

# Hybrid teaching intelligence: Lessons learned from an embodied mathematics learning experience

Giulia Cosentino<sup>1</sup>  | Jacqueline Anton<sup>2,3</sup>  | Kshitij Sharma<sup>1</sup>  |  
Mirko Gelsomini<sup>1,4</sup> | Michail Giannakos<sup>1</sup> | Dor Abrahamson<sup>2,5</sup> 

<sup>1</sup>Department of Computer Science, Norwegian University of Science and Technology, Trondheim, Norway

<sup>2</sup>Berkeley School of Education, University of California Berkeley, Berkeley, California, USA

<sup>3</sup>Department of Special Education, San Francisco State University, San Francisco, California, USA

<sup>4</sup>Department of Innovative Technologies, Scuola Universitaria Professionale della Svizzera Italiana, Lugano, Switzerland

<sup>5</sup>College of Education, Korea University, Seoul, South Korea

## Correspondence

Giulia Cosentino, Department of Computer Science, Norwegian University of Science and Technology, Trondheim, Norway.  
Email: [giulia.cosentino@ntnu.no](mailto:giulia.cosentino@ntnu.no)

## Funding information

Peder Sather Grant program

As AI increasingly enters classrooms, educational designers have begun investigating students' learning processes vis-à-vis simultaneous feedback from active sources—AI and the teacher. Nevertheless, there is a need to delve into a more comprehensive understanding of the orchestration of interactions between teachers and AI systems in educational settings. The research objective of this paper is to identify the challenges and opportunities when AI intertwines with instruction and examine how this hybrid teaching intelligence is being perceived by the students. The insights of this paper are extracted by analysing a case study that utilizes an AI-driven system (MOVES-NL) in the context of learning integer arithmetic. MOVES-NL is an advanced interactive tool that deploys whole-body movement and immediate formative feedback in a room-scale environment designed to enhance students' learning of integer arithmetic. In this paper, we present an in-situ study where 29 students in grades 6–8 interacted individually with MOVES-NL for approximately 1 hour each with the support of a facilitator/instructor. Mixed-methods analyses of multimodal data sources enabled a systematic multifaceted account of students' cognitive–affective experiences as they engaged with MOVES-NL while receiving human support (eg, by asking students to elaborate on their digital actions/decisions). Finally, we propose design insights

This is an open access article under the terms of the [Creative Commons Attribution-NonCommercial-NoDerivs](https://creativecommons.org/licenses/by-nc-nd/4.0/) License, which permits use and distribution in any medium, provided the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

© 2024 The Author(s). *British Journal of Educational Technology* published by John Wiley & Sons Ltd on behalf of British Educational Research Association.

for instructional and technology design in support of student hybrid learning. The findings of this research contribute to the ongoing discourse on the role of hybrid intelligence in supporting education by offering practical insights and recommendations for educators and designers seeking to optimize the integration of technology in classrooms.

#### KEYWORDS

embodied learning, hybrid intelligence, teacher–AI collaboration

### Practitioner notes

What is already known about this topic

- Students and teachers develop different relations with and through AI, beyond just interacting with it.
- AI can support and augment the teachers' capabilities.
- Hybrid intelligence (HI) has already demonstrated promising potential to advance current educational theories and practices.

What this paper adds

- This research identifies the important learning opportunities and adversities emerging when AI intertwines with instruction and examines how learners perceive those moments.
- The results show that the system and the facilitator's feedback were complementary to the success of the learning experience. AI-enabled students to reflect upon and test their previous knowledge and guided teachers to work with students to consolidate challenging topics.
- Findings provide insights into how the teacher–AI collaboration could engage and motivate students to reflect conceptually upon mathematical rules.

Implications for practice and/or policy

- This study encourages practitioners and scholars to consider hybrid teaching intelligence when designing student-centred AI learning tools, focusing on supporting the development of effective teacher–AI collaborative technologies.

## INTRODUCTION

The current landscape of educational practice is being significantly reshaped by the integration of AI, presenting a transformative potential to enhance teaching and learning methodologies (Luckin et al., 2022; McCalla, 2023). This underscores the need for educators to cultivate new competencies tailored to harness AI tools' capabilities within instructional contexts (Celik, 2023). To effectively utilize AI, teachers must be aware of the advantages that AI offers and how those advantages can transform their role in the classroom (Hrastinski et al., 2019). Besides the role of the teacher as a facilitator (Luckin et al., 2022), very little is known about teachers' and AI entanglements for instructional education (Kim et al., 2021).

Despite the recent ongoing initiatives (eg, by UNESCO<sup>1</sup>) in understanding and modelling AI knowledge, skills and attitudes that teachers should possess for effective teacher–AI collaboration, very little is known on the challenges and opportunities when AI intertwines with instruction and how students perceive such hybrid teaching intelligence.

While commendable efforts are being made to explore how AI competencies complement educators' roles, it is essential to recognize that the introduction of AI may fundamentally alter the pedagogical approaches employed in classrooms and, consequently, the way the students learn. This evolution underscores the necessity for concurrent exploration of co-teaching (Holstein et al., 2020) and co-learning (Huang et al., 2019) processes between teachers and AI. Such processes are necessary for combining the strengths of both teachers and AI and for mutual reinforcement, thereby culminating in the realization of hybrid intelligence (HI) paradigms, all while keeping in mind the paramount importance of student learning outcomes (Järvelä et al., 2023). In this work, we aimed to focus on how AI can support and augment the teacher work by giving instructions to students and the potential of hybrid teaching intelligence from the students' learning experience perspective. Therefore, we propose the following research questions (RQs):

- RQ1: What are the learners' opportunities and challenges that arise when AI intertwines with teachers' instructional practices, particularly in the context of an embodied mathematics learning experience?
- RQ2: How do learners perceive those opportunities and adversities, and how do the states of stress, engagement and fatigue evolve across moments of AI–teacher support?

To shed light on those questions, we utilized rich data collected from an in-situ study in which 29 students, aged 11–14, worked with an AI rule-based interactive system named: MOVES-NL. It brings movement and immediate formative feedback together in a room-scale environment designed to enhance students' learning of integer arithmetic. We unobtrusively collected students' multimodal data (ie, video recordings using a camera, physiological from wristbands and skeletal from Microsoft Kinect) while they were interacting with MOVES-NL and encouraged verbal reflection on their actions and mathematical reasoning. To examine the learners' opportunities and challenges that arise when AI intertwines with teachers' instructional practices in the context of an embodied mathematics learning experience (RQ1), we employed qualitative video coding and homed in on the moments when full-body movements either complemented or contrasted students' previously received instruction. Next, to examine how learners perceive those moments (RQ2), we computed students' engagement, fatigue and stress and compared them with their average levels of these states. Lastly, we conducted a time-series analysis using Markov process to explore the time dependence of those states (eg, if students familiarize themselves with the AI support over time and subsequently change the ways they sense make). This approach allowed us to understand how teachers and AI entangle to achieve effective hybrid teaching intelligence, which provides implications for the design of both AI and instruction. The contributions of this paper include the following:

1. Learners' opportunities and challenges that arise when teachers' instructional practices intertwine with an AI rule-based interactive system, in the context of embodied mathematics learning;
2. Identifying how learners perceive those opportunities and challenges by looking at their engagement, stress and fatigue;
3. A discussion on how insights from teacher–AI entanglement can inform instructional and technology design to support teaching and learning.

## RELATED WORK AND BACKGROUND THEORIES

AI is expected to function in learning spaces alongside traditional classroom activities; therefore, it is vital that human instruction and AI intertwine with each other. At the same time, students and teachers develop different relations with and through AI, beyond just interacting with it (Schoonderwoerd et al., 2022). Those relations should be understood and empowered as a means of expanding HI (instead of merely using AI to replace them), and this should be done by taking into account human expertise and intentionality (Akata et al., 2020). HI approaches have already demonstrated promising potential to advance current theories and practices in education (Järvelä et al., 2023). Holstein et al. (2020) synthesized a set of dimensions to capture human–AI hybrid adaptivity and constructed a conceptual framework that indicates distinct ways in which humans and AI can augment each other's abilities. Despite those important developments, further work is needed in understanding and shaping HI in education (Roschelle et al., 2020). To date, little is known about how teachers and AI intertwine and collaborate to achieve effective hybrid teaching.

To address this gap, our study delves into the co-evolution of students and teachers with AI, guided by the theoretical frameworks of Entanglement HCI (Frauenberger, 2019) and mediation theory (Verbeek, 2015). Both theories are instrumental in understanding the complex, intertwined relationships that develop through interactions with technology. Entanglement HCI examines the deep interdependencies and consequences that arise from these relationships, which is crucial for analysing how teachers and students adapt to and influence AI-driven environments (Frauenberger, 2019). Mediation theory builds on the assumption that interaction is not limited to the function and use of technologies by humans (Verbeek, 2015). It provides a philosophical point of view to our exploration, emphasizing that technology is not merely a facilitator of educational processes but actively transforms how educational relationships are formed and sustained.

AI contributes to this transformation through the co-teaching (Holstein et al., 2020) and co-learning (Huang et al., 2019) processes between teachers and AI. Teachers and AI have different capabilities and mental models. Efficient teacher–AI collaboration requires the development of a mutual understanding (ie, a shared mental model) between the two parts (Huang et al., 2019), and this cultivates a mutually benefiting relationship by complementing or augmenting each other's capabilities with a goal of achieving results that the teacher or AI cannot achieve alone. Over time, teachers and AI are co-evolving (Huang et al., 2019; Järvelä et al., 2022) and expanding their capabilities by self-reflecting and self-regulating their learning strategies; this allows them to develop new or revise their capabilities. AI-specific features such as automation and capabilities such as feedback influence the relationship between AI and the teacher and the ways this hybrid teaching intelligence materializes. In particular, AI mediates educational experiences by providing personalized learning paths, recommending different instructional strategies, offering real-time feedback support and reshaping pedagogical approaches. Such relational perspectives can broaden the view of co-learning and HI. In this work, we explore the dimensions and relevance of learner– and teacher–AI entanglements for understanding and growth of hybrid teaching intelligence.

Furthermore, AI-enabled learning systems may help students develop conceptual knowledge of learning content, specifically in mathematics learning. Indeed, Lampert (1990) explains how social and cultural practices influence students' perceptions of mathematical knowing and learning; she argues that mathematical knowing is too often associated with getting the correct answer and that 'these cultural assumptions are shaped by school experience, in which doing mathematics means following the rules laid down by the teacher' (p. 32). Furthermore, Thompson and Dreyfus (1988) reported that students typically struggle to generalize algebraic concepts and, instead, view upper level mathematics as a set of

rules to be followed. However, students will not be successful in their mathematical careers if they rely on an only surface level, algorithmic knowledge and may eventually experience what (Wilensky, 1997) called 'epistemological anxiety' or a feeling of being lost and lacking comprehension of the mathematical symbols which they are competently manipulating. AI has the potential to combat these typical school practices by providing constant feedback to prompt student thinking.

## METHODS

### The MOVES-NL design

In the context of this study, we designed and developed a technology named MOVES. MOVES is a portable multisensory environment (MSE)—enabling technology that overcomes the hardwired limitation of most environments. The hardware platform has a solidly and flexibly built structure with wheels that allow easy transport. The platform holds a mini-PC that reads motion-sensing and outputs to two Ultra Short Throw LED projectors using two independent video outputs (projecting the interacting area on the floor and on the wall). The platform is equipped with SENSEi software (Gelsomini, 2023), which is installed as a set of modules that interface to the sensing and actuation devices and a final viewable layer to which contents are displayed.

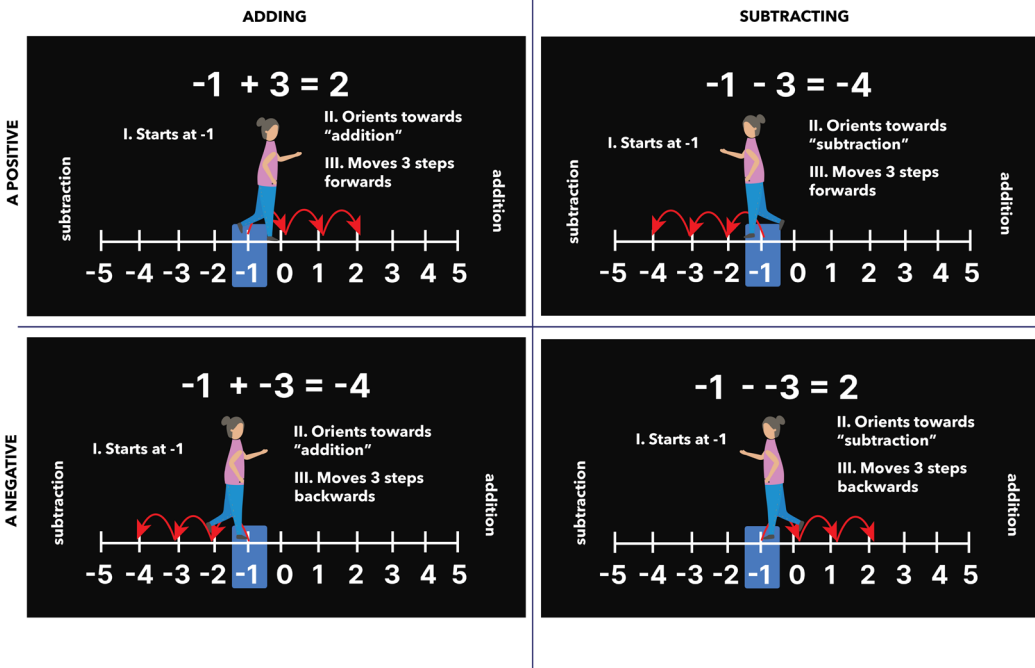
In the MOVES-NL educational design, students engage in two activities by walking on a body-scale number line (NL) in order to solve integer arithmetic problems. The NL is a beneficial pedagogical resource for teaching integer arithmetic because it represents integers as ordinal and spatially organized, where negative integers are a reflection of positive ones (Varma & Schwartz, 2011). Previous studies have shown that using the NL as a semiotic resource for teaching integer arithmetic influences leads to higher levels of numerical fluency when compared with other pedagogical methods (Bofferding & Hoffman, 2014; Nurnberger-Haag, 2018).

The basic instructions for how to solve integer arithmetic problems by walking on the NL are (see Figure 1) as follows:

- a. Start by standing on the first number in the problem;
- b. Turn to the right (positive side of the NL) for addition problems, or turn to the left (negative side of the NL) for subtraction; and
- c. Walk the number of steps indicated by the second number in the problem (forwards if the number is positive, backwards if the number is negative).

Figure 1 depicts a child's movements for the four potential combination schemes of adding or subtracting positive or negative integers on the Walking NL, as reflected by the three-move instructions above.

The first activity of the MOVES-NL (Figure 2) includes a Walking NL projected onto the floor ranging from  $-5$  to  $+5$  and an integer addition or subtraction problem projected onto the wall. The software SENSEi (Gelsomini, 2023) allows the dual wall and floor projectors to track students' movement, position and orientation while walking on the NL and provides feedback based on predetermined rules (eg, mirroring students' movements, indicating correctness). Specifically, when students step onto each hash mark along the NL, the corresponding number under their feet is highlighted and a small sound is played. This way, students receive auditory and visual feedback that the system is recognizing their position on the NL. In addition to capturing students' position on the NL, the motion sensor recognizes bodily orientation and movement. As the student performs the correct movements,



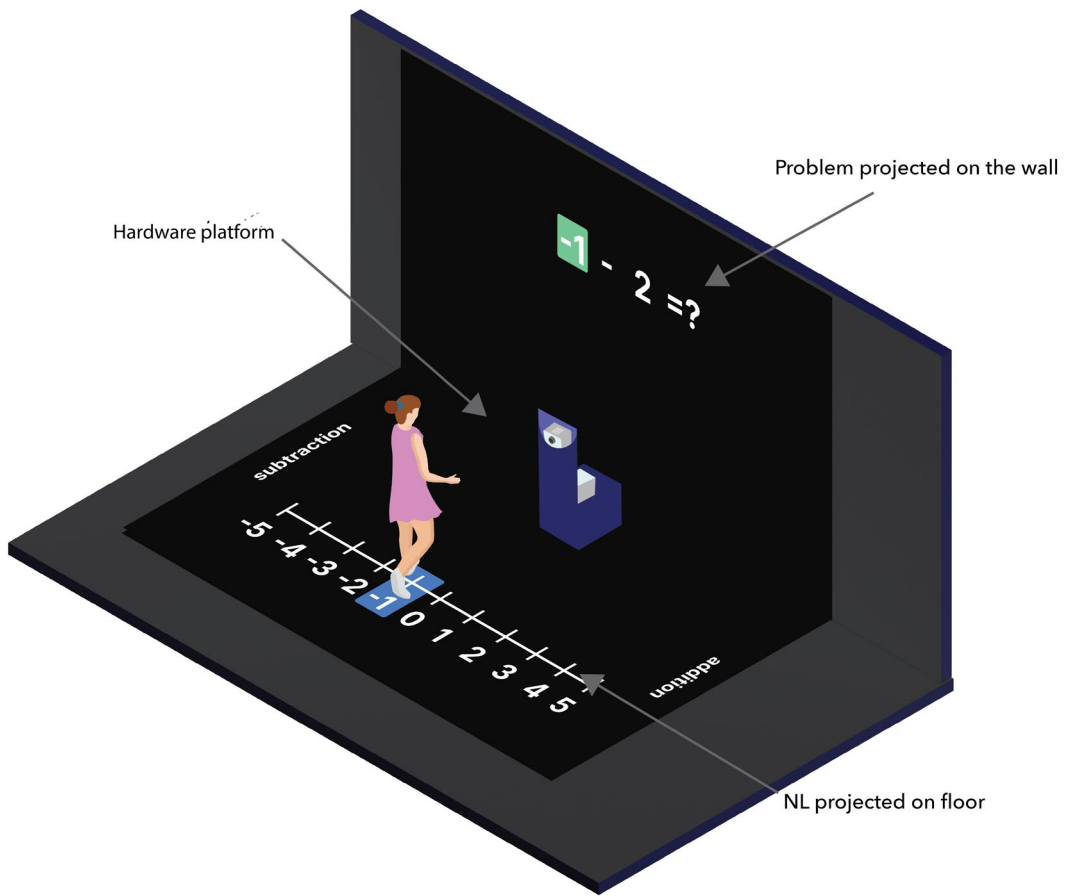
**FIGURE 1** Representation on how a student walks through the NL under the four different possible combination schemes of adding or subtracting positive or negative integers.

the problem projected onto the wall in front of them turns green and a congratulatory chime is played, which provides students with immediate feedback on their whole-body solution-oriented movements. For example, to solve the problem ‘ $-1 - 2$ ’, the student would begin by standing on the  $-1$  hash mark on the NL. As they stepped onto  $-1$ , the  $-1$  underneath their feet would turn blue. At the same moment, the  $-1$  on the wall in front of the student would turn green, and a small sound would be played. Next, the student would turn to the left, orienting themselves towards the subtraction direction. As soon as the student turns, the subtraction sign on the wall turns green, and another sound is played. Finally, the student would need to take two steps forward. Once the student has taken these two steps, they raise their hands above their head to signal that they have arrived at the solution. If they are correct, the entire problem with the solution on the wall turns green, and a congratulatory sound plays. If they are incorrect, the solution on the wall does not turn green and no sound is playing. Avoiding negative feedback is justified as it may demotivate or discourage students (Van Duijvenvoorde et al., 2008).

Differently from the first activity, the second activity introduces a virtual avatar projected onto the wall which mirrors the student's position and movements (Figure 3).

## Context and procedures

In collaboration with Mission Dolores Academy in San Francisco, the study lasted 2 weeks in October 2023. The teacher provided us with a list of participants for each day, and one of the researchers randomly called the participating students from the class list one by one; on average, each student's session lasted 40–50 minutes. Throughout the entire session, the facilitator engaged students in a semi-structured interview (Ginsburg, 1997) so as to gain

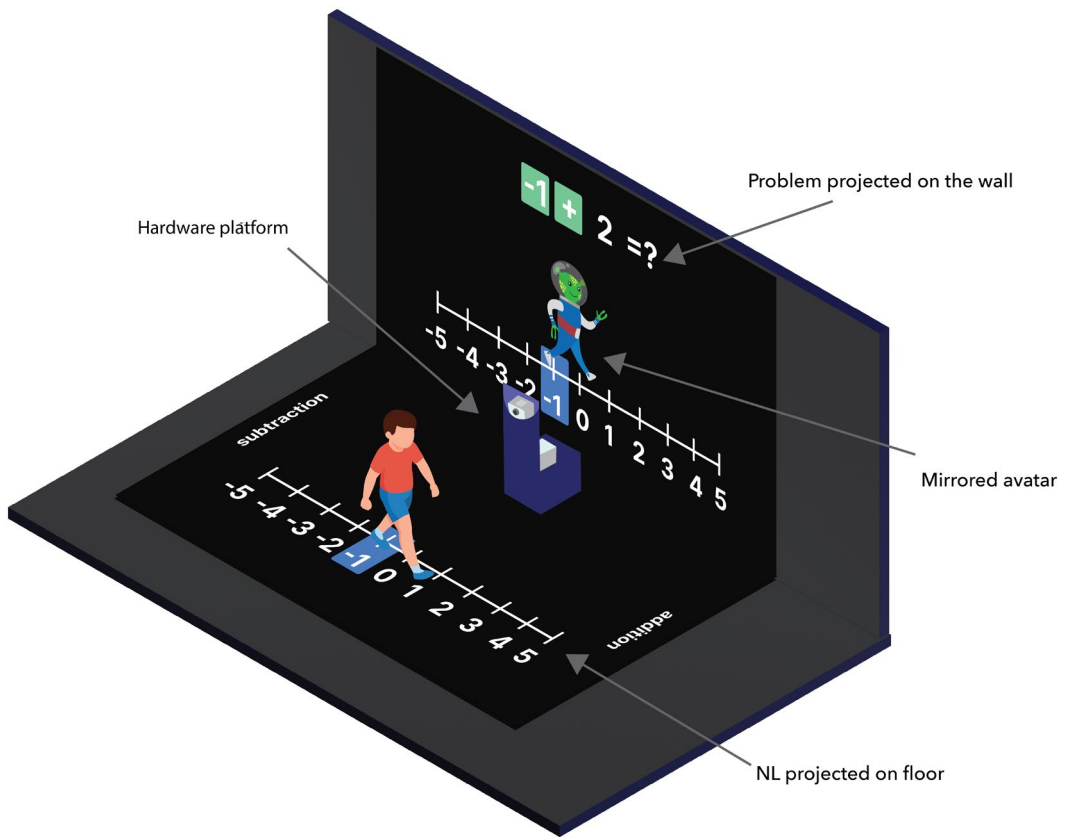


**FIGURE 2** First activity: Students walk a floor-projected interactive NL.

insight into their mathematical reasoning. We conducted a within-subjects study to investigate learning opportunities and adversities emerging when AI intertwines with instruction during the activities of the MOVES-NL (each student tried all the activities). The study room was set up in a dedicated classroom inside the school to avoid external distractions. Each study session consisted of the phases:

1. Facilitator's introduction covering our identity, planned activities and the data collection process (including camera recording and wristband setup).
2. The facilitator asked the participants questions about their experience with math in the classroom.
3. Introduction to the Walking NL: the facilitator showed students how to move to solve problems.
4. First activity, Walking NL: the student walks the NL to solve the problem projected on the front screen.
5. Second activity, Walking NL with mirrored avatar: the student walks the NL in order to solve arithmetic problems, while an avatar mirrors movements on the screen.

*Hybrid teaching intelligence:* The second author, who was a math teacher for 5 years, facilitated both activities by providing initial instructions and helping students work through moments of confusion. The teacher also engaged students in reflective questioning that



**FIGURE 3** Second activity: Students walk a floor-projected interactive NL, a screen-based avatar is introduced who mimics their whole-body movements.

encouraged them to reflect both upon their movements and on mathematical concepts (eg, 'do you think this answer is correct? Why?' or 'How did you remember to move?'). Therefore, students received support both from the teacher and from the AI system through automated feedback. The teacher decided when to change the difficulty of problems based on each student's performance. The difficulty level of the problems was ordered: easy, medium, and hard. The AI system automatically randomized problems on a specific level from the list provided by the educator.

## Participants

Our sample consisted of 28 students in grades 6–8 (ages 11–14). Students attended a private school in San Francisco, whose mission was to serve an economically disadvantaged community. Prior to their participation, a written informed consent was obtained from their legal guardians. All the ethical procedures were approved by the national human research ethics organization. Students' participation and data collection were conducted after approval from the Norwegian Agency for Shared Services in Education and Research (Sikt) and with Institutional Review Board approval (protocol ID 2022-10-15703), following all the regulations and recommendations for research with students.



## Data collection and measurements

We recorded students' interactions with MOVES-NL and employed sensing devices which allowed us to capture students' experience via multimodal data (eg, physiological data from the wristband and skeletal data from the motion sensor). The decision to use these data collection techniques was also influenced by the fact that they account for (to some extent) students' embodied learning and their importance in multisensory systems (eg, students externalize their actions with the use of their body/skeleton). The sensing devices and their respective multimodal data allowed us to closely monitor and understand how students experienced the received support, leveraging the key affordances of multimodal data (eg, temporality and direct access to indicators of students' cognitive and affective processes (Cukurova et al., 2020)). Students' activity sessions were recorded using two mobile cameras and two additional sensor devices: Empatica E4 wristbands and the Astra Pro camera. Below we outline the main data collections and measurements used in our analysis.

*Semi-structured interview:* We implemented a semi-structured interview during the session (see Appendix in supplementary files for the protocol) that was video-recorded using two mobile cameras: one in the front and one in the back.

*Skeleton data:* These data were collected using the Orbbec Astra motion-sensing device with the Nuitrack software (3DiVi, 2023) and recorded at a sampling rate of 10Hz. These represented the 3D position of 20 joints: head, neck, spine, hip-centre, left and right hands, wrists, elbows, shoulders, feet, ankles, knees and hips.

*Physiological data:* Students' physiological data were captured using an Empatica E4 wristband and measured four different variables: heart rate variability—HRV (1 Hz), electrodermal activity—EDA (64 Hz), skin temperature (4 Hz), and blood volume pulse—BVP (4 Hz).

## Multimodal measurements

When it comes to the skeleton and physiological data, we computed the following measurements:

*Stress (physiological data):* Computed as HRV's increasing slope. The more positive the slope of the HRV is in a given time window, the higher the stress is (Taelman et al., 2009). The HRV has been used to measure stress in educational (Sharma et al., 2019) and problem-solving (Mirjafari et al., 2019) contexts.

*Engagement (physiological data):* A linear combination of EDA's increasing slope and the arrival rate of EDA peaks. The more positive slope of the EDA and the higher the rate of arrival of peaks in a given time window is, the higher the engagement is (Hasson et al., 2008; Leiner et al., 2012).

*Fatigue (skeleton data):* Fatigue is proportional to energy spent. For moving objects, it can be shown that the trajectory with the lowest jerk (rate of change of acceleration) is the least energy consuming. Hence, greater jerk leads to greater fatigue. This is computed as the average jerk of all the joints (Guigon et al., 2007).

## Data analysis

In this sub-section, we describe the analysis used to answer each of the RQs. (Table 2).

To explore the educational opportunities and challenges presented by integrating AI with teaching (RQ1), we applied qualitative video coding, starting with an exploratory, inductive approach (Mayring et al., 2004; Patton, 1990). The first two authors independently reviewed

**TABLE 1** The code names and their descriptions used to annotate the videos.

Sub-code name	Description
Interaction—Complementary (ICM)	During the interaction: when students' movement solutions to the problems and the feedback from the facilitator–system were complementary
Interaction—Conflicting (ICN)	During the interaction: when students' movement solutions to the problems and the feedback from the facilitator–system were conflicting
Post–Interaction—Complementary (PCM)	After the interaction: when students made connections between their movement-based solutions and integer arithmetic rules learned in the classroom
Post–Interaction—Conflicting (PCN)	After the interaction: when students did not make connections between their movement-based solutions and integer arithmetic rules learned in the classroom

**TABLE 2** Research question and analysis performed.

Research question	Data collection and analysis
RQ1	<i>Data:</i> Video recordings from 28 students engaging with teacher–AI support <i>Analysis:</i> Qualitative video coding to identify key behavioural patterns and assessed the alignment of AI-driven feedback with classroom teaching, evaluating both complementary and discordant interactions
RQ2	<i>Data:</i> Multimodal measurements (see third section) <i>Analysis:</i> Computation of students' engagement, stress and fatigue and comparison with their average states

all videos and then collaboratively identified and coded patterns of students' interactions with the technology, labelling specific behaviours and utterances (Braun & Clarke, 2006), such as 'making sense' or 'refers to teacher rules'. This process, guided by established qualitative research principles, gradually became more interpretive, aiming to understand the implications of these patterns. The coding reliability was confirmed with a Kappa score of 0.74.

The first author then went through the video frames of the patterns coded, focusing on two recurring scenarios: students solving problems using the NL with support from both the teacher and the system's AI and subsequent reflections guided by the teacher on the solutions' relevance to classroom-taught rules. Further, we categorized interactions based on whether students' solutions and system feedback were complementary and whether students connected their solutions to classroom learning (see Table 1 for the description of the sub-codes). The selected coding addresses the important elements of the related works and theories that highlight the critical role of complementarity and intentionality (eg, Akata et al., 2020; Huang et al., 2019; Verbeek, 2015).

Next, to examine how students perceived those moments and identify lessons learned for technology and instruction design (RQ2), we conducted an analysis of variance (ANOVA), with the correction for different cardinalities, to find the differences in the affective states (independent variables: engagement, stress and fatigue) across the sub-codes (dependent variables). Prior to ANOVA, we conducted the Shapiro–Wilk test to determine the normality of the data and Breusch–Pagan test to determine the homoskedasticity across the sub-codes. For the pairwise testing, we conducted post hoc pairwise *t*-tests using the adjusted standardized residuals and an adjusted *p*-value (ie, Bonferroni method) (Beasley & Schumacker, 1995), which determine the pairwise differences between sub-codes. For each physiological measure, we had to conduct 15 pairwise *t*-tests, therefore the alpha

altered =  $0.05/15 = 0.003$  (rounded). This is less than 0.05 and alpha critical =  $1 - (1 - (\alpha \text{ altered})^{15}) = 0.044$  (rounded). Therefore, the significance level is set to 0.044.

Next, for time series analysis, we conducted a Markov process, which is a probabilistic model for representing probability distribution over sequences of observation (Vatsalan et al., 2022) analysis and then analysed the transitions between the multimodal states. These states were defined using engagement, stress and fatigue. Once we had the Markov models (MMs) fitted on the different episodes defined by the sub-codes, we conducted ANOVAs to differentiate the transition probabilities between the sub-codes. We followed the same process as we described above for ANOVA and post hoc tests and, once again, the acceptable  $p$ -value was set at 0.044. There were two main reasons for using MM here: (1) MMs are well-suited for analysing sequential data, such as sequences of events or states over time. They can capture dependencies between successive observations and model the evolution of a system through discrete states. (2) MM offers clear interpretations of transition probabilities, which can provide insights into the underlying dynamics of the system being modelled. This interpretability can be valuable for decision making and identifying patterns in data

## RESULTS

We further analysed the sub-codes described in Table 1 with the aim of identifying learning opportunities and adversities emerging when AI intertwines with instruction. For both the first and second activities, we identified a total of 200 moments where interaction or post-interaction was either complementary or conflicting to instruction (sub-codes). For clarity, we have divided the sub-codes for the first and second activities (eg, 1.ICM or 2.ICM).

The code ICN (eg, when one participant was solving the problem  $'-4 - 8'$ , she remained confused even after both system and teacher feedback) was only present in three moments (less than 1%); therefore, we removed this sub-code from the analysis. On the other hand, we identified many moments in which a student was able to solve the problem by interacting with the system, given both the teacher and system feedback (1.ICM—24.86%). Moreover, we noticed that when students made mistakes and consequently did not receive any feedback from the system, they often reflected on their movements and recognized their errors. The facilitator's intervention during these moments was crucial because she was able to help students link their full-body movements with the symbolic notation of the problem. In most cases, both the system and the facilitator's feedback were complementary to the success of the learning experience. For example, when P4 raised his hands to select what he thought to be the correct answer, he became confused when he did not experience congratulatory feedback. The teacher then asked him to explain why he was confused and helped him towards the solution by providing real-time feedback. In another example, P9 attempted to solve the problem  $'0 - 4'$  and began correctly by standing on zero and facing the left (subtraction direction). However, she erroneously walked backwards instead of forwards, towards positive 4. The system helped her realize *that* she had made an error by not playing a congratulatory chime, and the facilitator helped her realize *why* she had arrived at an erroneous solution by guiding her through reflection.

The second activity (2.ICM—20.10%) (walking NL with a projected avatar) added another level of real-time support. Indeed, the avatar's mirrored movements helped students understand their own bodily location and positively affected the learning experience; for example, when P2 was solving the problem  $'-4 + 2'$ , he continuously shifted his gaze between the avatar on the wall and his own feet, which he claimed helped him to accurately count his steps. After the students solved each problem, they were asked to explain how they solved them. When the students referred to the rules they had previously learned in class in order to make

sense of the solution and/or the way they had interacted with the system, we categorized it as PCM (1.PCM—19.04% and 2.PCM—24.33%). For example, after P5 correctly moved his body along the NL to solve the problem '3-2', he explained a rule he had previously learned in class, stating: 'the two minuses become a plus'. When the facilitator asked him if this classroom rule aligned with his movements on the NL, the student was confident that they did. In another example, P9 connected her movements to the classroom rules, saying: 'I think the movements are helping me think through it', and 'I think I will go back to the classroom thinking about moving left and right and back and forwards'.

However, there were many difficult moments in which, even though students experienced feedback from both the system and teacher and got to the right answer, they were not able to reason or explain why their movements might connect to the rules they had learned in the classroom. We categorized such events as PCN (1.PCN—7.93% and 2.PCN—3.70%). For example, after P9 solved the problem '-5-9', she was confused by what she had learned in the classroom (that two negatives make a positive) and the movements she did to reach the solution; she could not explain why her classroom rule made sense, even after walking along the NL, and she gave up when trying to verbally reason through it. To summarize, from the aforementioned coding, we found the following challenges and opportunities:

- The support provided to the student when the system intertwines with the instruction is mostly positive for the learning experience (complementary), while the facilitator amplifies the support received by the system (eg, asking students to think aloud and explaining/correct their thinking) making the students reflect on the movements they were making to reach the solution. Thus, *AI served as a powerful support tool for teachers to understand students' misconceptions and complement and amplify their teaching.*
- In the second activity, the continuous support through mirroring students' movements (via an avatar) helped students understand their own bodily location and interactions in a timely manner (eg, posture, first step). Besides supporting students' reflection, this also gave teachers early insights into students' misconceptions and positively affected the students' experience. Thus, *AI provided timely insights to both the student and the teacher, allowing them to reflect and focus their attention on conceptually important features of the activity.*
- The MOVES-NL AI-driven design helped students explain and expand upon many of their teachers' rules when the facilitator prompted them to reason through their answers. Thus, *AI-enabled students to reflect upon and test their previous knowledge and guided teachers to work with students in consolidating challenging topics.*
- There were numerous challenging moments when, despite receiving feedback from the AI and the teacher, students were unable to rationalize or justify their actions, and they were unable to make the connection to the rules they had previously acquired in the classroom. Thus, *hybrid teaching intelligence also resulted in moments where the support offered was not helpful for the student. This highlights two important areas required for effective hybrid teaching intelligence, namely, the student-centred design of AI learning tools and the need for supporting the development of effective teacher-AI collaboration.*

To understand how students perceive the moments where instruction intertwines with the system's support (RQ2), we conducted an ANOVA with post hoc pairwise *t*-tests between those moments and their affective states. Moreover, we used MM, to explore how students' states transition across those important moments. Below, we detail how students' experiences (based on the multimodal measurements described in third section) vary across different moments identified by sub-codes during video analysis.

From the ANOVA tests for engagement, stress and fatigue, we observed a significant difference between engagement across the sub-codes ( $F[5,23]=24.17, p<0.0001$ ,

Figure 4). Table 3 shows the pairwise comparisons. From the pairwise comparisons, we observed that irrespective of the interaction mode, ICM had the highest engagement, followed by PCM and then PCN. There is also a significant difference in engagement between the two activities. The engagement during the first activity is higher than the engagement during the second for all sub-codes. Considering stress, we observed a significant difference across the sub-codes ( $F[5,23]=39.08, p<0.0001$ , Figure 5). Table 4 shows the pairwise comparisons. From the pairwise comparisons, we observed that there is a significant difference between the sub-codes, but not between the first and second activities. Irrespective of the activities, the stress during PCM is the highest, followed by the stress during ICM and then the stress during PCN. Finally, considering the fatigue across sub-codes, we observed a significant relation ( $F[5,23]=37.31, p<0.0001$ , Figure 6). Table 5 shows the pairwise comparisons. From the pairwise comparisons, we observed that there is a significant difference between the sub-codes but not between the first and second activities. Irrespective of the activities, the fatigue during ICM is the highest, followed by the fatigue during PCM and then the fatigue during PCN. Due to a lack of space, the full analysis is available in the Appendix.

Next, based on the MM results, we observed significant differences between the modelled transition probabilities across the different sub-codes. Figure 7 shows the combined representations of the transitions that were significantly different across the sub-codes. The complete statistical tests are presented in Appendix A (ie, ANOVA tests and pairwise tests, Tables A1–A9). Apart from the transitions present in Figure 7, either the rest of the transitions were significantly not different across the sub-codes, or they did not appear in the model. Due to a lack of space, we mainly opted for presenting results that help us tackle RQ2. In Figure 7, the green arrows show the sub-code with the significantly highest transition probabilities (among all the sub-codes), and the red arrows show the sub-codes with

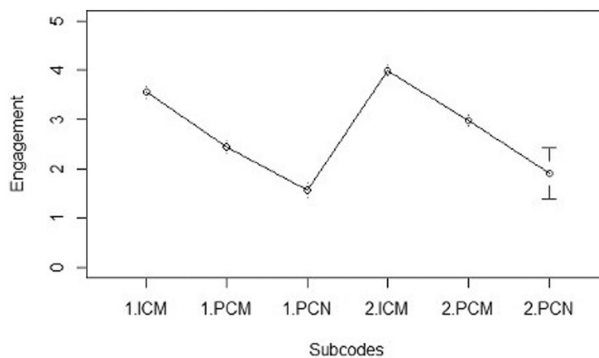


FIGURE 4 Students' engagement across the sub-codes.

TABLE 3 Pairwise one-way ANOVA, with correction for different cardinalities, comparisons for engagement.

Engagement	1.PCM	1.PCN	2.ICM	2.PCM	2.PCN
1.ICM	4.31***	10.25***	-3.25**	2.15*	2.09*
1.PCM		3.07**	-6.21***	-2.16*	1.09
1.PCN			-9.07***	-5.30***	-1.59
2.ICM				4.01***	3.09**
2.PCM					2.01*

\* $p<0.05$ ; \*\* $p<0.01$ ; \*\*\* $p<0.001$ .

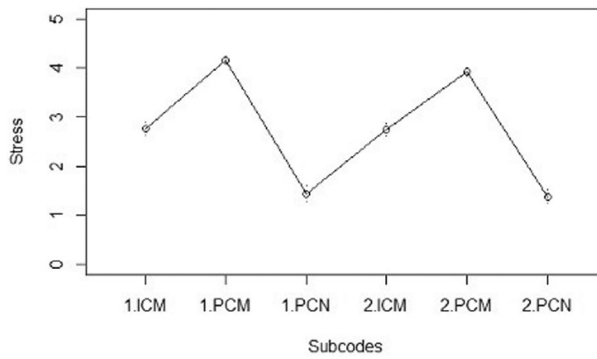


FIGURE 5 Stress across the sub-codes.

TABLE 4 Pairwise one-way ANOVA, with correction for different cardinalities, comparisons for stress.

Stress	1.PCM	1.PCN	2.ICM	2.PCM	2.PCN
1.ICM	-6.04***	4.24***	0.24	-5.15***	5.83***
1.PCM		10.18***	5.84***	1.63	12.62***
1.PCN			-4.02***	-9.15***	0.71
2.ICM				-5.10***	3.21***
2.PCM					12.12***

\* $p < 0.05$ ; \*\* $p < 0.01$ ; \*\*\* $p < 0.001$ .

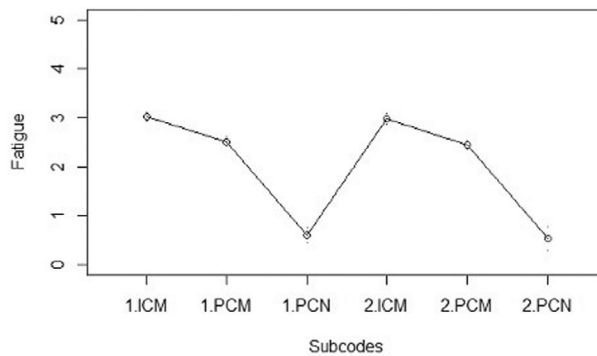


FIGURE 6 Fatigue across the sub-codes.

the significantly lowest transition probabilities (among all the sub-codes). We observed the following from Figure 7.

- 1.ICM: This sub-code has the lowest probability of remaining in a state of low engagement, high stress and low fatigue. Moreover, this sub-code has the highest probability of transitioning from high engagement, low stress and low fatigue to high engagement, low stress and high fatigue, thus indicating the increase in fatigue. Furthermore, this sub-code has maximum probabilities of remaining in states of (1) high engagement, low stress and high fatigue and (2) high engagement, high stress and low fatigue.

**TABLE 5** Pairwise one-way ANOVA, with correction for different cardinalities, comparisons for fatigue.

Fatigue	1.PCM	1.PCN	2.ICM	2.PCM	2.PCN
1.ICM	2.87**	9.81***	0.49	3.01**	7.89***
1.PCM		7.92***	-2.12*	1.04	6.17***
1.PCN			-8.68***	-7.62***	0.60
2.ICM				2.12*	7.25***
2.PCM					5.93***

\* $p < 0.05$ ; \*\* $p < 0.01$ ; \*\*\* $p < 0.001$ .

2. 1.PCM: This sub-code has the highest transition probability of going from low engagement, high stress and low fatigue to low engagement, high stress and high fatigue—indicating the increment in fatigue; and the lowest transition probability of going from high engagement, low stress and high fatigue to high engagement, low stress and low fatigue—indicating the lack of decrement in fatigue. Moreover, this sub-code has the highest probability of remaining in low engagement, high stress and high fatigue.
3. 1.PCN: This sub-code has the lowest probabilities of remaining in states: (1) high engagement, low stress and high fatigue and (2) high engagement, high stress and low fatigue; and the lowest transition probabilities for (1) going from high engagement, low stress and low fatigue to high engagement, low stress and high fatigue—indicating the increment in fatigue and (2) going from high engagement, low stress and high fatigue to high engagement, low stress and low fatigue—indicating the decrement in fatigue.
4. 2.ICM: This sub-code has the lowest probability of remaining in low engagement, high stress and low fatigue and the lowest transition probability of going from low engagement, high stress and low fatigue to low engagement, high stress and high fatigue—indicating the increment in fatigue. Moreover, this sub-code has the highest probability of remaining in high engagement, low stress and high fatigue and the highest transition probabilities of (1) going from high engagement, low stress and low fatigue to high engagement, low stress and high fatigue—indicating the increment in fatigue and (2) going from high engagement, low stress and high fatigue to high engagement, low stress and low fatigue—indicating the decrement in fatigue.
5. 2.PCM: This sub-code has the highest transition probability of going from low engagement, high stress and low fatigue to low engagement, high stress and high fatigue—indicating the increment in fatigue; this sub-code also has the highest probability of remaining in high engagement, low stress and high fatigue. Moreover, this sub-code has the lowest transition probability of going from high engagement, low stress and high fatigue to high engagement, low stress and low fatigue—indicating the lack of decrement in fatigue.
6. 2.PCN: This sub-code has the lowest probabilities of remaining in (1) high engagement, low stress and high fatigue and (2) high engagement, high stress and low fatigue; and the lowest transition probability for going from high engagement, low stress and high fatigue to high engagement, low stress and low fatigue—indicating the lack of decrement in fatigue.

## DISCUSSION AND IMPLICATIONS

In this study, we used qualitative video coding to identify a particular class of moments—when students engaged with MOVES-NL interactive technology while also having to process instruction—in order to highlight learning opportunities and challenges emerging when AI intertwines with instruction (RQ1). Next, we calculated students' involvement and affective states and compared them with their average states to look at how they perceived

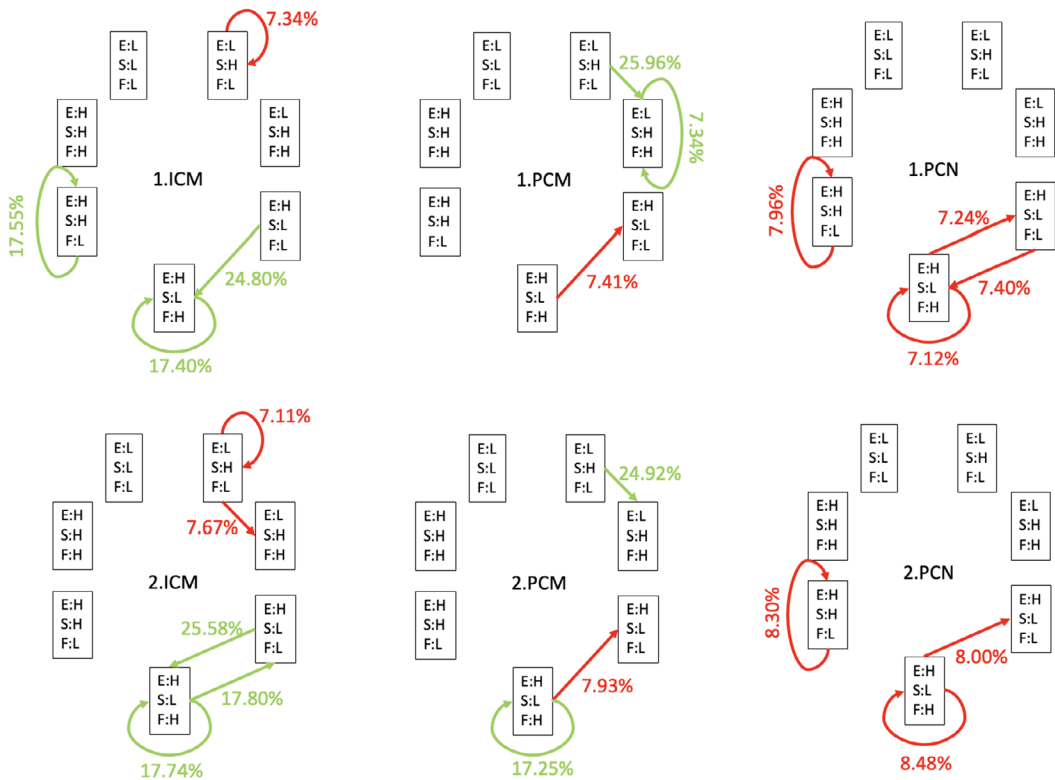


FIGURE 7 Modelled transition probabilities across the different sub-codes.

those moments. Then, we used MM to explore how they transition throughout the duration of every sub-code (when the system's support intertwines with instruction) (RQ2). Our study focuses on understanding the interplay between teacher–AI interaction for students' learning, grounded in the theoretical frameworks of Entanglement HCI and mediation theory. By exploring the integration of AI technology, exemplified by the MOVES-NL system, into teaching and learning processes, we shed light on the nature of teacher and AI entanglement, offering insights into the design and implementation of AI-driven educational systems.

Our results provide a multifaceted view of how students interact with both the AI system and human teachers, revealing subtleties in learning processes that are often invisible in less technologically integrated settings. Indeed, from both qualitative and quantitative results, we could see that when the students interact with the system and get support from both the AI and the teacher (1.ICM), and the learning experience is positive (ie, the feedback provided is complementary, and students tend to reach the correct solution). This corresponds with previous studies that recognized the strengths of the complementary collaboration between the teacher and the technology, highlighting the role of AI in facilitating teaching processes by enhancing teachers' instructions to facilitate student learning (Baker, 2016; Holstein et al., 2017; Kamar, 2016; Luckin & Cukurova, 2019). Also, students benefit from feedback and instruction that is represented in many modes (ie, kinesthetic, tactile, audible and visual; Tancredi et al., 2022). The physiological data from these moments suggest that student engagement was significantly higher than their average engagement and peaked during the second activity where the system offers continuous mirroring capabilities (2.ICM). This highlights *the importance of continuous and timely support from the system, as well as the importance of multiple modes of feedback, which cannot be provided by the teacher or the system alone*. Moreover, the feedback allowed the students to stop and reflect on their



actions, especially when they made a mistake. We believe that *under specific conditions* (ie, when students lack the information or abilities required to reason through the task but have not yet disengaged), students must receive specific, targeted support. Such support can be best provided under an HI approach where the machine can provide the needed insight to the teacher in a timely manner, and the teacher can use their expertise to decide how to best support the student (Hrastinski et al., 2019; Kim et al., 2021). The adoption of implicit continuous support provides new frames of reference for both students and teachers, allowing both parties to reflect on how students progress (Järvelä et al., 2020; Martinez-Maldonado et al., 2021; Soller et al., 2005). By detecting signs of physical fatigue (Figure 7—1.ICM) through posture analysis, the AI could inform the teacher of unseen affective states and propose a transition to other kinds of interaction paradigms (eg, voice interactions, prompting the student to respond verbally instead), slowing down, providing additional clarification or scheduling a break from instruction (Mavroudi et al., 2018; Sharma et al., 2020; Shute et al., 2021; Vesin et al., 2018). *This shift could not only mitigate the student's current negative state but also reinvigorate their engagement by introducing a novel mode of interaction and enable the teacher to provide the appropriate instruction in the right moment.*

When students did successfully connect classroom experience with AI-facilitated movement (coded as PCM), their stress was very high. This could be explained by the fact that the AI-driven MOVES-NL design surfaced learners' inability to explain and expand upon many of their teachers' rules. Usually, when given procedural rules, learners cling to them because of a fear of providing the wrong response to a problem (Staempfli, 2007; Weeks & Howell, 2012). Even if the students were successful in solving the problem, it is possible that they were not used to reflecting on their movements in relation to performing mathematical operations because such epistemic practices do not exist in their mathematics classroom (Feucht, 2010). Reasoning in relation to their full-body movements constituted a mental effort that students perceived as more burdensome in comparison to simply explaining the rules they had learned in their classroom (stress levels high) (Csikszentmihalyi, 2020). *The HI approach enables students to work through complexity and solidify their grasp of mathematical concepts* (Abrahamson, 2012; Saxe et al., 2013). Moreover, in Figure 7, we can see a difference between the two activities, indeed in 2.PCM. A low level of stress self-transition during the second activity could be justified but the fact that the students got used to connecting the rules they had learned with the movements they performed and internalized this process after 20 minutes of interacting with MOVES-NL. This finding suggests that *the teacher–AI collaboration could help students develop conceptual knowledge of the learning content beyond 'following the rules laid down by the teacher'* (Thompson & Dreyfus, 1988).

In the sub-code PCN (when students were not able to reason about why they had moved in a certain way, and they could not connect to the rules they had learned in the classroom), engagement, stress and fatigue were low. In Figure 7, this sub-code (both 1.PCN and 2.PCN) showed the lowest probability of transitioning to more 'high' affective states. This can be explained by considering that in these episodes, the students at first seemed to favour defining the arithmetic/ operative 'rules' rather than reasoning about them, and when asked to push their thinking, they were unwilling to question their classroom teacher's rules, even if they did not understand the rule conceptually. Eventually, students gave up, mostly saying 'I don't know, it doesn't make sense'. Reliance on mathematical rules can lead to short-term success, but conceptual understanding of these rules is critical for students to succeed in higher-level mathematics courses. *Therefore, it is crucial that, during these moments, the teacher–AI collaboration engages and motivates students to reflect upon the solution.* Examples of this include incorporating interactive elements such as gamification to increase engagement, encouraging physical activity to re-engage the student's attention and utilizing different modes of content delivery (eg, text, audio, visual) to cater to diverse learning preferences.

The theoretical contribution of our research lies in its exploration of the dynamics between human–AI interaction within educational settings (Frauenberger, 2019; Verbeek, 2015). These comprehensive data, which include multimodal inputs, demonstrate that students benefit significantly from the combined support of AI and human teachers. Additionally, our research provides theoretical insights into the complementary nature of AI and human teachers (Gibson et al., 2023). The AI's continuous and timely support enhances student engagement and helps in providing targeted feedback, which improves learning outcomes. The research highlights the importance of investigating teacher–AI co-learning processes (Huang et al., 2019) to provide students with enhanced and personalized learning experiences. Effective teacher–AI collaboration is critical for engaging and motivating students, particularly when they struggle to connect procedural rules with conceptual understanding, which is particularly important in subjects like mathematics, where understanding underlying concepts is crucial for long-term success.

Moreover, we practically contribute in the ongoing research on how multimodal data can inform educational processes with an analysis of the AI-enhanced learning sessions. In our work, *the utilization of multimodal data—encompassing video recordings, skeleton tracking and physiological measures—enables a comprehensive understanding of the learning environment* (Giannakos & Cukurova, 2023). The integration of advanced machine learning models can also enhance the AI's ability to understand and predict student needs, preventing difficulties and enriching the ways in which students engage with the learning content. This helps in mitigating negative states and reinvigorating student engagement, ensuring that the support provided is timely and effective. This continuous feedback loop between the AI and the teacher allows for more responsive and adaptive teaching strategies (Nazaretsky et al., 2023). A suitable interface that facilitates teachers and AI's reciprocal interaction may be necessary (van Leeuwen et al., 2021) for two-way communication: teachers can update the system and share their pedagogical actions and knowledge, and AI can diagnose and provide teachers with information based on data collected during the learning process (Holstein et al., 2018; Kim, 2023; van Leeuwen et al., 2021). *Incorporating feedback loops for students and educators into the AI's learning model can refine its approach, ensuring a continuously improving and adapting teaching strategy.*

Furthermore, the introduction of LiDAR technology to interact with projected walls offers an innovative approach to enriching student interaction and engagement. By allowing students to physically manipulate and engage with projected content through gestures and movements detected by LiDAR sensors and processed by dedicated software (Slamtec, 2024), this technology can bridge the gap between abstract concepts and tangible understanding. This hands-on interaction not only facilitates a more immersive learning experience but also encourages active participation and collaboration among students, making the learning process more dynamic and engaging.

## LIMITATIONS AND CONCLUSION

In this article, we present a comprehensive analysis of the role of HI (teacher–AI entanglement) in supporting education utilizing an AI-driven system. Our findings come from a mixed-method approach, including video recordings and sensing technology. In particular, we proposed learning opportunities and adversities emerging when AI intertwines with instruction as identified from the video coding; followed by a focus on how learners perceive those moments, through physiological analysis and time-series analysis (MM). Our results shed light on the potential of teachers and AI collaboration and how it will further build on the combined strengths of each other, which indicates that AI is not a mere tool but a collaborative agent to augment teachers' capacity (AlShaikh & Hewahi, 2021).

Our findings report on an in-situ study where students interacted with MOVES-NL in the context of embodied integer arithmetic learning. Despite the carefully selected case study and potential for producing generalized knowledge (by seeing our results through the lens of entanglement and mediation theories), we should also stress some limitations. Even though we believe that the study findings may have wider applicability, they are specific to the group of participants we investigated (28 students in grades 6–8 in a private school for economically disadvantaged community). In this sense, a larger generality of the insights should be carefully made by taking into consideration the specific context and artefacts. In the case study analysed, we did not provide negative feedback from the AI; indeed, the teacher facilitated when a child did not respond correctly or was confused. Implementing automated negative feedback has the potential to alleviate the burden on teachers by providing timely and consistent corrective guidance to students. However, it is important to acknowledge that automating negative feedback presents its own set of challenges and considerations. Negative feedback, if not delivered thoughtfully, could potentially demotivate or discourage students. Future research could explore the feasibility and effectiveness of incorporating automated negative feedback into AI-driven educational systems, considering the potential benefits and drawbacks of such an approach. Moreover, the AI rule-based interactive system we focused on for this study allowed for a robust and accurate set of rules easy to control from a teacher's perspective. We believe that contemporary generative AI-powered systems (whose data processing and decision making are not clear to the teachers) is expected to present additional challenges to the teacher (eg, lack of reliability, wide range of support and complexity in corroborating and orchestrating), resulting in more dynamic and complex teacher–AI entanglement. Thus, even though we believe that the study findings provide important early steps towards understanding teacher–AI entanglements and may have wider applicability, when interpreting our results, we need to take into account the nature of the system used and the group of participants whom we investigated.

### **ACKNOWLEDGEMENTS**

We would like to thank all the children and teachers who participated in this study. Without their enthusiasm and dedication, this work would not have been possible.

### **FUNDING INFORMATION**

This manuscript was funded by the Peder Sather Grant programme and has not been presented publicly.

### **CONFLICT OF INTEREST STATEMENT**

There is no potential conflict of interest in this study.

### **DATA AVAILABILITY STATEMENT**

As it is possible to identify participants from the data, ethical requirements do not permit us to share participant data from this study. Participation was voluntarily, and all the data collected anonymously.

### **ETHICS STATEMENT**

We confirm that the data discussed in this manuscript have been collected with the ethical approval of all relevant bodies. IRB approval was obtained before the collection of data (protocol ID 2022-10-15703). Written consent to publish potentially identifying information, such as details and quotations, was obtained from the participants and/or their legal guardians. Appropriate permissions and ethical approval were requested and approved.

## ORCID

Giulia Cosentino  <https://orcid.org/0000-0001-7569-1019>

Jacqueline Anton  <https://orcid.org/0000-0003-0667-6004>

Kshitij Sharma  <https://orcid.org/0000-0003-3364-637X>

Dor Abrahamson  <https://orcid.org/0000-0003-2883-4229>

## ENDNOTE

<sup>1</sup><https://www.unesco.org/en/digital-education/ai-future-learning>.

## REFERENCES

- 3DiVi. (2023). *Nuitrack full body skeletal tracking software*. Nuitrack. <https://nuitrack.com/>
- Abrahamson, D. (2012). Seeing chance: Perceptual reasoning as an epistemic resource for grounding compound event spaces. *ZDM*, 44, 869–881. <https://doi.org/10.1007/s11858-012-0454-6>
- Akata, Z., Balliet, D., De Rijke, M., Dignum, V., Dignum, F., & Eiben, G. (2020). A research agenda for hybrid intelligence: Augmenting human intellect with collaborative, adaptive, responsible, and explainable artificial intelligence. *Computer*, 53(8), 18–28. <https://doi.org/10.1109/MC.2020.2996587>
- AlShaikh, F., & Hewahi, N. (2021). AI and machine learning techniques in the development of intelligent tutoring system: A review. In Zallaq, Bahrain 2021 *International Conference on innovation and Intelligence for informatics, computing, and technologies (3ICT)* (pp. 403–410). IEEE. <https://doi.org/10.1109/3ICT53449.2021.9582029>
- Baker, R. S. (2016). Stupid tutoring systems, intelligent humans. *International Journal of Artificial Intelligence in Education*, 26, 600–614. <https://doi.org/10.1007/s40593-016-0105-0>
- Beasley, T. M., & Schumacker, R. E. (1995). Multiple regression approach to analyzing contingency tables: Post hoc and planned comparison procedures. *The Journal of Experimental Education*, 64(1), 79–93. <https://doi.org/10.1080/00220973.1995.9943797>
- Bofferding, L., & Hoffman, A. J. (2014). *Learning negative integer concepts: Benefits of playing linear board games*. International Group for the Psychology of Mathematics Education. <https://api.semanticscholar.org/CorpusID:231261619>
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Celik, I. (2023). Towards intelligent-tpack: An empirical study on teachers' professional knowledge to ethically integrate artificial intelligence (AI)-based tools into education. *Computers in Human Behavior*, 138, 107468. <https://doi.org/10.1016/j.chb.2022.107468>
- Csikszentmihalyi, M. (2020). *Finding flow: The psychology of engagement with everyday life*. Hachette UK. <https://doi.org/10.5860/choice.35-1828>
- Cukurova, M., Giannakos, M., & Martinez-Maldonado, R. (2020). The promise and challenges of multimodal learning analytics. *British Journal of Educational Technology*, 51, 1441–1449. <https://doi.org/10.1111/bjet.13015>
- Feucht, F. C. (2010). Epistemic climate in elementary classrooms. In L. D. Bendixen & F. C. Feucht (Eds.), *Personal epistemology in the classroom: Theory, research, and implications for practice* (pp. 55–93). Cambridge University Press. <https://doi.org/10.1017/CBO9780511691904.003>
- Frauenberger, C. (2019). Entanglement HCI the next wave? *ACM Transactions on Computer-Human Interaction (TOCHI)*, 27(1), 1–27. <https://doi.org/10.1145/3364998>
- Gelsomini, M. (2023). *Sensei—Experience in all senses. multisensory and multimodal research framework*. <https://sensei.space/>
- Giannakos, M., & Cukurova, M. (2023). The role of learning theory in multimodal learning analytics. *British Journal of Educational Technology*, 54(5), 1246–1267. <https://doi.org/10.1111/bjet.13320>
- Gibson, D., Kovanovic, V., Ifenthaler, D., Dexter, S., & Feng, S. (2023). Learning theories for artificial intelligence promoting learning processes. *British Journal of Educational Technology*, 54(5), 1125–1146. <https://doi.org/10.1111/bjet.13341>
- Ginsburg, H. (1997). Not a cookbook: Guidelines for conducting a clinical interview. In *Entering the child's mind: The clinical interview in psychological research and practice* (pp. 115–137), Cambridge: Cambridge University Press. <https://doi.org/10.1017/CBO9780511527777.005>
- Guigon, E., Baraduc, P., & Desmurget, M. (2007). Computational motor control: Redundancy and invariance. *Journal of Neurophysiology*, 97(1), 331–347. <https://doi.org/10.1152/jn.00290.2006>
- Hasson, U., Furman, O., Clark, D., Dudai, Y., & Davachi, L. (2008). Enhanced intersubject correlations during movie viewing correlate with successful episodic encoding. *Neuron*, 57(3), 452–462. <https://doi.org/10.1016/j.neuron.2007.12.009>

- Holstein, K., Alevan, V., & Rummel, N. (2020). A conceptual framework for human-AI hybrid adaptivity in education. In *Artificial Intelligence in Education: 21st International Conference, AIED 2020, Ifrane, Morocco, July 6–10, 2020, Proceedings, Part I* (pp. 240–254). [https://doi.org/10.1007/978-3-030-52237-7\\_20](https://doi.org/10.1007/978-3-030-52237-7_20)
- Holstein, K., Hong, G., Tegene, M., McLaren, B. M., & Alevan, V. (2018). The classroom as a dashboard: Co-designing wearable cognitive augmentation for k-12 teachers. In *Proceedings of the 8th International Conference on Learning Analytics and Knowledge* (pp. 79–88), New York <https://doi.org/10.1145/3170358.3170377>
- Holstein, K., McLaren, B. M., & Alevan, V. (2017). Intelligent tutors as teachers' aides: Exploring teacher needs for real-time analytics in blended classrooms. In *Proceedings of the Seventh International Learning Analytics & Knowledge Conference* (pp. 257–266), New York. <https://doi.org/10.1145/3027385.3027451>
- Hrastinski, S., Olofsson, A. D., Arkenback, C., Ekström, S., Ericsson, E., Fransson, G., Jaldemark, J., Ryberg, T., Öberg, L. M., Fuentes, A., Gustafsson, U., Humble, N., Mozelius, P., Sundgren, M., & Utterberg, M. (2019). Critical imaginaries and reflections on artificial intelligence and robots in postdigital k-12 education. *Postdigital Science and Education*, 1, 427–445. <https://doi.org/10.1007/s42438-019-00046-x>
- Huang, Y., Cheng, Y., Chen, L., & Hsu, J. Y. (2019). Human-AI Co-learning for data-driven AI. *arXiv preprint arXiv:1910.12544*. <http://arxiv.org/abs/1910.12544>
- Järvelä, S., Gašević, D., Seppänen, T., Pechenizkiy, M., & Kirschner, P. A. (2020). Bridging learning sciences, machine learning and affective computing for understanding cognition and affect in collaborative learning. *British Journal of Educational Technology*, 51(6), 2391–2406. <https://doi.org/10.1111/bjet.12917>
- Järvelä, S., MalMBERG, J., & Järvelä, H. (2022). Generation Z and beyond: Co-evolution of human capabilities and intelligent technologies in the twenty-first century. In: Lee, W.O., Brown, P., Goodwin, A.L., Green, A.(eds) *International handbook on education development in Asia-Pacific* (pp. 1–13). Springer. [https://doi.org/10.1007/978-981-19-6887-7\\_115](https://doi.org/10.1007/978-981-19-6887-7_115)
- Järvelä, S., Nguyen, A., & Hadwin, A. (2023). Human and artificial intelligence collaboration for socially shared regulation in learning. *British Journal of Educational Technology*, 54, 1057–1076. <https://doi.org/10.1111/bjet.13325>
- Kamar, E. (2016). Directions in hybrid intelligence: Complementing AI systems with human intelligence. In *IJCAI'16: Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence* (pp. 4070–4073), New York. <https://api.semanticscholar.org/CorpusID:2631842>
- Kim, J. (2023). Leading teachers' perspective on teacher-AI collaboration in education. *Education and Information Technologies*, 29, 8693–8724. <https://doi.org/10.1007/s10639-023-12109-5>
- Kim, S., Jang, Y., Choi, S., Kim, W., Jung, H., Kim, S., & Kim, H. (2021). Analyzing teacher competency with TPACK for K-12 AI education. *KI-Künstliche Intelligenz*, 35(2), 139–151. <https://doi.org/10.1007/s13218-021-00731-9>
- Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29–63. <https://doi.org/10.3102/00028312027001029>
- Leiner, D., Fahr, A., & Früh, H. (2012). Eda positive change: A simple algorithm for electrodermal activity to measure general audience arousal during media exposure. *Communication Methods and Measures*, 6(4), 237–250. <https://doi.org/10.1080/19312458.2012.732627>
- Luckin, R., & Cukurova, M. (2019). Designing educational technologies in the age of AI: A learning sciences-driven approach. *British Journal of Educational Technology*, 50(6), 2824–2838. <https://doi.org/10.1111/bjet.12861>
- Luckin, R., George, K., & Cukurova, M. (2022). *AI for school teachers*. CRC Press. <https://doi.org/10.1201/9781003193173>
- Martinez-Maldonado, R., Gašević, D., Echeverria, V., Fernandez Nieto, G., Swiecki, Z., & Buckingham Shum, S. (2021). What do you mean by collaboration analytics? A conceptual model. *Journal of Learning Analytics*, 8(1), 126–153. <https://doi.org/10.18608/jla.2021.7227>
- Mavroudi, A., Giannakos, M., & Krogstie, J. (2018). Supporting adaptive learning pathways through the use of learning analytics: Developments, challenges and future opportunities. *Interactive Learning Environments*, 26(2), 206–220. <https://doi.org/10.1080/10494820.2017.1292531>
- Mayring, P., et al. (2004). Qualitative Content Analysis. *Forum Qualitative Sozialforschung / Forum: Qualitative Social Research* <https://doi.org/10.17169/fqs-1.2.1089>
- McCalla, G. (2023). The history of artificial intelligence in education—the first quarter century. In *Handbook of Artificial Intelligence in Education* (pp. 10–29). Edward Elgar Publishing. <https://doi.org/10.4337/9781800375413.00010>
- Mirjafari, S., Masaba, K., Grover, T., Wang, W., Audia, P., Campbell, A. T., Chawla, N. V., Swain, V. D., Choudhury, M. D., Dey, A. K., D'Mello, S. K., Gao, G., Gregg, J. M., Jagannath, K., Jiang, K., Lin, S., Liu, Q., Mark, G., Martinez, G. J., ... Striegel, A. (2019). Differentiating higher and lower job performers in the workplace using mobile sensing. *Proceedings of the ACM on Interactive, Mobile, Wearable and Ubiquitous Technologies*, 3(2), 1–24. <https://doi.org/10.1145/3328908>
- Nazaretsky, T., Mikeska, J. N., & Beigman Klebanov, B. (2023). Empowering teacher learning with AI: Automated evaluation of teacher attention to student ideas during argumentation-focused discussion. In *LAK23: 13th International Learning Analytics and Knowledge Conference* (pp. 122–132), Arlington. <https://doi.org/10.1145/3576050.3576067>

- Nurnberger-Haag, J. (2018). Take it away or walk the other way? Finding positive solutions for integer subtraction. In: Bofferding, L., Wessman-Enzinger, N. (eds) *Exploring the integer addition and subtraction landscape: Perspectives on integer thinking* (pp. 109–141). [https://doi.org/10.1007/978-3-319-90692-8\\_5](https://doi.org/10.1007/978-3-319-90692-8_5)
- Patton, M. Q. (1990). *Qualitative evaluation and research methods*. Sage Publications, Inc. <https://doi.org/10.1002/nur.4770140111>
- Roschelle, J., Lester, J., & Fusco, J. (2020). *AI and the future of learning: Expert panel report*. Digital Promise. <https://doi.org/10.51388/20.500.12265/106>
- Saxe, G. B., Diakow, R., & Gearhart, M. (2013). Towards curricular coherence in integers and fractions: A study of the efficacy of a lesson sequence that uses the number line as the principal representational context. *ZDM*, 45, 343–364. <https://doi.org/10.1007/s11858-012-0466-2>
- Schoonderwoerd, T. A., Van Zoelen, E. M., van den Bosch, K., & Neerinx, M. A. (2022). Design patterns for human-AI co-learning: A wizard-of-oz evaluation in an urban-search-and-rescue task. *International Journal of Human-Computer Studies*, 164, 102831. <https://doi.org/10.1016/j.ijhcs.2022.102831>
- Sharma, K., Papamitsiou, Z., & Giannakos, M. (2019). Building pipelines for educational data using ai and multimodal analytics: A “grey-box” approach. *British Journal of Educational Technology*, 50(6), 3004–3031. <https://doi.org/10.1111/bjet.12854>
- Sharma, K., Papamitsiou, Z., Olsen, J. K., & Giannakos, M. (2020). Predicting learners' effortful behaviour in adaptive assessment using multimodal data. In *Proceedings of the Tenth International Conference on Learning Analytics & Knowledge* (pp. 480–489), Frankfurt. <https://doi.org/10.1145/3375462.3375498>
- Shute, V., Rahimi, S., Smith, G., Ke, F., Almond, R., Dai, C.-P., & Sun, C. (2021). Maximizing learning without sacrificing the fun: Stealth assessment, adaptivity and learning supports in educational games. *Journal of Computer Assisted Learning*, 37(1), 127–141. <https://doi.org/10.1111/jcal.12473>
- Slamtec. (2024). *Lidar sensors and software*. <https://www.slamtec.ai/>
- Soller, A., Martinez, A., Jermann, P., & Muehlenbrock, M. (2005). From mirroring to guiding: A review of state of the art technology for supporting collaborative learning. *International Journal of Artificial Intelligence in Education*, 15(4), 261–290. <https://dblp.org/rec/journals/aiedu/SollerMJM05>
- Staempfli, M. B. (2007). Adolescent playfulness, stress perception, coping and well being. *Journal of Leisure Research*, 39(3), 393–412. <https://doi.org/10.1080/00222216.2007.11950114>
- Taelman, J., Vandeput, S., Spaepen, A., & Huffel, S. V. (2009). Influence of mental stress on heart rate and heart rate variability. In *4th European Conference of the International Federation for Medical and Biological Engineering* (pp. 1366–1369), Antwerp. [https://doi.org/10.1007/978-3-540-89208-3\\_324](https://doi.org/10.1007/978-3-540-89208-3_324)
- Tancredi, S., Chen, R. S., Krause, C., & Siu, Y.-T. (2022). The need for SpEED: reimagining accessibility through special education embodied design. In Sheila L. Macrine, Jennifer M.B. Fugate *Movement Matters: How Embodied Cognition Informs Teaching and Learning*. <https://doi.org/10.7551/mitpress/13593.003.0021>
- Thompson, P. W., & Dreyfus, T. (1988). Integers as transformations. *Journal for Research in Mathematics Education*, 19(2), 115–133. <https://doi.org/10.5951/jresmetheduc.19.2.0115>
- Van Duijvenvoorde, A. C., Zanolie, K., Rombouts, S. A., Raijmakers, M. E., & Crone, E. A. (2008). Evaluating the negative or valuing the positive? Neural mechanisms supporting feedback-based learning across development. *Journal of Neuroscience*, 28(38), 9495–9503. <https://doi.org/10.1523/JNEUROSCI.1485-08.2008>
- van Leeuwen, A., Knoop-van Campen, C. A. N., Molenaar, I., & Rummel, N. (2021). How teacher characteristics relate to how teachers use dashboards: Results from two case studies in k-12. *Journal of Learning Analytics*, 8(2), 6–21. <https://doi.org/10.18608/jla.2021.7325>
- Varma, S., & Schwartz, D. L. (2011). The mental representation of integers: An abstract-to-concrete shift in the understanding of mathematical concepts. *Cognition*, 121(3), 363–385. <https://doi.org/10.1016/j.cognition.2011.08.005>
- Vatsalan, D., Rakotoarivelo, T., Bhaskar, R., Tyler, P., & Ladjal, D. (2022). Privacy risk quantification in education data using Markov model. *British Journal of Educational Technology*, 53(4), 804–821. <https://doi.org/10.1111/bjet.13223>
- Verbeek, P.-P. (2015). Cover story beyond interaction: A short introduction to mediation theory. *Interactions*, 22(3), 26–31. <https://doi.org/10.1145/2751314>
- Vesin, B., Mangaroska, K., & Giannakos, M. (2018). Learning in smart environments: User-centered design and analytics of an adaptive learning system. *Smart Learning Environments*, 5(1), 1–21. <https://doi.org/10.1186/s40561-018-0071-0>
- Weeks, J. W., & Howell, A. N. (2012). The bivalent fear of evaluation model of social anxiety: Further integrating findings on fears of positive and negative evaluation. *Cognitive Behaviour Therapy*, 41(2), 83–95. <https://doi.org/10.1080/16506073.2012.661452>
- Wilensky, U. (1997). What is normal anyway? Therapy for epistemological anxiety. *Educational Studies in Mathematics*, 33(2), 171–202. <https://doi.org/10.1023/A:1002935313957>

## SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

**How to cite this article:** Cosentino, G., Anton, J., Sharma, K., Gelsomini, M., Giannakos, M., & Abrahamson, D. (2025). Hybrid teaching intelligence: Lessons learned from an embodied mathematics learning experience. *British Journal of Educational Technology*, 56, 621–649. <https://doi.org/10.1111/bjet.13525>

## APPENDIX A

**TABLE A1** Pairwise post hoc comparisons for remaining in low engagement, high stress and low fatigue.

Sub-code 1	Sub-code 2	F-value	Df2 (corrected)	p-Value
1.ICM	1.PCM	205.90	53.49	<0.00001
1.ICM	1.PCN	68.87	16.29	<0.00001
1.ICM	2.ICM	0.40	81.95	0.52
1.ICM	2.PCM	252.28	70.14	<0.00001
1.ICM	2.PCN	31.55	6.56	0.001
1.PCM	1.PCN	0.02	22.72	0.87
1.PCM	2.ICM	217.69	52.95	<0.00001
1.PCM	2.PCM	0.46	77.36	0.49
1.PCM	2.PCN	0.73	8.17	0.41
1.PCN	2.ICM	72.98	16.31	<0.00001
1.PCN	2.PCM	0.37	21.88	0.54
1.PCN	2.PCN	0.39	13.03	0.53
2.ICM	2.PCM	265.86	68.91	<0.00001
2.ICM	2.PCN	33.71	6.56	0.0008
2.PCM	2.PCN	1.49	7.93	0.25

**TABLE A2** Pairwise post hoc comparisons for moving from low engagement, high stress and low fatigue to low engagement, high stress and high fatigue—indicating increment in fatigue.

Sub-code 1	Sub-code 2	F-value	Df2 (corrected)	p-Value
1.ICM	1.PCM	307.60	77.64	<0.00001
1.ICM	1.PCN	0.42	23.01	0.51
1.ICM	2.ICM	263.67	68.03	<0.00001
1.ICM	2.PCM	296.93	90.26	<0.00001
1.ICM	2.PCN	1.27	8.18	0.29
1.PCM	1.PCN	132.16	24.16	<0.00001
1.PCM	2.ICM	1363.66	50.71	<0.00001
1.PCM	2.PCM	3.18	73.48	0.07
1.PCM	2.PCN	73.83	8.52	<0.00001
1.PCN	2.ICM	107.11	16.31	<0.00001
1.PCN	2.PCM	114.99	21.28	<0.00001
1.PCN	2.PCN	0.27	12.97	0.60
2.ICM	2.PCM	1587.29	70.34	<0.00001
2.ICM	2.PCN	73.66	6.56	<0.00001
2.PCM	2.PCN	61.43	7.76	<0.00001

**TABLE A3** Pairwise post hoc comparisons for remaining in low engagement, high stress and high fatigue.

Sub-code 1	Sub-code 2	F-value	Df2 (corrected)	p-Value
1.ICM	1.PCM	186.22	75.43	<0.00001
1.ICM	1.PCN	0.37	24.72	0.54
1.ICM	2.ICM	0.72	81.47	0.39
1.ICM	2.PCM	0.38	90.83	0.53
1.ICM	2.PCN	0.93	8.54	0.36
1.PCM	1.PCN	116.40	27.59	<0.00001
1.PCM	2.ICM	161.00	70.72	<0.00001
1.PCM	2.PCM	211.25	72.87	<0.00001
1.PCM	2.PCN	52.57	9.36	<0.00001
1.PCN	2.ICM	1.51	25.10	0.23
1.PCN	2.PCM	0.03	23.58	0.86
1.PCN	2.PCN	1.62	12.81	0.22
2.ICM	2.PCM	2.21	79.45	0.14
2.ICM	2.PCN	0.21	8.68	0.65
2.PCM	2.PCN	1.75	8.27	0.22



**TABLE A4** Pairwise post hoc comparisons for moving from high engagement, low stress and low fatigue to high engagement, low stress and high fatigue—indicating increment in fatigue.

Sub-code 1	Sub-code 2	F-value	Df2 (corrected)	p-Value
1.ICM	1.PCM	0.57	80.41	0.45
1.ICM	1.PCN	171.37	49.39	<0.00001
1.ICM	2.ICM	311.62	81.62	<0.00001
1.ICM	2.PCM	0.02	90.96	0.86
1.ICM	2.PCN	1.48	8.20	0.25
1.PCM	1.PCN	160.30	42.98	<0.00001
1.PCM	2.ICM	354.94	71.85	<0.00001
1.PCM	2.PCM	0.85	79.67	0.35
1.PCM	2.PCN	2.66	7.96	0.14
1.PCN	2.ICM	1009.18	46.14	<0.00001
1.PCN	2.PCM	174.11	49.43	<0.00001
1.PCN	2.PCN	63.91	7.59	<0.00001
2.ICM	2.PCM	302.77	81.04	<0.00001
2.ICM	2.PCN	70.60	8.29	<0.00001
2.PCM	2.PCN	1.27	8.24	0.29

**TABLE A5** Pairwise post hoc comparisons for moving from high engagement, low stress and high fatigue to high engagement, low stress and low fatigue—indicating decrement in fatigue.

Sub-code 1	Sub-code 2	F-value	Df2 (corrected)	p-Value
1.ICM	1.PCM	264.70	73.66	<0.00001
1.ICM	1.PCN	126.95	20.77	<0.00001
1.ICM	2.ICM	267.50	81.80	<0.00001
1.ICM	2.PCM	246.27	90.48	<0.00001
1.ICM	2.PCN	87.53	8.69	<0.00001
1.PCM	1.PCN	0.11	23.59	0.74
1.PCM	2.ICM	963.90	69.40	<0.00001
1.PCM	2.PCM	2.27	75.61	0.13
1.PCM	2.PCN	1.21	9.92	0.29
1.PCN	2.ICM	451.12	20.85	<0.00001
1.PCN	2.PCM	1.88	21.73	0.18
1.PCN	2.PCN	1.37	16.14	0.25
2.ICM	2.PCM	988.17	81.83	<0.00001
2.ICM	2.PCN	359.07	8.74	<0.00001
2.PCM	2.PCN	0.01	9.08	0.89

**TABLE A6** Pairwise post hoc comparisons for remaining in high engagement, low stress and high fatigue.

Sub-code 1	Sub-code 2	F-value	Df2 (corrected)	p-Value
1.ICM	1.PCM	247.25	75.87	<0.00001
1.ICM	1.PCN	801.80	28.98	<0.00001
1.ICM	2.ICM	1.13	78.56	0.28
1.ICM	2.PCM	0.22	89.63	0.63
1.ICM	2.PCN	271.43	8.23	<0.00001
1.PCM	1.PCN	198.13	32.01	<0.00001
1.PCM	2.ICM	249.89	71.96	<0.00001
1.PCM	2.PCM	209.46	78.55	<0.00001
1.PCM	2.PCN	51.92	8.87	<0.00001
1.PCN	2.ICM	779.91	32.45	<0.00001
1.PCN	2.PCM	719.13	32.65	<0.00001
1.PCN	2.PCN	5.43	10.43	0.04
2.ICM	2.PCM	2.11	80.94	0.14
2.ICM	2.PCN	280.15	8.92	<0.00001
2.PCM	2.PCN	253.08	8.83	<0.00001

**TABLE A7** Pairwise post hoc comparisons for remaining in high engagement, high stress and low fatigue.

Sub-code 1	Sub-code 2	F-value	Df2 (corrected)	p-Value
1.ICM	1.PCM	270.13	76.57	<0.00001
1.ICM	1.PCN	475.31	25.25	<0.00001
1.ICM	2.ICM	236.62	82.98	<0.00001
1.ICM	2.PCM	259.08	90.87	<0.00001
1.ICM	2.PCN	263.25	8.48	<0.00001
1.PCM	1.PCN	76.98	27.38	<0.00001
1.PCM	2.ICM	6.88	68.53	0.01
1.PCM	2.PCM	2.46	74.31	0.12
1.PCM	2.PCN	39.30	9.07	0.0001
1.PCN	2.ICM	126.97	23.04	<0.00001
1.PCN	2.PCM	107.41	24.17	<0.00001
1.PCN	2.PCN	0.27	12.38	0.61
2.ICM	2.PCM	1.23	81.80	0.26
2.ICM	2.PCN	64.09	8.01	<0.00001
2.PCM	2.PCN	54.15	8.23	<0.00001

TABLE A8 Modelled Transition Probabilities (converted to percentages) for the different sub-codes.

Transition	1.ICM	1.PCM	1.PCN	2.ICM	2.PCM	2.PCN
t00	7.55	7.81	7.54	7.42	7.63	7.84
t02	7.46	7.46	7.50	7.43	7.51	7.16
t03	7.50	7.06	8.16	7.77	7.84	7.26
t04	7.63	7.10	7.33	7.04	7.78	7.80
t05	7.11	7.81	6.91	7.35	7.63	8.56
t06	7.55	7.86	7.61	7.81	7.61	7.57
t07	7.32	7.36	7.37	7.43	7.23	6.75
t20	7.14	7.24	7.50	7.72	7.54	7.17
t22	7.34	14.93	14.77	7.12	15.37	13.87
t23	15.28	25.97	15.85	7.68	24.93	16.51
t24	7.87	7.85	7.34	7.50	8.02	6.82
t25	7.55	7.19	7.51	7.71	7.58	7.31
t26	7.54	7.67	8.17	7.14	7.50	7.39
t27	7.79	7.49	7.75	8.02	7.41	7.29
t30	7.82	7.34	7.90	7.48	7.58	6.78
t32	7.66	7.66	7.21	7.29	7.36	8.02
t33	15.38	24.76	14.84	15.93	14.99	16.46
t34	7.25	7.77	7.27	7.56	7.42	8.27
t35	7.64	7.32	6.90	7.59	7.65	7.54
t36	7.48	7.34	7.38	7.69	7.60	8.09
t37	7.42	7.73	6.80	7.61	7.58	7.41
t40	7.72	7.49	7.92	7.92	7.52	7.17
t42	7.39	7.40	7.91	7.03	7.38	7.48
t43	7.39	7.23	7.58	7.53	7.57	7.82
t44	7.81	7.42	7.50	7.57	7.53	6.47
t45	24.81	14.36	7.41	25.58	14.91	16.17
t46	7.68	7.97	7.30	7.47	7.17	8.24
t47	7.45	7.61	7.77	7.32	7.47	8.44
t50	7.80	7.39	7.72	7.75	7.63	7.38
t52	7.41	7.37	7.98	7.07	7.53	6.99
t53	7.57	7.36	7.87	7.29	7.67	7.86
t54	12.84	7.41	7.24	17.81	7.93	8.00
t55	17.41	12.46	7.12	17.74	17.26	8.48
t56	7.56	7.40	7.58	7.10	7.48	8.23
t57	7.23	7.29	7.31	7.11	7.29	6.47
t60	7.66	7.35	7.44	7.92	7.38	7.14
t62	7.22	7.90	8.00	7.27	7.28	7.31
t63	7.83	7.59	7.40	7.60	7.48	7.61
t64	7.63	7.59	7.65	7.77	7.54	7.30
t65	7.60	7.26	7.40	7.57	7.37	8.12
t66	17.55	11.94	7.97	12.80	12.47	8.30

TABLE A8 (Continued)

Transition	1.ICM	1.PCM	1.PCN	2.ICM	2.PCM	2.PCN
t67	7.44	7.31	7.61	7.32	7.38	6.73
t70	7.53	7.43	7.62	7.74	7.66	8.21
t72	7.46	7.12	7.32	7.63	7.51	6.33
t73	7.18	7.29	7.01	7.18	7.45	7.47
t74	7.57	7.64	7.27	7.50	7.59	7.73
t75	7.72	7.43	7.86	7.34	7.60	7.57
t76	7.10	7.46	7.50	7.74	7.38	7.95
t77	7.41	7.67	8.34	7.59	7.95	8.14

Note: The probabilities are not normalized, and therefore, they do not add up to one. The coloured ones match the ones in Figure 7.

TABLE A9 ANOVA, with corrections for different cardinalities, for the transition probabilities across different sub-codes.

Transition	Df1	Df2 (corrected)	F-value	p-Value (corrected)
t00	5	41.92	0.34	0.88
t02	5	42.34	0.07	0.99
t03	5	42.5	1.97	0.11
t04	5	48.7	2.14	0.08
t05	5	42.49	2.2	0.07
t06	5	41.89	0.27	0.92
t07	5	41.9	0.29	0.91
t20	5	41.96	0.81	0.54
<b>t22</b>	<b>5</b>	<b>40.47</b>	<b>99.84</b>	<b>&lt;0.00001</b>
<b>t23</b>	<b>5</b>	<b>40.96</b>	<b>458.48</b>	<b>&lt;0.00001</b>
t24	5	41.98	1.45	0.22
t25	5	41.51	0.58	0.71
t26	5	42.56	1.14	0.35
t27	5	41.84	0.99	0.43
t30	5	42.34	0.96	0.45
t32	5	42.12	0.75	0.58
<b>t33</b>	<b>5</b>	<b>42.62</b>	<b>51.43</b>	<b>&lt;0.00001</b>
t34	5	42.92	1.2	0.32
t35	5	42.93	0.94	0.45
t36	5	41.29	0.42	0.82
t37	5	41.61	0.78	0.56
t40	5	42.64	0.68	0.63
t42	5	41.77	0.9	0.48
t43	5	43.38	0.39	0.85
t44	5	42.6	1.62	0.17
<b>t45</b>	<b>5</b>	<b>44.14</b>	<b>191</b>	<b>&lt;0.00001</b>

TABLE A9 (Continued)

Transition	Df1	Df2 (corrected)	F-value	p-Value (corrected)
t46	5	42.23	1.87	0.11
t47	5	45.89	1.83	0.12
t50	5	43.16	0.47	0.79
t52	5	42.35	1.04	0.4
t53	5	41.91	0.58	0.71
<b>t54</b>	<b>5</b>	<b>42.59</b>	<b>289.11</b>	<b>&lt;0.00001</b>
<b>t55</b>	<b>5</b>	<b>43.15</b>	<b>253.57</b>	<b>&lt;0.00001</b>
t56	5	43.7	1.1	0.37
t57	5	42.99	0.62	0.68
t60	5	43.15	1.01	0.42
t62	5	42.07	1.65	0.16
t63	5	42.05	0.32	0.89
t64	5	42.3	0.15	0.97
t65	5	42.16	0.6	0.7
<b>t66</b>	<b>5</b>	<b>42.46</b>	<b>128.17</b>	<b>&lt;0.00001</b>
t67	5	41.84	0.29	0.91
t70	5	42.66	0.5	0.77
t72	5	43.34	1.99	0.09
t73	5	41.93	0.32	0.89
t74	5	41.81	0.17	0.97
t75	5	42.04	0.43	0.82
t76	5	43.24	1.15	0.34
t77	5	43.27	1.65	0.16

Note: The bold rows have the statistical significance, and the connected post hoc tests are presented in Tables A1–A7.