare with Lilienthal & Schindler, 2017; Schneider et al., 2016) is that we are able to overlay the scan paths of both

FIGURE 6.3
Dual eye-tracking experimental setting where two participants discuss an image on a shared screen

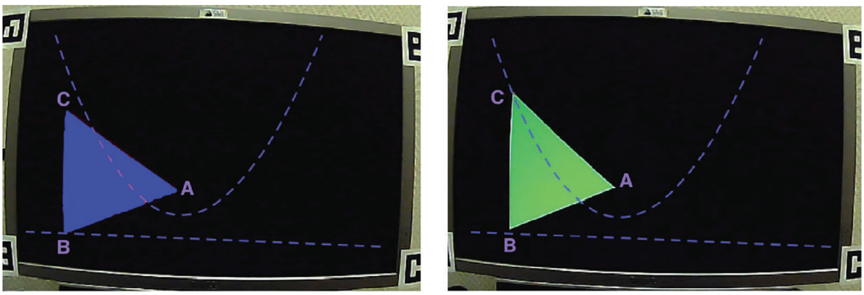
participants' eye movements on the video of their actions in the shared visual scene, including manipulation and gesture (Shvarts et al., 2018). This technical solution makes qualitative frame-by-frame analysis possible and efficient. The sample rates of the eye-trackers (60 Hz) and the world scene camera (30 Hz) allow us to grasp fairly well most of the events of interpersonal communication. Data records of the two participants' eye-tracking events are synchronized at about 1ms.

3.2 *Procedure and Participants*

Four 17–21-year-old students took part in our dual eye-tracking study. All the students had completed secondary school at a middle level of mathematics proficiency. There were two pairs, and each of them went through the following procedure: The tutor and the student wore eye-tracking goggles and took turns going through a pre-task five-point calibration procedure. Then the task was run, and the student was free to manipulate a target object on the screen by finger movements on a touchpad lying on the desk. The three task stages (see below) were run consecutively. The first student performed a parabola activity, with the researcher (first author) serving as tutor. Next, the first student became a tutor for a second student, who had not been present during the first stage.

3.3 *Learning Activity*

In this study, we constructed an activity following the principles of the action-based embodied design genre (Abrahamson, 2014). In this activity genre, the student interacts physically with a responsive technological platform. The student is tasked to discover on the screen a set of loci that satisfy a target state, such as causing an object to take on a particular mathematical property. In this study, students manipulated the vertex of a triangle (Point C in Figure 6.4) in an attempt to make the triangle green. The collection of points that satisfy this task objective are located along a parabola. This solution results from the geometrical properties of parabolas that have been programmed into the activity as task constraints bearing particular numerical values: To make the triangle green, Point C should be positioned on the screen so that it is equidistant from a straight horizontal line, namely the parabola's directrix (see Figure 6.4, Point B directly below Point C, running along the line) and a separate point (see Point A fixed on the parabola's focus); in this case the triangle ABC is isosceles, as BC = AC. Consistent with the design principle that semiotic symbols should be absent in the initial embodied activity (Abrahamson, 2014), only a triangle is featured on the screen during the first stage of the task (the dashed lines in Figure 6.4 are for illustration only and are never shown to the students). Our activity follows earlier work that has used the topic of parabolas as a productive resource that generates interesting empirical data related to the development of mathematical conceptualization from an embodied activity (Brown et al., 2013; Zagorianakos & Shvarts, 2015).



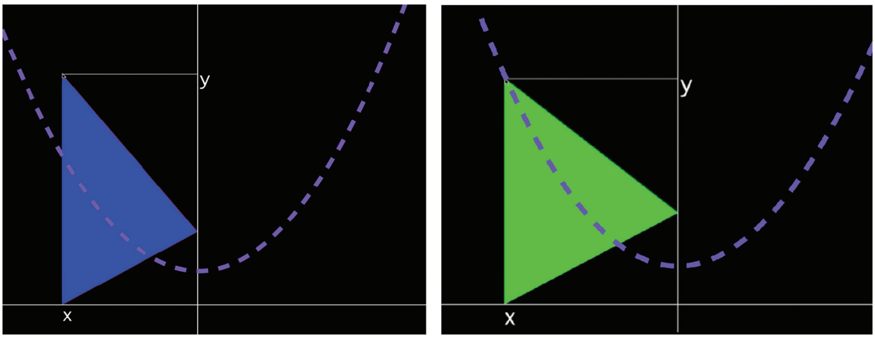
a. b.

FIGURE 6.4 An action-based embodied design for parabolas. The triangle is green when it is isosceles with BC = AC, where B runs along the horizontal dashed line, A is the parabola's focus, and the student manipulates Vertex C. By keeping the triangle green while moving Vertex C, the student would effectively be inscribing a parabola. 6.4a presents a non-target state and 6.4b presents a target state of the screen. Note that the labels (A, B, C) as well as the dashed lines in this figure are only used here to illustrate the design for readers of this text—these lines did not appear for the students as they engaged in the activity

As discussed earlier, collaboration with a tutor plays an essential part in students' transitioning from prospectively mathematical sensory-motor experiences to normative mathematical concepts. Note that the tutors in our study were instructed to help students without giving them explicit solutions.

In the first stage of the activity, the tutor asked the student to "make the triangle green" (the triangle's non-goal state is blue); once they succeeded, the tutor suggested "move the triangle in a way that it will always stay green." The students are initially unaware of the rule that determines the triangle's color. Rather, they solve a situated motor task of keeping the triangle green while manipulating Vertex C. Once the task is solved, the tutors ask, "What is the rule that determines the color of triangle?", thus shifting the students' goals towards reflecting on their activity. The teaching/learning process transpires as the joint activity of iteratively dialogue and enactment, eventually leading to the student discovering the rule through active exploration.

In the second stage, mathematical symbolization is introduced, including the appearance of the orthogonal axes of the Cartesian plane and X and Y markers for projections of Vertex C on the axes (Figure 6.5). The students were scaffolded by the tutors to determine the formula of the curve traced by the manipulated Vertex C in keeping the triangle green. In this process, a tutor explained to the student that the distance AY can be calculated as $\sqrt{y^2 - x^2}$ (Figure 6.6). With varying levels of support, eventually the student noticed that the distance from A to the (0,0) point is constant, wrote the equation $\sqrt{y^2 - x^2} + d = y$, and, through algebraic manipulation, arrived at the formula $y = \left(x^2 - d^2\right)/2d$.



a. b.

FIGURE 6.5 The second stage of the action-based embodied design for parabolas. 6.5a presents a non-target state and 6.5b presents a target state of the screen. Axes and markers for coordinates are introduced. (The dashed line was not exposed to the participants)

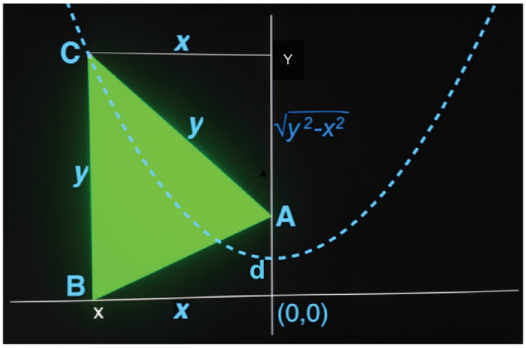


FIGURE 6.6
An intermediate stage of the solution. (The blue inscriptions were not exposed to the participants, but discussed with the tutor)

3.4 *Data Analysis*

All data were analyzed following principles of micro-ethnography (Streeck & Mehus, 2005) in search of patterns across student actions, student/tutor gaze parameters, and the dyad's multimodal utterance. As we will now explain, we draw on Vygotsky's idea of theory–method dialectics to constitute our methodology within the triplet of theoretical principles, methods, and research questions (Radford & Sabena, 2015).

Our theory identifies the unit of our analysis as the *intercorporeal* system of a tutor and a student engaged in a teaching–learning activity rather than their respective *individual* teaching and learning processes (see also Newman et al., 1989, pp. 59–75).

Our dual eye-tracking method and embodied design enable us to unpack this theoretical unit through the focus on multimodal intercorporeal coordination and discoordination between: (a) student attention and tutor attention; and (b) student action and tutor attention. Moreover, given the theoretical definition of a functional system as emerging under a particular goal, we also focus our analysis on the tasks that participants solve and share in each coordinated mode. Thus, aiming to uncover the process of functional system development, we were tracing the changes in intercorporeally-coupled behavior as these manifested independently in different modalities: motor actions and perceptual strategies. Finally, with the focus on the emergence of a mZPD, we analyzed with great detail when and how the tutor elicits the student's sensory-motor strategies and facilitates the student's mathematical ideas.

Analyzing the video, audio, and eye-movements data from all four tutor–student pairs (of which two were researcher–participant pairs) we iteratively used diverse scales, jumping from distinguishing major changes in the process to very detailed frame-by-frame analysis of eye-tracking and video data and back.

4 Results and Discussion

We report on episodes from Stage 1 and Stage 2 of the activity. These stages present contrasting examples of student–tutor communication. Stage 1 exemplifies a discovery-based embodied activity that almost entirely relies on the student's sensory-motor experience, and Stage 2 exemplifies a traditional process of scaffolded problem solving: a tutor provides as much support as needed for the solution of the problem.

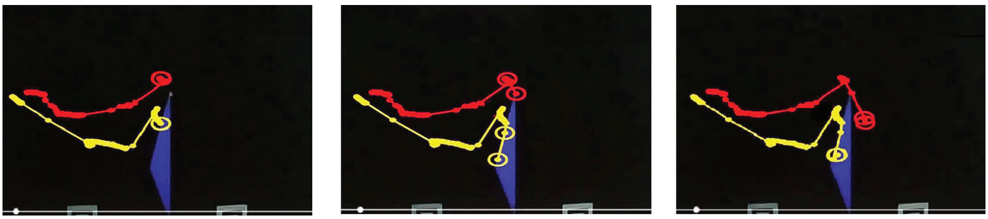
4.1 *Stage 1. An Intercorporeal System and Micro-Zone of Proximal Development: A Focus on the Student's Embodied Actions*

Analysis of Stage 1 of the learning activity elicited four phases that we detail below. These phases were observed in all student–tutor pairs, whether the researcher or an instructed student served as tutor. Thus, whereas we use the first-person singular pronouns, these descriptions obtain to all cases.

4.1.1 Phase 1: Convergence: Establishing *Inter*corporeal Dynamic Functional System

The student begins exploring the problem space for "green spots," and so the triangle is blue most of the time. While the blue triangle is moved, student and tutor gaze paths are tightly synchronized (see Figure 6.7 for an example).

Recall that one person (the student) is performing the screen action, while the other person (the tutor) is only observing. The tutor's gaze lags after the student's gaze by about 17 ms, thus following precisely the student's movements of the triangle on the screen. When the triangle's movement is completely



a b c

FIGURE 6.7 Synchronous eye movements of the student (yellow, lighter) and the tutor (red, darker), as the student manipulates Vertex C (the blue triangle's top vertex). Each node-like point on a gaze-path line represents one sample, and the duration between two points is about 17 ms. The circles represent current eye-positions at the moment of this video frame (two circles mean that the video camera is slower than the eye tracker), thus Figures 6.7a, 6.7b, and 6.7c show the sequential development of gaze path unexpected, the tutor's gaze delays for several hundreds of a second, then immediately regains synchronization.

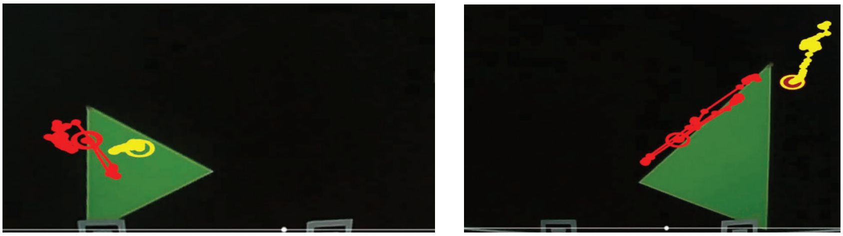
The dynamic synchronization makes evident that joint attention is not determined by gaze-following (Brooks & Meltzoff, 2014) or mind-guessing but by attending to the dynamic scene that naturally provides an opportunity for joint experience. Thus, joint attention emerges naturally through shared orientation to a manipulated object (see also Yu & Smith, 2017). However, passively following the object would not result in the level of precision observed in the data. Rather, these results correspond to findings that people tacitly predict the movements of others while observing their actions (Flanagan & Johansson, 2003; Gredebäck & Falck-Ytter, 2015).

An individualistic perspective on our data would model two separate subjects—the student and tutor—each following the same triangle on a screen. However, our analysis conceptualizes the student and tutor as a single unit. Moreover, by treating the tutor's gaze as anticipating—not just following—the student's action, we have grounds to analyze this dyad's activity as manifesting a *coupling between two bodily systems*, where an external dynamic object (viz. the virtual figure on the screen) facilitated the dynamical self-organization of this system (Dale et al., 2014; Spivey et al., 2009). Finally, this coupling arguably constitutes a *functional system*. Namely, the coupling is not an automatic outcome of visual constraints. Rather, it emerges under the pressure of *intentional synthesis* between a student and a tutor. If she were not intending to teach, the tutor could choose to gaze elsewhere, with little if any regard to the student's learning process.

4.1.2 Phase 2: Divergence: The Tutor Tunes Away to Anticipate an Innovation in the Student's Dynamics

Phase 2 begins once the student has found several green locations and has partially succeeded in keeping the triangle green while moving it but is not yet fluent in these movements, has not yet constituted a new sensorimotor coordination, and has not yet formulated their sensorimotor strategy in terms of a mathematical rule for keeping the triangle isosceles.

Unlike Phase 1, Phase 2 is not characterized by intercorporeal gaze synchronization. On the contrary, while the student's gaze mostly continues to follow the operational point (Vertex C), the tutor changes their perceptual strategy to a cultural form as soon as the triangle is green: the tutor's eye-gaze patterns suggest she is construing the green triangle as isosceles, thus manifesting a *cultural form* of perception. The tutor, who is not physically manipulating the triangle, thus activates the attentional anchors (AAs) that they would employ



a. b.

FIGURE 6.8 The tutor's (red) AA s are manifested in rapid, iterated, back-and-forth saccadic eye movements, while the student's eye movements (yellow) are initially restricted to continuously tracing only the focal object they are manipulating (Vertex C of the moving triangle)

if they themselves were manipulating the triangle from its current location, whereas, in fact, the triangle is entirely under the student's command.

Figure 6.8 exemplifies the tutors' AA as deployed either along the triangle's median or along its side. While the tutors were not actually performing any action, their perceptive systems were coordinated with the actions of the students, consistently "launching" from the constantly shifting location of the triangle under the student's command, as though the tutor was practicing how they would perceive the triangle if they were manipulating it themselves. Thus, despite discoordination in *their perceptions*, the tutor's AAs reveal *coordination between the tutor's perception and the student's actions*, thus again supporting a model of student and tutor as dynamically coupled in a single distributed intercorporeal system.

In Phase 1 we observed a natural coupling of the student's and tutor's perceptions that emerged under the tutor's general intention to attend to the student's performance. However, the tutor's eye-movements in Phase 2 cannot be explained as such. If the tutors were about to perform this very action themselves, they would normally utilize AAs to boost enactment. Surprisingly, our data evidence that AAs were initiated despite the fact that tutors were only observing the student's performance. Apparently, the tutor's sensorimotor coordinations were elicited by actively observing the student's goal-oriented actions, a phenomenon which might be explained by the mirror neurons hypothesis (e.g., Rizzolatti et al., 1996). If in the previous phase we could claim only general congruency of teaching and learning intentionalities, here we observe *intentional synthesis*, as the tutor is tuning into the performance of a student's particular actions.

Two very different perceptual strategies—the tutor's AAs and the students' tracing of the manipulated vertex of the triangle—are both tightly coupled with the student's motor actions, revealing a complex architecture of the intercorporeal system at this moment. The tutor's perception of the green triangle is radically different from their perception of the blue triangle, while the student's perceptual strategy does not change with the change of color. We may suspect that the manipulated triangle bears different subjective meanings at this moment: The tutor immediately saw the triangle as an isosceles and kept tracking it as such, while for the student the triangle was still a non-specific triangle that needed to be kept green in accordance with the task request.

The tutor's perception reveals innovation in the teacher's perception–action dynamics. However, unlike in the example of piano improvisation, where a tutor would explicitly suggest innovation by performing it within joint enactment (Laroche & Kaddouch, 2014), this innovation does not change the immediate dynamics of the system, because the tutor's perceptual strategy is invisible to the student.

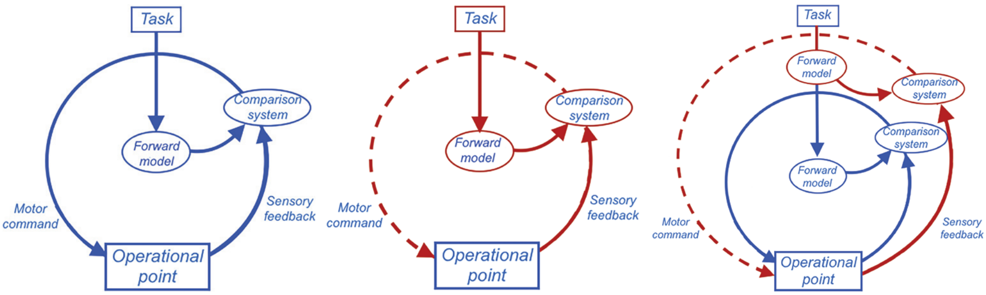
The question as to the specific pedagogical significance of Phase 2 remains: What could possibly be the pedagogical advantage of this temporal lapse in perceptual coordination? We speculate that the student's perception–action processes need to become self-organized through their own activity, under the pressure of the keeping-green task and within the design constraints. Effective tutors intuitively provide this opportunity by avoiding premature intervention. We suggest that the tutor's perceptual behavior—that is, the tutor's activated AAs—allowed the tutor to detect the student's perception–action self-organization, as they were thus able to compare the student's actions with their own anticipated fluent performance. As we will see later in Phase 4, below, the tutor's detection of the student's self-organization moment provided the tutor an opportunity to productively intervene in the student's activity within a mZPD.

4.1.3 Phase 3: Embodied Discovery: New Dynamic Patterns in the Student's Functional System

How did the AAs finally emerge in the student's activity, and what was the role of the tutor in this? Phase 3 is marked by the tutors encouraging the students by expressing appreciation of their movements and inviting them to move faster, slower, or more accurately. The video data manifest students' increasing motor fluency in manipulating the virtual objects, while the eye-tracking data reveal the systematic emergence of new AAs facilitating the students' actions. Students' AAs are evidenced in their rapid, iterative saccadic eye movements either along the median of the triangle (see the yellow activity in Figures 6.9a, 6.9b) or along one of its sides (Figure 6.9c), concurrent with enhanced motor performance (greater accuracy and speed).

a b c

FIGURE 6.9 A student's AA s are reflected in rapid iterative saccadic eye movements either along the median of the triangle (a, b) or along one of its sides (c). (In the interest of this figure's clarity, the tutor's overlapping gazes—the red lines—have been removed from this image)



a b c

FIGURE 6.10 Intercorporeal coupling between student and tutor: forming a distributed perception–action system. The dashed line indicates imaginary simulation of action. (a) A student's model—a perception–action loop in regulation of enactment; (b) a tutor's model—a perception–simulated-action loop; and (c) coupling of two perception–action systems upon the joint operational point

In Phases 1–3 the tutor's and the student's perception–action systems stay intercorporeally coupled: in each case the actual perception–action regulation of the student is coordinated with the tutor's perception, which regulates the simulated enactment (see Figure 6.10). However, this coupling develops from phase to phase. Very simple synchronization between gaze data is observed in Phase 1: the natural smooth tracking of the moving object by both participants guaranties the coordination of perceptual experiences. Yet, the student's and tutor's perception serve different functional roles in Phase 2. While the student is still attached to the "keeping green" task, the tutor jumps forward and distinguishes the triangle as isosceles. Notwithstanding, the intercorporeal system is preserved, because both their perceptual strategies are tightly coupled with the student's motor actions.

In Phase 3 our empirical data support the conjecture that in each pair the student's and tutor's perceptual strategies develop independently (different

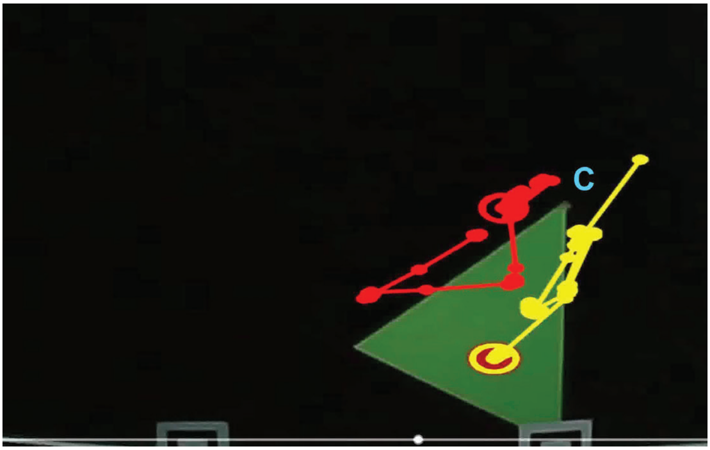
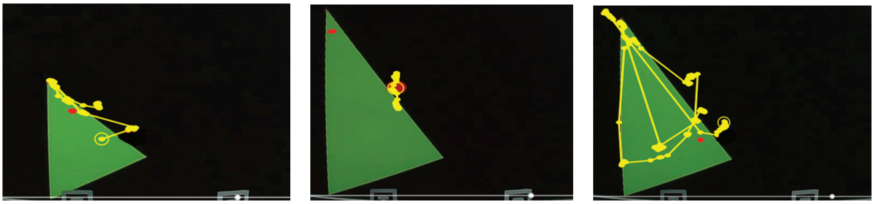


FIGURE 6.11 The student (yellow) and tutor (red) perform iterative eye movements, but these movements are different, signifying different AAs: Whereas the student performs saccades along the median of the triangle, the tutor performs a three-step repetitive eye-movements: she traces one side of the triangle (on the left) and then goes to the other side and back to the manipulated Vertex C. (Note that the tutor's red gaze path should be interpreted as offset to the left of its actual position, due to constraints of instrument sensitivity. The "C" notation of the vertex was added to this figure for clarity)

sensory-motor routines are shown in Figure 6.11). However, their locally diverse strategies remain functionally coupled to the object of their joint action. *Whereas the tutor and student may not construe the situation in the same way, they need somehow to codify the situation that they jointly lived through, for further references. They need to coordinate the mathematical labeling of their respective eye-movement strategies.* In the next section we will trace this emergence of a coordinated verbal reference to the established sensory-motor routine during Phase 4.

4.1.4 Phase 4: Micro-Zone of Proximal Development: Inviting the Student to Reflect on the Fluent Performance

Phase 4 begins where the student is manipulating the triangle quite fluently, and both tutor and student are enacting eye-movement path patterns corresponding to their respective AA. However, the emergence of an AA does not imply an articulated *expression* of the rule. Rather, for the tutor, witnessing the student's fluent motor performance accord with task specifications (keeping the triangle green) marks the student's readiness for the next steps of the educational interaction. The tutor introduces a new goal: reflecting on the performance and expressing the strategy in the form of a verbal rule.



a b c

FIGURE 6.12 The student focuses attention on the triangle: (a) She explores the right side of the triangle; (b) The student momentarily pauses her actions while attempting to recall the geometrical term for the class of triangles that her actions have (unwittingly) been generating (viz. isosceles triangles); (c) The student answers the tutor's question about the meaning of the term "isosceles triangle." (In the interest of this figure's clarity, the tutor's overlapping gazes—the red lines—have been removed from this image)

At this point, the tutor invites the student to reflect, as the following transcription exemplifies.

T1: Could you think about the triangle? How do you manage to keep it green?
S1: Alright … The triangle is obviously … ((She explores only the right side of the triangle in Figure 6.12a)) Oh, I am bad with geometrical terminology …
T1: It's alright, you can explain, I will help you with the word.
S1: It is not equilateral … but isosceles ((Figure 6.12b)). I think that's how it's called.
T1: Yes, what does it mean?
S1: It means … that it has two sides of equal length ((Figure 6.12c)).

Interestingly, the student looked directly along the triangle's sides only *after* verbalizing the rule; she apparently already knew that the sides were equal and looked there perhaps to confirm that her verbal explanation is correct.

The transcription, above, illustrates a pedagogically desirable outcome. In other cases, the tutors queried the student's behavior prematurely—before the student had established a fluent sensory-motor routine for manipulating the triangle. The student was consequently unsuccessful in articulating the rule, and the collaboration would subsequently further iterate between practice and dialogue.

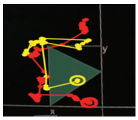
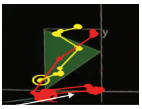
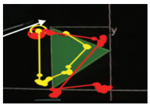
We theorize the successful interventional event as exemplifying a mZPD created by the student–tutor previous coordination in Phases 1–3 as well as the tutor's scaffolding activity. In this micro-zone, a tutor's intervention caused significant transformation to the intercorporeal system's dynamics: the student expressed mathematically the rule she had earlier enacted bodily. As we have seen, the micro-zone is brought about through a tutor's timely invitation to pursue a new goal—the verbal articulation—thus establishing for the system a new intentional synthesis. At this pedagogical moment, the tutor's intervention transforms the student–tutor system's dynamics beyond the personal student's capability.

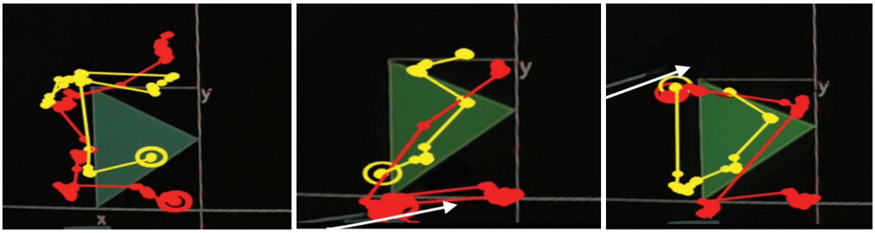
For the intercorporeal system, the essential innovation of Phase 4 is in establishing a common label—the isosceles triangle—that henceforth substitutes joint experience and, in so doing, unifies the partially dis-aligned dynamics of Phase 3. Later (see Section 4.2) this consensual labeling of the triangle enables the dyad to invoke, and thus co-refer, to the previously manipulated green triangle as inherently isosceles.

4.2 *Stage 2. An Intercorporeal System and Micro-Zone of Proximal Development: Focus on a Tutor's Explanations*

In this section we will focus on one of the pairs to present a short excerpt from the second stage of the activity. This stage consists of the student, with the tutor's support, figuring out the formula of the curve, which she had enacted in the previous stage. We delve deeply into details of this process to see contingencies in the tutor's responses. We argue that student–tutor coupling, viewed as an *inter*corporeal *functional system, enables* rapid sensitive multimodal co-adjustments during this scaffolding process that steer the collaboration towards the ultimate educational objective, the student's new conceptualization.

In its entirety, Stage 2 lasted 10 min 18 sec. During the first 3 min 27 sec, the student and the tutor dialogued via bi-directional exchange of phrases and pointing gestures towards an instance of the previously manipulated triangle. Then the tutor apparently realized that the student needed greater support and, so, switched to an explanation mode. The rest of the activity features alternating episodes of the tutor explaining and the student thinking aloud. Below, we provide part of the transcription from the episodes. See Appendix 1 for the transcription key.

01 T: So, for example, all of it is a rectangle ((Figure 6.13 a)) 
02 S: (0.6) yes-yes
03 T: (0.5) Consequently we may say, that this distance ((Figure 6.13 b))  is ((Figure 6.13 c)) X 
04 S: (0.4) Uhm=
05 T: =And this is (0.4) Y.
06 S: (0.2) this is yes.



a b c

FIGURE 6.13 (a, Turn 01) The student (yellow) and the tutor (red) run their eyes in synchrony along the rectangle's vertices. (b, Turn 03) The student misses a tutor's gesture along the *x*-axis. (c, Turn 03) The student wrongly conjoins the tutor's gesture along the vertical side of the triangle with the tutor's verbal notation X in Turn 03. The white arrow overlaid on these eye-tracking images indicates the location of the tutor's pointer, which appears as a gray line. All gray lines shown here actually appeared on the screen

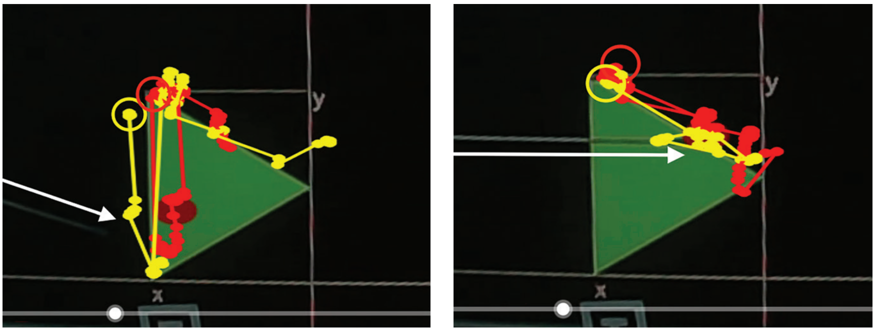
The excerpt begins with the tutor introducing the rectangle (see the faint gray vertical and horizontal lines in Figure 6.13). The student and tutor then follow the tutor's pointing gestures in coordination (Figure 6.13a captures eye movements that coincide in time with the tutor's gesture along the rectangle's perimeter). The tutor then indicates with the computer's on-screen pointer (Turn 03, see also white arrow on the Figures 6.13b and 6.13c) the section along the x-axis that equals to *x* as determined by the location of Vertex C. However, the student fails to notice the tutor's next pointing gesture along the rectangle's horizontal side (see the tail-end of this gesture in Figure 6.13b) and consequently misinterprets the explanation (Figure 6.13c): Whereas the tutor considers the rectangle's *horizontal* sides as equal to *x* (as evident in her eye-movements presented in Figures 6.13b and 6.13c), the tutor's vertical gesture might be interpreted as indicating the *vertical* side. This brief gap in synchrony (38 frames over 1.3 sec) leads to miscomprehension in Turn 03. Notwithstanding, the student confirms the tutor's explanation (Turns 04–06), because she coordinates the tutor's gesture and verbal utterance in some meaningful way. The misunderstanding is revealed further in Turn 10 and later resolved in Turns 11–13.

07 T: (1.3) And, consequently, we know
that for example the distances -
THESE (0.5) sides are equal
because the triangle is isosceles.
((the student moves the triangle
and makes it green))

08 S: (0.8) Uhm
09 T: (0.3) And consequently we can say that all these distances, all these distances are [equal to] Y.
10 S: (1.2) Which distances, once more?
11 T: (0.4) This one is (0.2) Y ((Figure 6.14a))
12 S: (0.4) Yes =
13 T: = and this one is also Y ((Figure 6.14b))
14 S: (0.9) Oh, yes.

In Turn 07 the tutor relies on the equivalence of the triangle's two sides, which had earlier been established during the embodied enactment (Stage 1) that had led to perceiving the triangle as isosceles. This reference is clear for the student. Later (Turn 09), the tutor uses the fact that had been mentioned in Turn 05, namely, that the triangle's vertical side is equal to *y*. As we have seen earlier, this fact had not been established thus for the student. The student asks for a re-explanation (Turn 10). This time, the student understands the tutor. As Figures 6.14a and 6.14b demonstrate, the student follows the tutor in full synchrony and anticipates the gestures similar to the tutor: their eyes arrived at the end of the gesture ahead of the pointer.

In sum, the dyad's activity evidences that individuals are both closely synchronized with and anticipating the actions of the other individual. The student confirms understanding of the tutor's utterance in the cases when the



a b

FIGURE 6.14 Visual joint attention to the sides of the triangle, while the tutor moves the pointer along them and verbally points at them in Turns 11 and 13

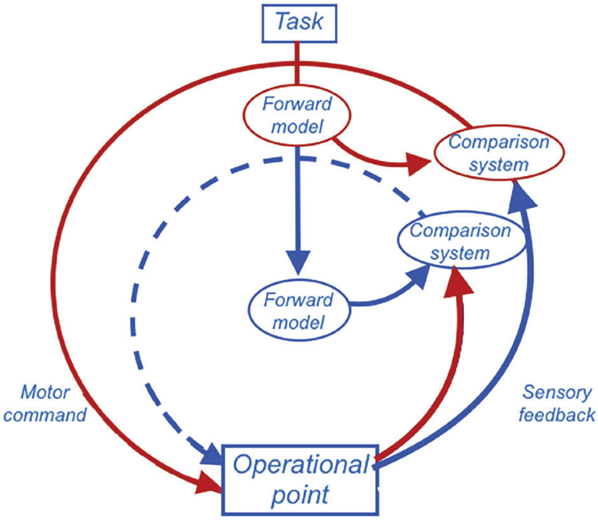


FIGURE 6.15
In the circle of mutual co-regulation, the student simulates (dashed blue line) the tutor's statements (red line) and confirms whether the utterance matches the simulation or asks for re-explanation (blue line)

actual gesture matches with her anticipation. Theoretically, we suggest that the student and tutor form an *intercorporeal functional system*, in which their perception and action mechanisms interlace. This interpersonal interlacing is analogous to intrapersonal perception–action interlacing in the regulation of one person's motor control.

This time it is the student who anticipates the tutor's actions in a feed-forward model. She regulated her utterances through feedback injections and questions. Figure 6.15 offers a model of the dyad's dynamical constitution: the tutor plans and performs an action (in red), while the student, in order to understand, also plans the same action yet only simulates it (dashed line); the real movements of a tutor provide feedback to the student on the correctness of her understanding, while the student's utterances provide feedback to the tutor on whether the student understood her explanations or whether some adjustment is needed.

In order to see the regularity and the iterative character of this coupled process, we present the same piece of the dialogue but now focus on the analysis of the audio and especially on the length of the pauses that preceded each turn (indicated in single parentheses). See Appendix 1 for the transcription key.

01 T: So, for example, all of it is a rectangle ((Figure 6.13a))
02 S: (0.6) yes-yes
03 T: (0.5) Consequently we may say that = this distance ((Figure 6.13b)) is
((Figure 6.13c)) X

04 S: (0.4) Uhm=
05 T: =And this is (0.4) Y.
06 S: (0.2) this is yes.
07 T: (1.3) And, consequently, we know that for example the distan- THESE
(0.5) sides are equal because the triangle is isosceles.
08 S: (0.8) Uhm ((the student moves the triangle and makes it green))
09 T: (0.3) And consequently we can say that all these distances, all these
distances are [equal to] Y.
10 S: (1.2) Which distances, once more?
11 T: (0.4) This one is (0.2) Y ((Figure 6.14a))
12 S: (0.4) Yes =
13 T: = and this one is also Y ((Figure 6.14b))
14 S: (0.9) Oh, yes.
15 T: (1.0) Thus we know this side
16 S: (0.6) Uhmm
17 T: (0.3) We know (0.8) the constant. (1.4) And NOW we can try somehow
express (1.2) °this one°
18 S: (3.2) ((the tutor looks at the eyes of the student)) Uhmm
19 T: (0.3) using (0.8) right triangle
20 S: (0.6) A-A-A! ((An expression of a sudden understanding, similar to insight)) using right triangle

There is a repeated pattern in the verbal utterances: one partner makes a statement and, as soon as it is finished, the other participant confirms it with a positive interjection such as "Yes" or "Uhmm" (the confirmatory character of these ingestions, underlined in the transcription, is clear in the audio record). This pattern is present during almost the entire activity, and we could distinguish 39 positive interjections in about 7 min 30 sec of activity. Analysis of the pauses between phrases shows that these confirmations follow the statement in a regular way (M = 613 ms, SD = 341 ms, after excluding three outlying cases beyond two standard deviations).

In the extract above the tutor makes statements, and the student confirms them. The longer pauses (1.2 sec in Turn 10 and 3.2 seconds in Turn 18, marked by italic) appear when the tutor's explanation does not match the student's anticipation. There might be verbal information from the student (in Turn 10), or no additional verbal information (in Turn 18); in any case the change of the regular pattern informs the tutor that adjustment is required. The longer pauses also provide the tutor with time to prepare additional explanations, which immediately follow the student's utterances, as quickly as the regular explanations that followed positive confirmation (the pause before Turn 11 is 0.4 sec; before Turn 19 it is 0.3 second). So, the change in the regular pattern of the pauses, namely the absence of the usual confirmatory reaction, directs the tutor towards preparing additional guidance. Drawing an analogy with the runner in our snake example, where the body adjusted the performance without the girl's full awareness of the snake, we suggest that the tutor does not consciously attend to pauses by thinking something like "this pause is too long, apparently the student did not understand something". Instead, the tutor's bodily regulation detects a longer pause and initiates re-explanation.

The confirmatory interjections enable the two participants to sustain joint attention during the entire dialogue and restore it as quickly as possible when any discoordination emerges. The eye-tracking data reveal that key moments of the dialogue are accompanied by joint *visual* attention (see Figures 6.13a and 6.14a, 6.14b), as the participants' perception is coupled in coordinating a gesture, a verbal utterance, and a visual inscription. Concordant with our previous findings (Shvarts, 2018a), as well as other analyses (Jornet & Roth, 2015; Zagorianakos & Shvarts, 2015), the coincidence of different presentations appears as an active coordination between modalities, while, as we can see here, the *inter*corporeal system serves the coordination of these multimodal nodes between two participants.

Iterative feedback from the student allows the pair to maintain the communication within a zone of proximal development by carefully establishing a mZPD before each new statement. And yet, the entire activity unfolds as iterative steps forward and backward in a search of mZPD: Turns 09 and 17 appear to transgress the boundaries of the micro-zones, as the tutor's explanations do not move the system forward, and the steps backwards are needed.

Each forward step within the mZPD is characterized by bootstrapping: intercorporeal anticipation in one modality allows development of intercorporeal coordination in the other modality (Flood, 2018). In Section 4.1, the tutor succeeds in triggering the student's verbal reflection, when the student and tutor are coordinated in sensory–motor fluent performance; in Section 4.2, it is only when the student is capable of anticipating the tutor's gestures that new meaning might be achieved through verbal labeling. Pointing gestures are successful when verbal references are clear.

The idea of a mZPD might explain why guiding the student's attention may have only limited effectiveness, especially in mathematics (see meta-analysis on visual cueing, Schneider et al., 2018). When a visual field needs to be approached in a particular way to see mathematical relations, the movement within mZPD requires transforming the student's functioning in multiple modalities: new perceptual strategies need to develop together with new verbal references. This requires of the tutor to carefully probe for the student's anticipations, in establishing the moves that would preserve tight coupling between a student and a tutor.

5 Summary and Conclusion

We have presented a theoretical discussion that seeks to bridge between two views on learning, the culture–historical approach and complex dynamic systems. Our discussion began with analyzing the notion of a functional system, which was introduced in Russian physiology and used in psychology and neuropsychology, and the notion of a complex dynamic system, which was initially elaborated to describe complex processes of thermodynamics and was later applied to the description of motor and cognitive activity. These two systems have similar features, such as dynamical self-organization of multiple processes. The difference between these two theoretical models, which can be attributed to their respective historical developments, lies in interpreting their self-organization processes: Whereas complex dynamic systems self-organize in accordance with environmental constraints driven exclusively by thermodynamic laws, functional systems assemble in the process of iterative attempts to realize some task, as it is conceptualized by an observer. Although both models point at physically the same processes, we suggest that this difference in theoretical modeling is important specifically for educational endeavors. Focusing on a functional system, an educational researcher ought to discern a learning task, which causes self-organization, and consider the student's goal as a form of subjective anticipation of task fulfillment. We emphasized that the systemic notion of anticipation, as realized by an entire non-centrally regulated organism, departs from the traditional views of intention as a subject's deliberate and conscious planning, instead corresponding to ideas of operative, motor, or enactive intentionality (Gallagher & Miyahara, 2012; Merleau-Ponty, 2002) and, in the field of physiology, physical anticipation of future sensory feedback (Bernstein, 1967)—an anticipation fulfilled through the peripheral coupling of diverse contemporaneous processes (Latash, 2015; Stepp & Turvey, 2010) for the purpose of prospective control (Bernstein, 1967; Turvey, 2019).

Bridging the cultural–historical and complex-dynamic-systems theories of learning supports educational applications in viewing a student–tutor dyad's sensory-motor coupling as an *intercorporeal dynamic functional* system. This system emerges as a coordination between a student's action and a tutor's perception (or vice versa), a coordination that can be traced through the methodology of dual eye-tracking, enabling researchers to demonstrate specific processes and forms of intercorporeal coupling. In this process, we trace the emergence of joint attention to manipulated objects (see also Pagnotta, 2018; Yu & Smith, 2016). Importantly for educational concerns, the object itself, as a sensory–motor perceptual entity, transforms during the teaching–learning process, in the sense that it is approached differently and thus acquires new meaning. The notion of *micro-zone of proximal development* (*mZPD*) helps us understand how this re-approaching emerges and how the dynamics of this joint system of two bodies are changed over the process of engaging with the educational design.

We presented empirical illustrations of an intercorporeal functional system, as it assembled and played out in two contrasting episodes from the implementation of an embodied action-based activity for parabolas. One episode consisted of a student's embodied activity that was only slightly guided by a tutor, while the other episode exemplified a more classical teaching situation, namely a tutor's explicit explanation of new material. Dual eye-tracking served as the main method in this micro-ethnographical study.

As we have demonstrated in the cases of four student–tutor pairs in the first episode, a tight temporal and spatial coupling between student–tutor perceptions and actions emerges as they both follow and anticipate the movements of the triangle on the screen by smooth pursuit. This coordination into a united intercorporeal system is transformed as the students come to discover the target performance and keep the triangle green: although being coordinated with the student's actions, the tutor's and student's perceptual strategies decouple, revealing natural (smooth pursuit) and cultural (AAs) ways of perceiving the scene. The tutor does not just follow the student's performance, but *as-if*-enacts the student's motor activity, as we could distinguish from the tutor's monitoring eye-movements that evidenced AAs. We assume that this coupling with the student's actions while enacting efficient strategies of perception enables the tutor to distinguish a moment when the student's perceptual strategies undergo spontaneous transformation towards an efficient pattern under the fluent-performance demand. Following this transformation, both student and a tutor actualize AAs in following and regulating the movement of the triangle. Thus, the three observed phases demonstrate the following dynamics of an *intercorporeal functional system*: tight coupling of a student's and tutor's natural strategies of perception with the student's motor actions; de-coupling of natural and cultural perceptual strategies and yet coupling of both strategies with the student's motor actions; and coupling of both participants' cultural strategies with student's motor actions.

When students' perception is spontaneously transformed, a *mZPD* can be established, as it is an optimal moment for a verbal intervention. Tutors introduce to students the next goal of coordinating their personal sensory-motor experience, which had self-organized within the task constraints, with a cultural labeling and verbal referencing. This new goal transforms the functional system, and a word—isosceles—becomes a new means of coordinating the student and tutor's dynamics. This word will henceforth stand in for the *de facto* enactment of the joint dynamics that had been previously achieved.

The second case showed an *intercorporeal functional system*, as a student followed a tutor's explanations. Similar to the first case, merely following explanations is not sufficient for understanding. The student's attention becomes coupled with the tutor's gestures, thus anticipating the tutor's subsequent explanatory move. The student iteratively either confirms or denies she is grasping the tutor's multimodal utterances, thus regulating the tutor's expressions and soliciting re-explanations. What happens is that the intercorporeal system is steered by the dyad's joint and coupled perception–action process. This process iteratively accommodates to re-establish *mZPDs* and keep joint dynamics developing towards the final goal of new mathematical enculturation. In these micro-zones, a student can both anticipate the tutor's gestures and base an understanding of the verbal utterances on her own embodied experience.

Both episodes are characterized by multi-level coupling of two bodies into one intercorporeal system, thus following and anticipating each other's motor actions, gestures, and verbal utterances. This coupling enables contingent co-adjustments within the scaffolding process (Van de Pol et al., 2010); namely, the system moves forward in dynamical changes through iteratively re-establishing mZPDs. These two episodes provided two examples of the tutors' efficient moves of this nature: in the first episode, a new task was efficiently introduced when the student had accomplished fluent performance of the previous task; in the second episode the tutors bootstrapped the development of the system's dynamics by utilizing the student's gesture anticipation to build new verbal moves, and utilizing the student's anticipation of verbal expressions to build new gestures.

By demonstrating the conceptual and predictive similarity of the explanatory models offered by two historically disparate and yet sometimes close streams of scholarship—the cultural–historical approach and dynamic systems theory—we hope to encourage the field of educational research to nurture theoretical models from both approaches, as well as their methodological traditions. Further investigation of teaching moves that transform the dynamics of student–tutor systems is needed to clarify the notion of mZPD and describe efficient teaching strategies in the context of joint multimodal dynamics. Another open question concerns the future of the intercorporeal system in the learning process: "When teaching according to the program had ended, development began," says Vygotsky (Vygotsky's Notebooks, Zavershneva & van der Veer, 2018, p. 355). We assume that after a tutor's scaffolding fades and the student's perception–action system has differentiated from the student–tutor collaboration, a self-scaffolding of a student herself within their *intra*corporeal system retains traces of the intercorporeal system and helps the student fully acquire new dynamics (Shvarts & Bakker, 2019).

These theoretical elaborations push educational practice towards nonlinear pedagogy that implies teaching by creating conditions—such as tasks and environmental constraints—for guided mathematics learning in the action-based genre of embodied design (Abrahamson, 2014), as such environments provide rich opportunities for student's embodied discoveries and embodied collaborations with the tutors (Abrahamson & Sánchez–García, 2016). At the same time, findings of tight coupling between perception–action systems raise numerous questions for engineers of artificial tutorial systems. Designers of interactive virtual teachers should appreciate the effectiveness of one-to-one collaboration. Simulating such dyadic learning requires multimodal interaction and learning analytics (Abdullah et al., 2017; Pardos et al., 2018).

While applying our findings to classroom teaching is still an open research area, clearly the notion of a mZPD calls for attention to the student's multimodal processes as they unfold within particular tasks.

Acknowledgments

We would like to thank Katharine Young for suggesting the term *intercorporeal* in this context, Michael Cole for drawing our attention to the term *functional system,* and Anatoly Krichevets for years of productive discussions.

Notes

1 The notion of intentional synthesis is drawn from the Husserlian phenomenological perspective on mathematics education (Zagorianakos, 2015).

2 The chapter presents a substantial elaboration on materials previously published in a conference paper (Shvarts, 2018b) and an article (Shvarts & Abrahamson, 2019).

References

Abdullah, A., Adil, M., Rosenbaum, L., Clemmons, M., Shah, M., Abrahamson, D., & Neff, M. (2017). Pedagogical agents to support embodied, discovery-based learning. In J. Beskow, C. Peters, G. Castellano, C. O'Sullivan, I. Leite, & S. Kopp (Eds.), *Proceedings of 17th International Conference on Intelligent Virtual Agents* (*IVA 2017*) (pp. 1–14). Springer.

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child-Computer Interaction, 2*(1), 1–16.

Abrahamson, D., Gutiérrez, J., Charoenying, T., Negrete, A., & Bumbacher, E. (2012). Fostering hooks and shifts: Tutorial tactics for guided mathematical discovery. *Technology, Knowledge and Learning, 17*(1–2), 61–86.

Abrahamson, D., Lee, R. G., Negrete, A. G., & Gutiérrez, J. F. (2014). Coordinating visualizations of polysemous action: Values added for grounding proportion. *ZDM Mathematics Education, 46*(1), 79–93.

Abrahamson, D., & Sanchez-Garcia, R. (2016). Learning is moving in new ways: The ecological dynamics of mathematics education. *The Journal of the Learning Sciences*, *25*(2), 203–239.

Abrahamson, D., Shayan, S., Bakker, A., & Van der Schaaf, M. F. (2016). Eye-tracking Piaget: Capturing the emergence of attentional anchors in the coordination of proportional motor action. *Human Development, 58*(4–5), 218–244.

Abrahamson, D., & Trninic, D. (2011). Toward an embodied-interaction design framework for mathematical concepts. In P. Blikstein & P. Marshall (Eds.), *Proceedings of the 10th Annual Interaction Design and Children Conference* (*IDC 2011*) (Vol. "Full papers," pp. 1–10). IDC.

Anderson, M. L. (2010). Neural reuse: A fundamental organizational principle of the brain. *Behavioral and Brain Sciences*, *33*(4), 245–266. https://doi.org/10.1017/S0140525X10000853

Anokhin P. K. (1935/1975). Problema centra i periferii v sovremennoj fiziologii nervnoj deyatel'nosti [The problem of the center and periphery in the contemporary physiology of neural system]. In P. K. Anokhin (Ed.). Ocherki po fiziologii funkcional'nyh system (pp. 63–108). Medicina.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241.

Atmaca, S., Sebanz, N., Prinz, W., & Knoblich, G. (2008). Action co-representation: The joint SNARC effect. *Social Neuroscience, 3*(3–4), 410–420.

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective. In L. D. English, M. G. Bartolini Bussi, G. A. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of international* *research in mathematics education, 2nd revised edition* (pp. 720–749). Lawrence Erlbaum Associates.

Becchio, C., Manera, V., Sartori, L., Cavallo, A., & Castiello, U. (2012). Grasping intentions: from thought experiments to empirical evidence. *Frontiers in Human Neuroscience, 6*(May), 1–6.

Bernstein, N. A. (1947). *O postroenii dvizheniy* [*On the construction of movements*]. Meditsina.

Bernstein, N. A. (1967/1967). *The co-ordination and regulation of movements.* Pergamon Press.

Brooks, R., & Meltzoff, A. N. (2014). Gaze following: A mechanism for building social connections between infants and adults. In M. Mikulincer & P. R. Shaver (Eds.), *Mechanisms of social connection: From brain to group* (pp. 167–184). American Psychological Association.

Brown, T., Heywood, D., Solomon, Y., & Zagorianakos, A. (2013). Experiencing the space we share: Rethinking subjectivity and objectivity. *ZDM Mathematics Education, 45*(4), 561–572.

Chapman, S. (1968). Catching a baseball. *American Journal of Physics*, *36*(10), 868–870. http://doi.org/10.1119/1.1974297

Dale, R., Fusaroli, R., Duran, N. D., & Richardson, D. C. (2014). The self-organization of human interaction. In B. H. Ross (Ed.), *The psychology of learning and motivation* (pp. 43–95). Elsevier Inc.: Academic Press.

de Freitas, E., & Sinclair, N. (2013). New materialist ontologies in mathematics education: The body in/of mathematics. *Educational Studies in Mathematics*, *83*(3), 453–470. http://doi.org/10.1007/s10649-012-9465-z

de Freitas, E., & Sinclair, N. (2018). The quantum mind: Alternative ways of reasoning with uncertainty. *Canadian Journal of Science, Mathematics and Technology Education*, *18*(3), 271–283. http://doi.org/10.1007/s42330-018-0024-1

Duijzer, C. A. C. G., Shayan, S., Bakker, A., Van der Schaaf, M. F., & Abrahamson, D. (2017). Touchscreen tablets: Coordinating action and perception for mathematical cognition. *Frontiers in Psychology, 8*, 144.

Dumas, G., Lachat, F., Martinerie, J., Nadel, J., & George, N. (2011). From social behaviour to brain synchronization: Review and perspectives in hyperscanning. *Irbm*, *32*(1), 48–53. https://doi.org/10.1016/j.irbm.2011.01.002

Flanagan, J. R., & Johansson, R. S. (2003). Action plans used in action observation. *Nature, 424*(6950), 769–771.

Flood, V. J. (2018). Multimodal revoicing as an interactional mechanism for connecting scientific and everyday concepts. *Human Development*, *61*, 145–173.

Flood, V. J., & Abrahamson, D. (2015, April 16–20). *Refining mathematical meanings through multimodal revoicing interactions: The case of "faster"* [Paper presentation]. Annual Meeting of the American Educational Research Association. Chicago, IL, United States.

Flood, V. J., Harrer, B. W., & Abrahamson, D. (2016). The interactional work of configuring a mathematical object in a technology-enabled embodied learning environment. In C.-K. Looi, J. L. Polman, U. Cress, & P. Reimann (Eds.), *"Transforming learning, empowering learners," Proceedings of the International Conference of the Learning Sciences* (*ICLS 2016*), *Vol. 1* (pp. 122–129). International Society of the Learning Sciences.

Fogel, A. (2006). Dynamic systems research on interindividual communication: The transformation of meaning-making. *The Journal of Developmental Processes*, *1*, 7–30.

Fogel, A., & Garvey, A. (2007). Alive communication. *Infant Behavior & Development*, *30*, 251–257. http://doi.org/10.1016/j.infbeh.2007.02.007

Fuchs, A., & Kelso, S. J. A. (2018). Coordination dynamics and synergetics: From finger movements to brain patterns and ballet dancing. In S. C. Müller, P. J. Plath, G. Radons, & A. Fuchs (Eds.), *Complexity and synergetics* (pp. 301–316). Springer. https://doi.org/10.1007/978-3-319-64334-2_23

Gallagher, S., & Miyahara, K. (2012). Neo-pragmatism and enactive intentionality. In J. Shulkin (Ed.), *Action, perception and the brain. New directions in philosophy and cognitive science* (pp. 117–146). Macmillan. http://doi.org/10.1057/9780230360792_6

Gallagher, S. (2011). Interactive coordination in joint attention. In A. Seemann (Ed.), *Joint attention: New developments in psychology, philosophy of mind, and social neuroscience* (pp. 293–305). MIT Press.

Goodwin, C. (1994). Professional vision. *American Anthropologist, 96*(3), 606–633.

Gredebäck, G., & Falck-Ytter, T. (2015). Eye movements during action observation. *Perspectives on Psychological Science, 10*(5), 591–598.

Hutto, D. D. (2011). Elementary mind minding, enactivist-style. In A. Seemann (Ed.), *Joint attention: New developments in psychology, philosophy of mind, and social neuroscience* (pp. 307–341). MIT Press.

Hutto, D. D. (2019). Re-doing the math: Making enactivism add up. *Philosophical Studies, 176*(3), 827–837. https://doi.org/10.1007/s11098-018-01233-5

Hutto, D. D., & Sánchez-García, R. (2015). Choking RECtified: Embodied expertise beyond Dreyfus. *Phenomenology and the Cognitive Sciences, 14*(2), 309–331. http://doi.org/10.1007/s11097-014-9380-0

Jefferson, G. (2004). Glossary of transcript symbols with an introduction. In G. H. Lerner (Ed.), *Conversation analysis: Studies from the first generation* (pp. 13–31). John Benjamins.

Jermann, P., N, M.-A., Nussli, & Li, W. (2010). Using dual eye-tracking to unveil coordination and expertise in collaborative Tetris. In T. McWewan & L. McKinnon (Eds.), *Proceedings of the 24th BCS Interaction Specialist Group Conference*. British Computer Society.

Jornet, A., & Roth, W.-M. (2015). The joint work of connecting multiple (re)presentations in science classrooms. *Science Education*, *99*(2), 378–403. http://doi.org/10.1002/sce.21150

Jones, M. (2018). Numerals and neural reuse. *Synthese, 197*, 3657–3681. https://doi.org/10.1007/s11229-018-01922-y

Jurgens, A., & Kirchhoff, M. D. (2019). Enactive social cognition: Diachronic constitution and coupled anticipation. *Consciousness and Cognition, 70*, 1–10. http://doi.org/10.1016/J.CONCOG.2019.02.001

Kassner, M., Patera, W., & Bulling, A. (2014). Pupil: An open-source platform for pervasive eye tracking and mobile gaze-based interaction. In M. Kassner, W. Patera, & A. Bulling (Ed.), *Proceedings of the 2014 ACM International Joint Conference on Pervasive and Ubiquitous Computing Adjunct Publication* (*UbiComp '14*) (pp. 1151–1160). ACM Press.

Kazansky, A. B. (2015). Agential anticipation in the central nervous system. In M. Nadin (Ed.), *Anticipation: Learning from the past. Cognitive systems monographs, 25* (pp. 101–112) Springer. http://doi.org/10.1007/978-3-319-19446-2_6

Kelso, J. A. S. (1982). *Human motor behavior*. Psychology Press. http://doi.org/10.4324/9781315802794

Knoblich, G., Butterfill, S., & Sebanz, N. (2011). Psychological research on joint action: Theory and data. In B. Ross (Ed.), *The psychology of learning and motivation, Vol. 54* (pp. 59–101). Academic Press.

Kostrubiec, V., Zanone, P.-G., Fuchs, A., & Kelso, J. A. S. (2012). Beyond the blank slate: Routes to learning new coordination patterns depend on the intrinsic dynamics of the learner – experimental evidence and theoretical model. *Frontiers in Human Neuroscience, 6*(222), 1–14. http://doi.org/10.3389/fnhum.2012.00222

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic Books.

Laroche, J., & Kaddouch, I. (2014). Enacting teaching and learning in the interaction process: "Keys" for developing skills in piano lessons through four-hand improvisations. *Journal of Pedagogy*, 5(1), 24–47. http://doi.org/10.2478/jped-2014-0002

Latash, M. L. (2015). Bernstein's "desired future" and physics of human movement. In M. Nadin (Ed.), *Anticipation: Learning from the past. Cognitive systems monographs, 25* (pp. 287–299). Springer. http://doi.org/10.1007/978-3-319-19446-2_16

Lilienthal, A. J., & Schindler, M. (2017). Conducting dual portable eye-tracking in mathematical creativity research. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education, Vol. 1* (p. 231). PME.

Litchfield, D., Ball, L. J., Donovan, T., Manning, D. J., & Crawford, T. (2010). Viewing another person's eye movements improves identification of pulmonary nodules in chest x-ray inspection. *Journal of Experimental Psychology: Applied, 16*(3), 251–262.

Liu, N., Mok, C., Witt, E. E., Pradhan, A. H., Chen, J. E., & Reiss, A. L. (2016). NIRS-based hyperscanning reveals inter-brain neural synchronization during cooperative Jenga game with face-to-face communication. *Frontiers in Human Neuroscience, 10*, 1–11. https://doi.org/10.3389/fnhum.2016.00082

Luria, A. R. (1973). *The working brain. An introduction to neuropsychology*. Allen Lane.

Mechsner, F., Kerzel, D., Knoblich, G., & Prinz, W. (2001). Perceptual basis of bimanual coordination. *Nature, 414*(6859), 69–73.

Merleau-Ponty, M. (1942/1963). *The structure of behavior*. Beacon Press.

Merleau-Ponty, M. (1945/2002). *Phenomenology of perception*. Routledge.

Meyer, C., & Von Wedelstaedt, U. (Eds.). (2017). *Moving bodies in interaction—Interacting bodies in motion: Intercorporeality, interkinaesthesia, and enaction in sports*. John Benjamins.

Newman, D., Griffin, P., & Cole, M. (1989). *The construction zone: Working for cognitive change in school.* Cambridge University Press.

Okazaki, S., Muraoka, Y., & Osu, R. (2019). Teacher-learner interaction quantifies scaffolding behaviour in imitation learning. *Scientific Reports*, *9*(1), 1–13. https://doi.org/10.1038/s41598-019-44049-x

Oullier, O., de Guzman, G. C., Jantzen, K. J., Lagarde, J., & Kelso, J. A. (2008). Social coordination dynamics: Measuring human bonding. *Social Neuroscience, 3*(2), 178–192.

Pagnotta, M. (2018). *Living and learning together: Integrating developmental systems theory, radical embodied cognitive science, and relational thinking in the study of social learning* [Doctoral dissertation, University of St Andrews]. Research Repository St. Andrews. https://research-repository.st-andrews.ac.uk/handle/10023/16386

Pan, Y., Dikker, S., Goldstein, P., Zhu, Y., Yang, C., & Hu, Y. (2020). Instructor-learner brain coupling discriminates between instructional approaches and predicts learning. *NeuroImage*, *211*, 116657. https://doi.org/10.1016/j.neuroimage.2020.116657

Pardos, Z. A., Hu, C., Meng, P., Neff, M., & Abrahamson, D. (2018). Characterizing learner behavior from high frequency touchscreen data using recurrent neural networks. In D. Chin & L. Chen (Eds.), *Adjunct Proceedings of the 26th Conference on User Modeling, Adaptation and Personalization* (UMAP '18). ACM.

Pesquita, A., Chapman, C. S., & Enns, J. T. (2016). Humans are sensitive to attention control when predicting others' actions. In S. T. Fiske (Ed.), *Proceedings of the National Academy of Sciences, 113*(31), 8669–8674.

Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics, 30*(2), 2–7.

Radford, L. (2016). Mathematics education as a matter of labor. In M. A. Peters (Ed.), *Encyclopedia of educational philosophy and theory* (pp. 1–6). Springer. http://doi.org/10.1007/978-981-287-532-7_518-1

Radford, L., & Roth, W.-M. (2011). Intercorporeality and ethical commitment: An activity perspective on classroom interaction. *Educational Studies in Mathematics*, *77*(2–3), 227–245. http://doi.org/10.1007/s10649-010-9282-1

Radford, L., & Sabena, C. (2015). The question of method in a Vygotskian semiotic approach. In A. Bikner-Ahsbahs, C. Knipping, & N. C. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 157–182). Springer.

Richardson, D. C., Dale, R., & Kirkham, N. Z. (2007). The art of conversation is coordination. *Psychological Science*, *18*(5), 407–413. http://doi.org/10.1111/j.1467-9280.2007.01914.x

Richardson, M. J., Marsh, K. L., Isenhower, R. W., Goodman, J. R. L., & Schmidt, R. C. (2007). Rocking together: Dynamics of intentional and unintentional interpersonal coordination. *Human Movement Science, 26*(6), 867–891.

Rizzolatti, G., Fadiga, L., Gallese, V., & Fogassi, L. (1996). Premotor cortex and the recognition of motor actions. *Brain Research. Cognitive Brain Research, 3*(2), 131–41.

Roth, W.-M., & Radford, L. (2010). Re/thinking the zone of proximal development (symmetrically). *Mind, Culture, and Activity, 17*(4), 299–307.

Roth, W.-M., & Radford, L. (2011). *A cultural-historical perspective on mathematics teaching and learning.* Sense Publishers.

Schmidt, R. C., Carello, C., & Turvey, M. T. (1990). Phase transitions and critical fluctuations in the visual coordination of rhythmic movements between people. *Journal of Experimental Psychology: Human Perception and Performance*, *16*(2), 227–247. http://doi.org/10.1037/0096-1523.16.2.227

Schmitz, L., Vesper, C., Sebanz, N., & Knoblich, G. (2017). Co-representation of others' task constraints in joint action. *Journal of Experimental Psychology: Human Perception and Performance, 43*(8), 1480–1493.

Schneider, B., Sharma, K., Cuendet, S., Zufferey, G., Dillenbourg, P., & Pea, R. D. (2016). Detecting collaborative dynamics using mobile eye-trackers. In C.-K. Looi, J. L. Polman, U. Cress, & P. Reimann (Eds.), *Proceedings of the International Conference of the Learning Sciences* (*ICLS 2016*) *Vol. 1* (pp. 522–529). International Society of the Learning Sciences.

Schneider, S., Beege, M., Nebel, S., & Rey, G. D. (2018). A meta-analysis of how signaling affects learning with media. *Educational Research Review*, *23*, 1–24. https://doi.org/10.1016/J.EDUREV.2017.11.001

Shvarts, A. (2018a). Joint attention in resolving the ambiguity of different presentations: A dual eye-tracking study of the teaching-learning process. In N. Presmeg, L. Radford, W.-M. Roth, & G. Kadunz (Eds.), *Signs of signification: Semiotics in mathematics education research* (pp. 73–102). Springer.

Shvarts, A. (2018b). A dual eye-tracking study of objectification as student–tutor joint activity appropriation. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (pp. 171–178). PME.

Shvarts A. (2019). Theory, methodology and design as an insightful bundle: A case of dual eye-tracking the student-tutor collaboration on an embodied mathematical task. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for research in Mathematics Education* (pp. 3104–3105). Utrecht University.

Shvarts, A., & Abrahamson, D. (2019). Dual-eye-tracking Vygotsky: A microgenetic account of a teaching/learning collaboration in an embodied-interaction technological tutorial for mathematics. *Learning, Culture and Social Interaction*, *22*, 100316. https://doi.org/10.1016/J.LCSI.2019.05.003

Shvarts, A., & Bakker, A. (2019). The early history of the scaffolding metaphor: Bernstein, Luria, Vygotsky, and before. *Mind, Culture, and Activity*, *26*(1), 4–23. http://doi.org/10.1080/10749039.2019.1574306

Shvarts, A., Stepanov, A., & Chumachenko, D. (2018). Automatic detection of gaze convergence in multimodal collaboration: A dual eye-tracking technology. *The Russian Journal of Cognitive Science, 5*(3), 4–17.

Shvarts, A., & Zagorianakos, A. (2016). Theoretical perception of the Cartesian plane: A dual eye-tracking study through double theoretical lenses. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of 40th Annual Meeting of the International Group for the Psychology of Mathematics Education* (p. 287). PME.

Spivey, M. J., Richardson, D. C., & Dale, R. (2009). The movement of eye and hand as a window into language and cognition. In E. Morsella, J. Bargh, & P. M. Gollwitzer (Eds.), *The psychology of action*, *Vol. 2* (pp. 225–249). Oxford University Press.

Stephen, D. G., Dixon, J. A., & Isenhower, R. W. (2009). Dynamics of representational change: Entropy, action, and cognition. *Journal of Experimental Psychology: Human Perception and Performance*, *35*(6), 1811–1832. http://doi.org/10.1037/a0014510

Stepp, N., & Turvey, M. T. (2010). On strong anticipation. *Cognitive Systems Research*, *11*(2), 148–164. http://doi.org/10.1016/j.cogsys.2009.03.003

Stetsenko, A. (2017). *The transformative mind*. Cambridge University Press.

Streeck, J., & Mehus, S. (2005). Microethnography: The study of practices. In K. Fitch & R. Sanders (Eds.), *Handbook of language and social interaction* (pp. 381–404). Erlbaum.

Szymanski, C., Pesquita, A., Brennan, A. A., Perdikis, D., Enns, J. T., Brick, T. R., … Lindenberger, U. (2017). Teams on the same wavelength perform better: Inter-brain phase synchronization constitutes a neural substrate for social facilitation. *NeuroImage*, *152*, 425–436. https://doi.org/10.1016/j.neuroimage.2017.03.013

Tenenberg, J., Roth, W.-M., & Socha, D. (2016). From I-awareness to we-awareness in CSCW. *Computer Supported Cooperative Work* (CSCW), *25*(4–5), 235–278. http://doi.org/10.1007/s10606-014-9215-0

Thelen, E. (2000). Motor development as foundation and future of developmental psychology. *International Journal of Behavioral Development*, *24*(4), 385–397. http://doi.org/10.1080/016502500750037937

Tomasello, M., & Carpenter, M. (2007). Shared intentionality. *Developmental Science, 10*, 121–125. http://doi.org/10.1111/j.1467-7687.2007.00573.x

Tomasello, M., & Farrar, M. J. (1986). Joint attention and early language. *Child Development, 57*(6), 1454–1463.

Turvey, M. T. (1977). Preliminaries to a theory of action with reference to vision. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting, and knowing: Toward an ecological psychology* (Vol. 211–265). Erlbaum.

Turvey, M. T. (2019). *Lectures on perception: An ecological perspective*. Routledge.

Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher-student interaction: A decade of research. *Educational Psychology Review*, *22*(3), 271–296.

van Gog, T., Jarodzka, H., Scheiter, K., Gerjets, P., & Paas, F. (2009). Attention guidance during example study via the model's eye movements. *Computers in Human Behavior, 25*(3), 785–791. http://doi.org/10.1016/j.chb.2009.02.007

Vaziri-Pashkam, M., Cormiea, S., & Nakayama, K. (2017). Predicting actions from subtle preparatory movements. *Cognition, 168*, 65–75.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press. (Original work published 1930).

Vygotsky, L. S. (1987). *The collected works of L. S. Vygotsky* (Vol. 3: Problems of the Theory and History of Psychology.) (R. W. Rieber & A. S. Carton, Eds.). Plenum Press.

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem-solving. *Journal of Child Psychology and Psychiatry*, *17*(2), 89–100. http://doi.org/10.1111/j.1469-7610.1976.tb00381.x

Yu, C., & Smith, L. B. (2016). Multiple sensory-motor pathways lead to coordinated visual attention. *Cognitive Science, 41*, 1–27.

Yu, C., & Smith, L. B. (2017). Hand-eye coordination predicts joint attention. *Child Development, 88,* 2060–2078.

Zagorianakos, A. (2015). *The study of intuition as an objectifying act from a Husserlian perspective, in the cases of three prospective teachers of mathematics* [Doctoral dissertation, Manchester Metropolitan University]. E-space MMU https://e-space.mmu.ac.uk/595477/1/Zagorianakos-PhD%20Thesis_final.pdf

Zagorianakos, A. (2016). The study of the intuition of essence of a prospective teacher of mathematics from a phenomenological perspective. *Mediterranean Journal for Research in Mathematics Education*, *15*, 69–94.

Zagorianakos, A., & Shvarts, A. (2015). The role of intuition in the process of objectification of mathematical phenomena from a Husserlian perspective: A case study. *Educational Studies in Mathematics, 88*(1), 137–157.

Zavershneva, E., & van der Veer, R. (Eds.). (2018). *Vygotsky's notebooks*. *Perspectives in cultural-historical research* (Vol. 2). Springer. http://doi.org/10.1007/978-981-10-4625-4

APPENDIX 1

Transcription Key

((comment))	Double parentheses surround transcriber's descriptions.
(0.6)	Numbers in parentheses indicate time intervals between utterances, timed in tenths of a second.
=	Equal signs indicate no break or nearly overlapping turns at talk.
-	Dash indicates an abruptly ended or "cut off" utterance.
CAPS	Capitals mark a talk that is louder than surrounding talk.
word	Degree signs indicate words that are quieter than surrounding talk.

The transcription conventions are adapted from Jefferson, 2004.