A lukewarm defence of the mathematics lecture

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The lecture is an often-criticised format of teaching that nevertheless prevails in university mathematics classrooms. This paper offers empirically based perspectives on the qualities of a lecture in advanced linear algebra. Using ethnographic thick description based on third person video recordings, the aim is to examine the analytical relevance of psychoanalytic theories when making sense of lectures. I find that this interpretative lens forefronts how the teacher creates a learning environment that affords durational pleasure, thus highlighting affective values of the lecture.

Keywords: Lecture, university mathematics education, embodiment, resonance, erotics.

Introduction

In the beginning of the semester where I conducted my observations, the vice dean sent out a letter addressed to all teaching staff at the Faculty of Science. He wrote:

> The three-phase model at the Faculty of Science has had its time […] We cannot separate well-being and teaching […] Lectures may well be an element that we continue to use, but as a form of teaching, lectures have their natural limitations. (Nielsen, 2024)

The three-phase model, i.e. lectures, homework and tutorials, has been standard in the mathematics department for decades. In the letter, the vice dean seems to distinguish between forward-looking teaching that promotes well-being of the students, and lectures that are implied to be outdated. This plea for student-centred, active learning approaches is consistent with literature on university pedagogy and general didactics, where the legitimacy of lectures is called into question (Jones, 2007). For example, lectures are criticised for their reliance on transmitting knowledge to students who are rendered passive. Regardless, the lecture prevails in mathematics classrooms at universities (Viirman, 2021, pp. 467–468).

Literature on the lecture is of course not monolithic. Based on interviews on first-year mathematics students' reflections on why lectures matter, Melissa Rodd (2003) forefronts the 'spiritual purpose' of the lecture, in that the lecture may inspire wonder, develop identity, and stimulate imagination. She rejects the 'transmission' criticism as a false premise and highlights instead how lectures direct desire for learning. As such, she problematises the juxtaposition of valid participation and social interaction. Rodd asks the question: "if there were no mathematics lecture, what would be lost?" (Rodd, 2003, p. 15), pointing to how the qualities of lectures are difficult to replicate in other communication or teaching formats. Her sentiment is echoed by Steve Jones: "This live experience cannot be replicated through texts or recorded media; the immediacy, the corporality and the subtle dialogue between students and lecturer are irreproducible and ephemeral" (Jones, 2007, p. 402). This calls for qualitative research on lectures that makes sense of them in more ways than their form. As David Pritchard points out in reference to the debate on the legitimacy of lectures, "different disciplines must be treated differently" (Pritchard, 2010, p. 609). The lecture is a diverse practice, not least across disciplines, and according to Pritchard, some of the criticism of lectures appears

superficial when applied to mathematics. He outlines three functions of the mathematics lecture: communicating, modelling and motivating. This paper follows the path of motivation, although motivation is not the affective term, I will be using. Olov Viirman, who has followed the path of modelling in his work on the function of lectures, inquires after arguments about the value of lectures that are based on empirical investigation:

> [A]ny discussion of the role of the lecture in future mathematics teaching at universities, whether among researchers or practicing lecturers, needs to be informed by research on actual lecturing practice. (Viirman, 2021, p. 487)

Within this choir of lecture-apologists, this paper finds its voice. I contribute with empirical research on the qualities of a university mathematics lecture, making sense of the lecturing practice through a theoretically eclectic interpretative lens of embodiment, resonance, and erotics. By trying out these theories, I contribute with ways of understanding university mathematics teaching practice that forefronts alternative perspectives on it. These perspectives might benefit both the university mathematics education research community as well as the local context of my study.

Methods

The lecture analysed in this paper is part of the data for my PhD project. I work ethnographically (Van Maanen, 2011) in the learning environment of the institution where I studied and am currently employed. This makes my study akin to an autoethnography, which requires both intellectual and political considerations about taking 'native' viewpoints for granted and balancing organisational loyalty (Alvesson, 2003, pp. 167–174). As part of my ethnographic work, I have videorecorded 25 lectures given by four different teachers in courses on calculus and linear algebra for first-year students in pure and applied mathematics. The lectures were recorded with a video camera from the back of the class to my side and a microphone placed on the front row pointed at the teacher. The video and audio files have been synchronised and transcribed. While observing the lectures, I took extrospective and introspective notes to support my later writing.

The lecture is the thirteenth and second-to-last lecture in the course *Advanced Linear Algebra* that took place in the spring of 2024. It was transcribed using four tracks of information: The first records timestamps, the second categorises the situation, the third transcribes the speech, and the fourth holds stills from the video recordings, showing the blackboard, the slideshow, or the teacher. A timestamp was made every time the teaching element shifted in a meaningful way. For example, categories are: Going through a proof, giving examples on the board, answering questions from students, and so on.

The transcription serves a heuristic purpose, since I have worked mostly inductively without predefined categories or codes adhering to a specific theory. Instead, in general terms, my empirical approach is exploratory and descriptive, with a mix of hermeneutic, phenomenological and pragmatic viewpoints: Hermeneutic because I interpret the lecture while trying to be conscious about the context of my observations as well as my own prejudice. Phenomenological because I try to understand the social phenomenon of mathematics teaching from a human perspective. And lastly, pragmatic because I work from the assumption that the methods and theoretical notions I employ, produce the reality I describe (cf. Jackson & Mazzei, 2017). My ideal is to first wonder about the data and then

bring in appropriate theories and their vocabularies. In this way, describing a situation is part of analysing and explaining it (Geertz, 1973).

To make the choice of theories transparent, I interweave the sections on theory and results. In doing so, I write in the style of 'confessional tales' and 'impressionist tales' (Van Maanen, 2011), where one's own lived experiences become part of the object of research. I must emphasise that I do not use my own situatedness as a substitute for the student point of view, but for illuminating the research process as an interpretative act. The lecture that I will analyse was in fact a turning point for the theoretical framework I employ, so the interwoven structure reflects the temporal order of events.

Theoretical outset

My ideal may be to wonder first, but I did not set out theoretically emptyheaded when I began my observations. Having just been a student myself, my initial interest was student-centred, and I directed my attention to the learning environment as a site for experiences where students actively construct knowledge. I drew on the theory of embodied cognition (Lakoff & Núñez, 2000), which remains relatively modest in research on the tertiary mathematics education compared to lower levels, not least in combination with design-based methods (Abrahamson et al., 2020).

The theory of embodied cognition holds that human thought is inseparable from our bodies, and that abstractions are metaphorical and based on primary, bodily experiences (Lakoff & Johnson, 1980). This has several implications for mathematics education, some of them being that learning is situated in multimodal experiences (Shapiro & Stolz, 2019), and that a focus on gesture and other extra-linguistic modalities is beneficial for interpreting students' understanding of mathematics (Edwards et al., 2014). With these perspectives in mind, I went into the empirical phase of my research.

The case of the Jordan Decomposition Theorem

As stated, a turning point in my theoretical framework and research questions happened near the end of my empirical work. In the thirteenth lecture on *Advanced Linear Algebra*, the teacher ('Liam') proved the Jordan Decomposition Theorem, a result so major that it was basically the climax of the course. Liam put it like this:

> Liam: All right. Så lad os se på beviset, for vi har faktisk bygget op til det her bevis over de sidste tre, fire forelæsninger. Faktisk igennem hele kurset.
> *All right. So let us look at the proof, because we have actually built up to this proof for the last three, four lectures. Actually, through the entire course.*

I went into this lecture with a brewing sense of ambivalence toward lecturing as a practice. In last week's lecture on nilpotent matrixes, a streak of eight questions from students ended when Liam, in the interest of time, requested the students hold their questions so that they could move on to an example. I was quick to judge this hushing and disengaging in dialogue as bad pedagogical practice, but at the same time, I noticed in myself an excitement that he would keep on lecturing. I also noticed that the students, unsurprisingly, seemed more hesitant about asking questions, and although the lectures had certainly not turned into a completely monologic practice, elements like proofs seemed to be surrendered to Liam's initiative. Now I sat at the back of the class, ever ready to jot down notes on Liam's metaphors, body language, and visualizations, as the thirteenth lecture began.

About fifteen minutes of prefatory ideas, or what Liam called the 'baby version' of the decomposition, led up to the proof of the theorem. The first minute of the lecture was spent by students requesting two recap lectures, of which he agreed to do one before the written exam. Three minutes went with introductory motivations for the theorem. Liam then spent six minutes presenting both the general and the complex case of the decomposition, showing an illustration of the decomposed matrix:

Liam: Denne her blok nederst til højre […] den her matrix har den egenskab, at den ikke har nogen egenværdier. Man kan tænke på det som at vi har alle de her pæne blokke her, her og her, der går ned, og så har vi en matrix, som består af … skrald. Det her, det er en ubehagelig matrix, som ikke har nogen egenværdier. Det kan vi generelt ikke lide. Det er ikke noget, vi er særligt glade for, så det er sådan, det er der, vi prøver sådan at putte alt vores skrald ned i det her højre hjørne.
This block at the bottom to the right […] this matrix has the property that it has no eigenvalues. You can think of it like we have all these nice blocks here, here, and here, going down, and then we have a matrix consisting of…trash. This one, it is an unpleasant matrix that has no eigenvalues. That is something we generally do not like. It is not something we are particularly pleased with, so it is like, it is where we try to put all our trash, down into this right corner.

After this illustration, two and a half minutes went with students' questions. The next minute, Liam presented the theorem in general, non-matrix terms, i.e. the 'actual' Jordan Decomposition Theorem, after which he declared the proof to begin, as quoted at the beginning of this section. He spent five minutes proving the first lemma needed, and then spent one and a half minutes outlining the idea of the proof of the second lemma needed. Finally, seven minutes of uninterrupted proving followed.

I observed the multimodal aspects of the proof: Liam faced the class at a standing desk with his tablet on it, as the slides appeared on a canvas to his left. The proof manifested one line at a time, and from his desk, Liam moved the cursor as a pointer on the slides while talking. Once, after passing his gaze over the students, he turned to the blackboard to elaborate on a detail.

A recurrent metaphor throughout the proof was 'a matrix without eigenvalues is trash,' which Liam introduced in the quote above. He spat out the word 'trash' with a disgusted facial expression every time he said it, while he spoke of the field of complex numbers with corresponding fondness, since the decomposed matrix does not have a trash corner over this field. Most of the proof was about validating that the lowest right corner did indeed consist of trash over a non-complex field. Apart from laughing at Liam's apparent abhorrence of matrixes with no eigenvalues, the students had turned into mute non-data, until Liam concluded the proof with the remark:

Liam: Så nu har I altså set et af de absolut vigtigste resultater inden for lineær algebra: Jordan-dekompositionen. Ja?
So, now you have seen one of the absolute most important results in linear algebra: The Jordan decomposition. Yes?
Student: Du sagde, nulvektoren kunne aldrig være, hvad?
You said the zero vector could never be what?
Liam: En egenvektor.
An eigenvector.
Student: Men er det vektoren, der består af nuller, eller er det den generelle?
But is that the vector consisting of zeroes or is it the general one?
Liam: Det er nulvektoren for vores vektorrum. Nulvektoren for vores vektorrum er aldrig en egenvektor per definition.

It is the zero vector for our vector space. The zero vector for our vector space is never an eigenvector per definition.
[Six seconds of silence]

Liam: Så der har I altså Jordan-dekompositionen, som fortæller at alle matricer kan skrives som denne her blokdiagonale udgave, som jeg nævnte før. Eller som jeg nævnte i starten af forelæsningen.
So, there you have the Jordan decomposition that tells us that all matrices can be written as this block diagonal version, as I mentioned before. Or as I mentioned at the beginning of the lecture.

Later, I sat at my desk to write up the episode, struggling to describe the nuances of the proof with the vocabulary of embodied cognition. I found that I was missing perspectives on time and affect, and so I realized that embodied cognition was necessary but not sufficient for making sense of the lecturing practice I had observed.

Toward a networked framework of mathematical pleasure

I therefore present a tentative networked theoretical framework to forefront alternative perspectives on the case of the Jordan Decomposition Theorem (cf. Bikner-Ahsbahs & Prediger, 2009). To make sense of my observations, I want to take my framework in the direction of psychoanalysis. The versions of psychoanalysis I find particularly relevant for university mathematics education, are the ones that synthesize with critical theory. Here, Herbert Marcuse is an original torchbearer who dealt with erotics and pleasure as societal matters, relevant outside the private sphere.

Marcuse's pleasure principle arises from a critique of Sigmund Freud's belief that gratification is incompatible with progress (Marcuse, 1955). Marcuse disputes that repressing pleasure is necessary in a functioning society. In fact, he builds his Marxian utopia on a freed principle of pleasure, where the resources afforded by the industrial society, if used wisely, create the opportunities to live in accordance with the emancipated human nature, in an erotic relationship to the world.

A contemporary contribution to Marcuse's school of thought is Hartmut Rosa's notion of resonance, which describes a successful way of relating to the world, in that resonance is the counterpart to alienation in the Marxian sense of the word (Rosa, 2019). According to Rosa, a resonant experience will touch you, move you, and even transform you. Resonance requires self-efficacy but is at the same time uncontrollable: You can never guarantee resonance, but some environments might afford resonance more than others. Rosa formulates a critique of the conditions for resonance in our world, which according to him is characterized by acceleration, competition, and a fundamental attitude of manipulation and control of the world. Hence, to Rosa, the experience of being in the world is intimately related to time.

Resonance is a useful phenomenological term for describing learning situations that 'spark' (Rosa & Endres, 2017). Resonance pedagogy deals with the prerequisites for a resonant didactic triad between the subject, teacher, and students: in short, there must be an atmosphere of trust and genuine engagement with the material from both the teacher and the students. Through Marcuse, Rosa also references the erotic relationship to the world as an exemplary attitude in resonance pedagogy, stating that: "Eroticism, to me, is the opposite of trying to manipulate the world" (ibid., p. 35).

Queering the notion of erotics, I finally network KJ Cerankowski's notion of durational pleasures (Cerankowski, 2021). I find their terminology extremely relevant to my observations because it dethrones the orgasm as the pinnacle of pleasure and explores nonsexual pleasures that do not peak by ending. They write: "I want to stake a claim for an erotics of pleasure, of orgasm even, that does not rely on a teleological model of building tension until one arrives at the release" (ibid., p. 136). By removing the end goal, Cerankowski reframes the temporality of pleasure, broadening our understanding of which pleasures may be legitimate and valuable.

Revisiting the case of the Jordan Decomposition Theorem

In the case of the Jordan Decomposition Theorem, it is clear that Liam's lecture does in fact build on a 'teleological model of building tension until one arrives at the release'. In the first quote, he draws attention to this very build-up, which according to him has lasted through the entire course. It is also noticeable that he spends quite a bit of time on prefatory ideas, motivating the need for the theorem, visually illustrating its meaning and laying down all steps necessary before completing the proof in a streamlined manner. However, the release is disrupted when a student immediately asks a clarifying question at the finish line, afterwards prompting Liam to reiterate the 'ta-da', as it were.

Based on the dramaturgy of the lecture, I interpret that Liam intended for the climax to have a certain impact, brought about by half a semester's worth of prelude. The prefatory elements just mentioned can be seen as efforts to create an environment that affords wonder and vicarious feelings of pleasure. Through speech, slides, body language, and chalk, Liam performs multimodal expressions of embodied metaphors that I would argue create the conditions for a resonant didactic triangle. The 'trash corner', for example, is a metaphor whose function is equal parts communicating the idea and evoking a sense of beauty enabled by the 'trash-free' complex matrix, which excites Liam to a contagious degree. The build-up to the proof can then be seen as a way of persuading the students of its relevance, or "to arouse curiosity and a desire to know" (Sinclair & Watson, 2001, p. 40), instead of immediately stating it as a fact. Liam's lecturing reflects an attitude to mathematics not unlike Marcuse's idea of an erotic relationship, where the proof of the Jordan Decomposition Theorem for quite a while gets to be a potential awaiting realisation. Hence, I would argue that Liam's lecture performance is 'awe-inspiring' (Rodd, 2003, p. 17).

The ruining of the climax in turn disrupts who controls the pleasure, a power-smoothing that is made possible by the immediacy of the lecture, where Liam and the students gather in this specific time and place. With Cerankowski's terminology, the pleasure becomes durational in a temporality where the journey is the act itself. The lecture thus operates on two different teleological models at once, building tension and ending 'not with a bang but a whimper'.

Now, a lot of this discussion has hinged on verbal data, and as already mentioned, the students become non-data to me when they listen. However, it would be a fallacy to then conclude that nothing of note happens. For instance, Liam is evidently not just speaking 'at' the students. His gaze before he turns to the blackboard to elaborate on the proof can be interpreted as subtle communication, and while I do not have access to the students' experience, Rodd's (2003) findings suggest that more might go on in a lecture than meets the eye. It is also worthwhile to question whether looking for observable activity as wholesome learning qua activity is the be-all and end-all. As Miya Tokumitsu argues:

> Many critics dismiss lecture attendance as "passive learning," arguing that students in lectures aren't doing anything. Today, declaring something passive completely delegitimizes it […] under capitalism, anything that doesn't immediately appear as productive, even if only in the most superficial way, is dismissed as a waste of time. (Tokumitsu, 2017)

The students in the lecture analysed in this paper are calm and quiet most of the time. Are they learning? Maybe not this moment, here. But later, somewhere else, the desire to know instilled in them by the lecturer carries on. I argue here that learning is mostly unobservable in lectures on a surface level because learning might be displaced in spacetime. A value of the lecture may thus be excitation, understood as "an elevation in energy" (Porsager, 2021, p. 22). Seen through this lens, the apparent passivity of students instead appears as a build-up of desire for knowledge awaiting satisfaction.

Conclusion

Being an analysis of just one lecture out of the 25 I have observed, this paper serves as a pilot study for my doctoral dissertation. It is nonetheless a feature of ethnographic thick description that "small facts speak to large issues" (Geertz, 1973, p. 23). I have demonstrated the analytical relevance of psychoanalytical perspectives when discussing the value of lectures by reading Liam's proof of the Jordan Decomposition Theorem in its erotic dramaturgy, and further speculating about what goes on under the surface. Combining embodiment, resonance, and erotics has enabled me to appreciate the lecture as a learning environment that affords excitation and durational pleasure. Examining whether this perspective is consistent with students' experience is a relevant endeavour for further research.

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